Fuzzy Logic and Neural Networks
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Lecture – 08
Applications to Fuzzy Sets (Contd.)

Now, we have discussed that for a set of inputs, with the help of fuzzy inference engine, we will be able to find out, which are the rules to be fired.

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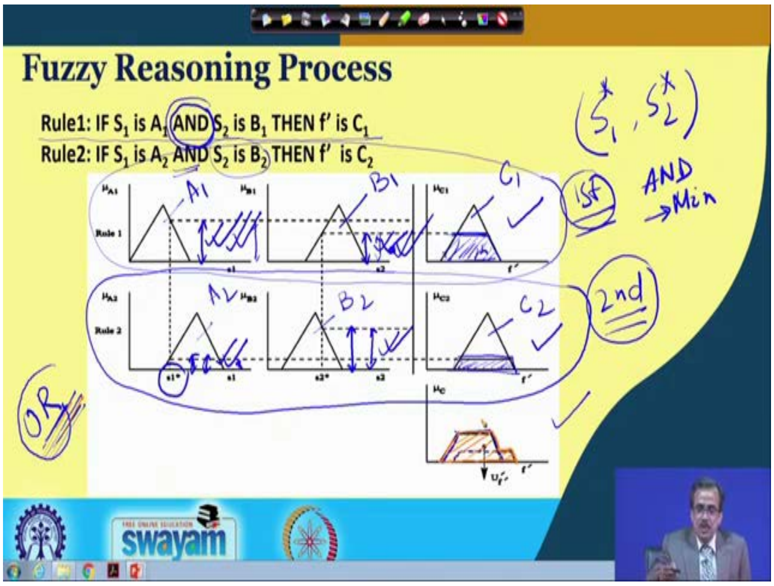

Now supposing that these two rules are going to be fired and here, we have got the membership function distribution for the inputs and outputs. Now, let me read the rules and supposing that these two rules are going to be fired and we will have to find out the output of these two rules.

Now, the first rule is nothing but, if S_1 is A_1 AND S_2 is B_1 then f' is C_1. Now, let us see, how to represent in the figure. So, this indicates actually the first rule if S_1 is A_1. So, this is nothing but the membership function distribution for A_1 and this is nothing but the membership function for your S_2 and S_2 is B_1 then f prime is C_1. So, this is nothing but the membership function distribution for the output.

Now, let me repeat. So, if S_1 is A_1, if S_1 is A_1 AND S_2 is B_1 then f^prime that is the output is nothing but C_1. So, this indicates actually the first fired rule, similarly, the second fired rule if S_1 is A_2. So, this is nothing but the membership function distribution for A_2 and S_2 is your B_2. So, this is nothing but the membership function distribution for B_2 then f' is C_2. So, this is nothing but is your C_2. So, the second rule is actually represented by; so this particular part of the figure.

So, this is the first rule, first fired rule and this is the second fired rule and supposing that I am passing one set of inputs, the inputs are nothing but S_1^* and your S_2^*. So, this is the set of inputs. So, this is nothing but two inputs-one output process, now here, I am just going to pass this S_1^* that is the first input. Now, if I pass this particular the first input, corresponding to the first fired rule; I will be getting one membership function value here, and I will be getting another membership function value here, and corresponding to this particular S_2^*, I will be getting one membership function value here and I will be getting another membership function value here. Now we concentrate on this particular the first rule the rule is as follows: if S_1 is A_1. So, here I will be getting some membership function value corresponding to S_1 and S_2 is B_1; so here corresponding to this S_2^*. So, I will be getting some membership function value then f' is C_1; that means this is nothing but the membership function distribution for this particular output.

Now, here, once again, if we concentrate on the rule 1, the first fired rule; so here, we have got one AND operator and AND operator is nothing but the minimum operator. So, this is nothing but the min operator. So, what I do is, to find out the output of the first fired rule, we compare this particular μ value and this particular μ value. So, this μ value is corresponding to S_1^* and this μ value is corresponding to your S_2^* and we compare these two μ values and we try to find out the minimum because we have got the AND operator.

So, if I compare this μ value and that particular μ value, this is the minimum; so corresponding to that, I can find out. So, this is nothing but the fuzzified output corresponding to the first fired rule. Now, by following this similar procedure, for the second fired rule, which states if S_1 is A_2 and S_2 is B_2, then f' is your C_2. So, this indicates the second rule, now corresponding to this S_1^*. So, I have got this particular membership function value and corresponding to this S_2^*.

I have got this particular membership function value and now, I will have to compare this numerical value of membership and this numerical value of membership. And, if I compare, this is found to be the smaller and there is AND operator. So, we will have to consider the minimum value of μ and corresponding to that minimum value of μ. So, I will be getting some output here and this shaded portion is nothing but the output of the second fired rule. So, we have got the output of the first fired rule, we have got the output of the second fired rule.

Now, we will have to combine. Now, to combine these particular outputs, we take the concept of the OR operator. Now, in the rule base supposing that we have got a large number of rules say there are nine rules. So, we say that either the first rule has got fired or the second rule has got fired or the third rule has got fired, and so on. So, there is one OR operator in between the rules and that is why, if we want to combine these two outputs, what we are going to do is, we are going to use the OR operator and by OR operator, we know that this is nothing but the max operator.

So, what we do is, we superimpose. So, this particular shaded portion and that particular shaded portion and we try to find out the maximum for example, say. So, this particular thing I have copied it here and this shaded area, I have copied it here. Now, if I just try to find out what should be actually the combined control action, the output of the combined control action is decided by this particular output. So, this is actually the area that indicates the combined control action considering both the fired rules.

So, this is the way actually, we combine the output of the two fired rules using the OR operator; now you see. So, this particular output is nothing but actually the fuzzified output. So, this is an area and corresponding to this particular area, we will have to find out what is the crisp value and that is why, we will have to go for some sort of your the defuzzification.

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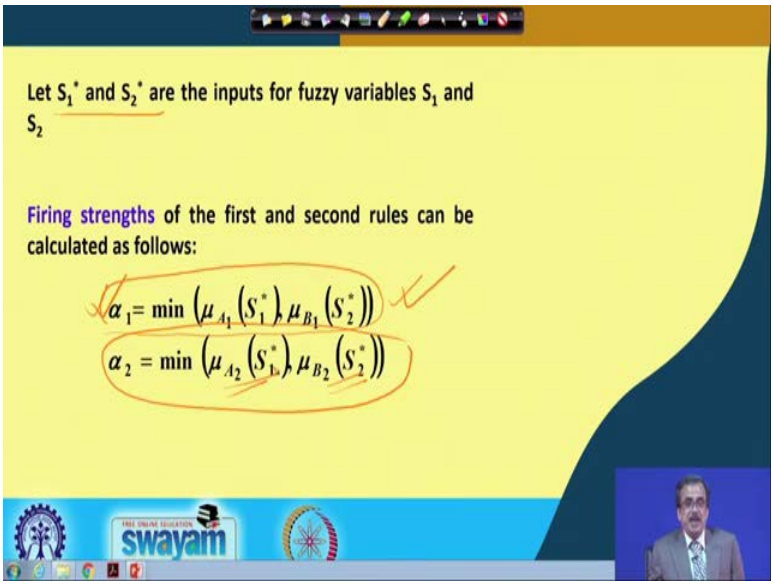

So, if you see like whatever we discuss, the same thing I have written it here. So, let me just read it out. Now, I am passing S_1^* and S_2^*; so these two inputs. So, these two inputs and corresponding to this S_1^* and S_2^* for the first fired rule, I will be getting the firing strength and that is nothing but the minimum of $\mu_{A1}(S_1^*)$ and $\mu_{B1}(S_2^*)$ this I have already discuss and you will have to find out the minimum of these two μ values and that is nothing but the firing strength of the first fired rule; similarly the firing strength of the second rule, we can find out. So, using this particular expression, that is, α_2 is nothing but the minimum between $\mu_{A2}(S_1^*)$, $\mu_{B2}(S_2^*)$. So, we are going to compare these two μ values and we will try to find out the minimum.

So, this is the way actually we actually determine the firing strength of each of the fired rules.

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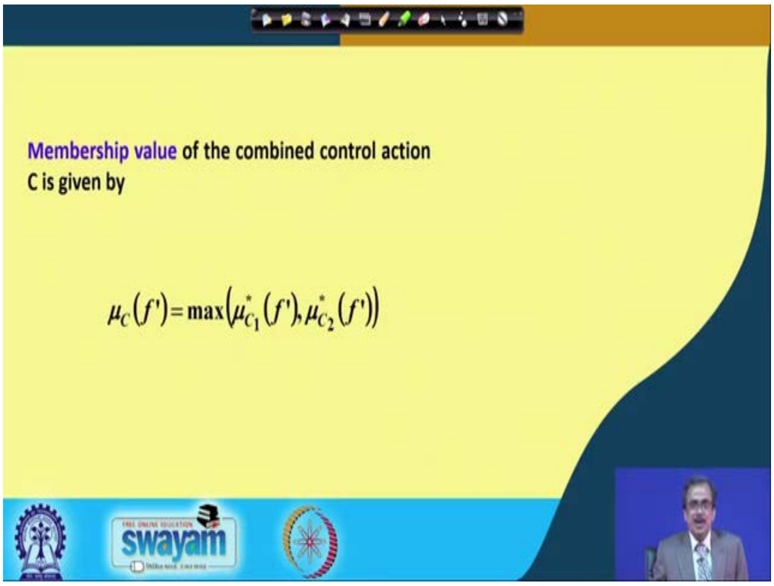

And, once you have got the firing strengths, now we are going for combining. So, we try to find out the combined control action considering both the fired rules. And, as we have already discussed that we take the help of some sort of max operator and that is nothing but the OR operator; and by using the concept of max operator, which I have already discussed, we can find out what should be the output, but this particular output is nothing but the fuzzified output.

So, we will have to take the help of some sort of defuzzification.

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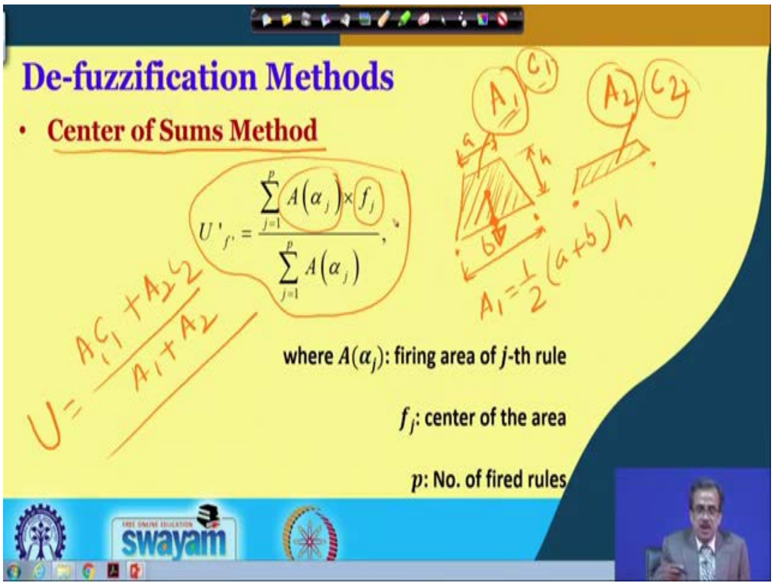

Now, we are going to discuss the defuzzification methods. Now, if you see the literature, we have got a number of methods for defuzzification and I am just going to use three methods for defuzzification, which are very popular. Now, here, the first method is the center of sums method. Now, according to this particular method, the center of sums method so, what you do is, supposing that we have got two fired rules.

Now, for the first fired rule supposing that I have got this type of the output, that is nothing but the truncated triangle, and for the second fired rule, supposing that I have got this type of output, that is another truncated triangle and of course, here, there are some numerical values, there are some numerical values here. So, those things I am not writing. Now, supposing that I am getting this particular truncated triangle corresponding to the first fired rule. So, what I am going to do is, I will try to find out what is the area of this particular truncated triangle or the trapezium and what should be the center of area denoted by your the C_1.

Now, for this trapezium, very easily we can find out the area, if I know, this particular dimension say a, and if I know the dimension say b and if I know this h. So, very easily, I can find out that A_1 is nothing but half a plus b multiplied by h. So, very easily, you can find out what should be the area of this particular trapezium; and what should be the center of area along this particular direction? The center of area from symmetry I can find out. So, this could be the center of area.

So, actually this along this dimension; so this is going to indicate the center of area, truly speaking, the center of area could be here and along the direction of b. So, along the variable; so I can find out this particular numerical value. So, I know this particular A_1 and C_1 and following the same procedure, I can also find out what is A_2 and what is your C_2 and once you have got your A_1, C_1, A_2, C_2. So, very easily we can find out the crisp output that is denoted by your U and that is nothing but is $\frac{A_1C_1 + A_2C_2}{A_1 + A_2}$. So, this way is going to give the crisp output, now here, in this mathematical formulation, in this formula actually, I have used the slightly different notations. So, it indicates the crisp value, that is, $U'_f = \frac{\sum_{j=1}^{p} A(\alpha_j) f_j}{\sum_{j=1}^{p} A(\alpha_j)}$. Now, this A (alpha_j) is going to indicate your A_1 and

A_2 and this f_j that is going to indicate your this C_1 and C_2. So, using this particular expression; so you can find out actually what should be the crisp output.

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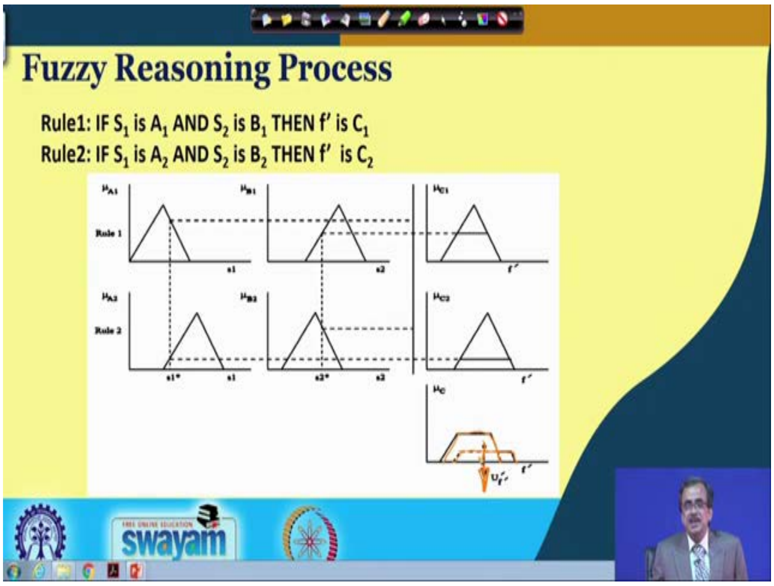

Now, if I just go back to the slide, I can find out. So, if this is one area, I know this area and center of area, this is another area. So, I know the area and center of area. So, this will be actually the crisp output. So, we can find out the crisp output and while controlling the process. So, we will have to depend on the crisp output. So, this is actually the way, the center of sums method works.

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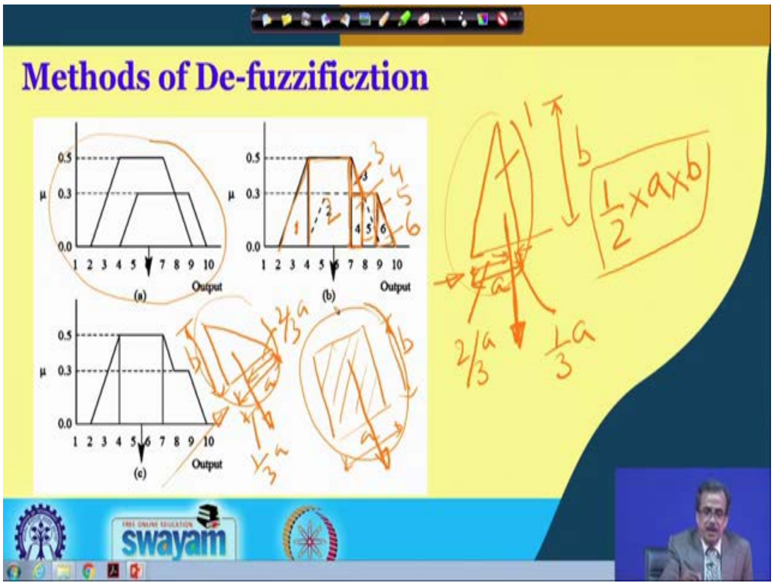

The same thing actually, I have explained here. So, this area, this particular figure has been used to explain that center of sums method, which I have already discussed.

So, center of sums method, I have already discussed. So, this is the center of sums method. Now, I am just going to discuss another technique and that is called the Centroid Method. Now, here actually, what we do is, we take the help of the centroid method. Now, the combined this output is divided into a few. So, this is actually the output of the combined control action and that is divided into that is divided into a few standard sub-regions.

So, what we do is, this whole area is divided into a number of sub-regions. So, this is nothing but the first sub-region, this is nothing but the second sub-region, this particular triangle is nothing but the third sub-region, this rectangle is the fourth sub-region, this particular rectangle is the fifth sub-region. And, we have got a triangle that is the sixth sub-region now, for each of this particular sub-region, very easily you can find out what should be the area and the center of area, for example, say if I consider this particular triangle that is denoted by 1 and very easily, I can find out what is the area. So, if I know this, this particular dimension say a and if I know, so this particular dimension say b.

So, very easily, I can find out, the area and this area is nothing but $\frac{1}{2}(a \times b)$. So, this is nothing but is your area for this particular the triangle and the center of this particular area can be determined very easily. Now, if it is a, if the total dimension is a. So, if I just try to measure from here, this would be a one-third, two-third. So, this will be two-third. So, this will be your two-third of a and this is nothing but is your one-third a. So, I can find out the center of area very easily for this particular right angled triangle.

Now, if I have got one say rectangle, very easily I can find out this particular area, if I know these particular dimensions, say a and b. So, area is nothing but a multiplied by b, and very easily I can find out the center of area. Now, if I have got this type of triangle, for example, say, this type of right angled triangle and I am measuring from this particular side. So, what I will have to do is if this is a and if this is your the b, very easily I can find out the center of area and could be here and this is nothing but is your one-third a and this is nothing but is your two-third a.

So, one-third, two-third a you can find out. So, either we have this type of triangle or you have got this type of triangle or you have got this particular the rectangle. And, we can find out for each of this particular sub-region, what is the area and what is the center of area. Now, once you have got this area and center of area,

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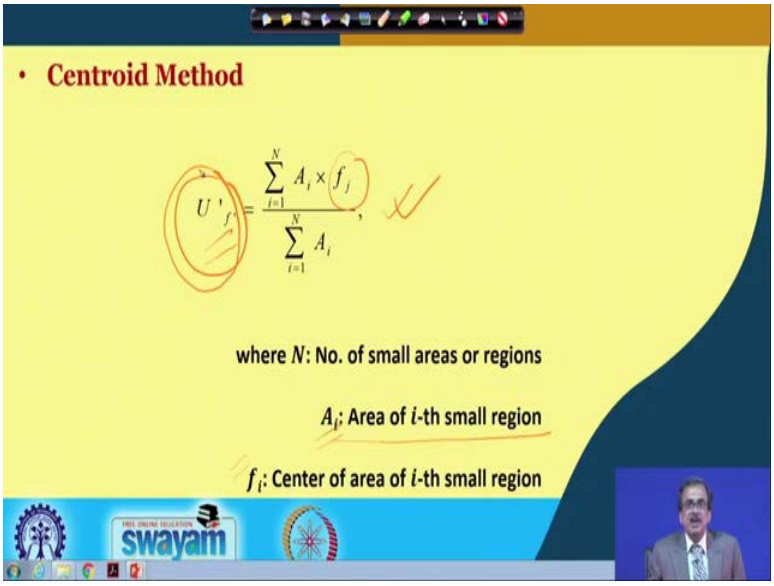

what you can do is now, you can use this centroid method to find out what should be the crisp output. Now, the crisp output, that is nothing but U'_f is nothing but summation i equals to 1 to N, A_i multiplied by f_j divided by summation i equals to 1 to n A_i, where A_i is nothing but the area of i-th small region and f_j is nothing but the center of area of j-th small region.

Now, I have already discussed how to determine the area and the center of area for each of these regular sub-regions and once you have got those information, by using this particular formula; you can find out what should be the crisp output considering all the combined outputs. So, this is the way, using the centroid method, what you can do is, you can find out the crisp value and that is nothing but the control action.

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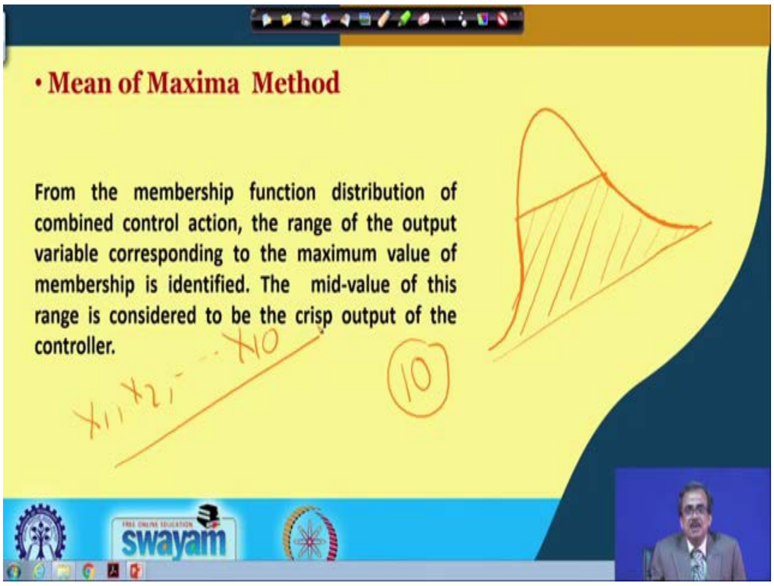

Now, I am just going to discuss another method, that is called the mean of maxima method.

Now, mean of maxima method, I can explain with the help of this figure very easily. Now, let me repeat that considering both the fired rules supposing that I am getting, this type of the fuzzified output and this shows actually the combined output of your both the fired rules. Now, our aim is to find out a crisp value. Now, what I do is, we start from here and try to move in this particular direction, and we try to find out the value of μ considering this combined control action this fuzzified output. So, if I move, if I am here, this is nothing but my μ; now if I am here, this is nothing but my μ, now if I just follow this particular principle that starting from here. So, I am just going to move along this particular direction.

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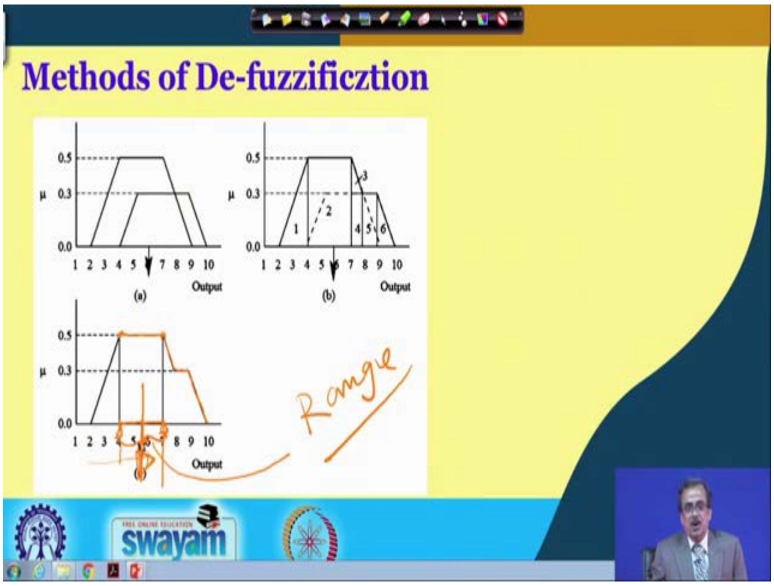

The value of μ is going to vary and corresponding to this particular value of the variable.

So, I will reach the maximum value of μ and after that, it will remain constant. So, this will remain the same up to this value of the output variable. That means, starting from here up to this, the value of this μ will remain constant and that is the maximum value, the range for getting the maximum value of the μ, that is the membership value. And, after that μ is going to be reduced, and then once again it will remain constant and it is going to be reduced.

Now, we try to find out the range for the output variable, where the μ reaches the maximum. Now, here, it reaches the maximum and after that it will remain the same up to this. That means, from here to here in this particular range, the μ will reach the maximum and that will remain constant. So, we try to find out a range for the output, where we get the maximum value for this membership, and once you have got this particular range for the output variable, for which we get the maximum value for the μ, we try to find out the center of this particular range very easily.

So, we try to find out the midpoint of this particular range and that is nothing but the crisp output. So, this is the way actually, we try to find out the crisp value corresponding to the fuzzified output. So, these three methods are very popularly used to determine the crisp value corresponding to the fuzzified output. And, let me repeat, the first method is your center of sums method. The second method is the centroid method and we have got the mean of maxima method.

Now, if I compare, these three methods, now question may arise, out of these three methods which one is the best. Now, it is bit difficult to say, which one is the best, but if you see in terms of the computational complexity, the mean of maxima method is computationally the fastest one and this center of sums method and the centroid method will be difficult in terms of computation, particularly if I consider the non-linear membership function distribution. Now, supposing that I am just going to consider, say one Gaussian distribution as the membership function distribution for the output.

So, this is nothing but the Gaussian distribution for the output variable, now supposing that I have got. So, this is one Gaussian, there could be another Gaussian here and let me consider for simplicity only one Gaussian. So, what I will have to do is, you will have to find out what is this particular area; and if you want to find out the area of the shaded portion. So, there is no way out, but you will have to take the help of integration because this is actually a non-linear one. So, I will have to find out the area of this particular shaded portion and we will have to take the help of integration and integration, you know, it is computationally very expensive. And, that is why, the center of sums method and the centroid method are computationally very heavy and we can rely on this mean of maxima method.

Now, I am just going to come back to the same query, that out of these three, which one is the best. The way I answered that it is bit difficult to declare that this is the best. The reason I am just going to tell you, I am just going to take a very practical example just to understand that it is bit difficult to declare which one is the best. Now, let me take this particular very practical example. Now, this particular course I hope will be taken by some undergraduate students also.

Now, supposing that after completing your degree, so you are going to join some industry and supposing that say in a particular organization there are 10 people. So, 10 people have joined at a time and these 10 people supposing that are coming from 10 different institutes, say X_1, X_2 up to say X_10. Now, the institutes are different and 10 people have joined, 10 graduate engineers have joined a particular organization. Now, your performance is going to tell you the output. So, your output will be decided by your performance, whether you are coming from the institute X_1 or X_2 and or X_10 that is not the thing which has to be considered. The main considerations would be your performance, your output, the same is true here whether you are using the center of sums method or centroid method or mean of maxima method, anyone is good.

Because based on that we are going to develop the database and the optimized rule base, so that this particular fuzzy reasoning tool is going to give you that output for a set of inputs as accurately as possible. So, based on this particular method of defuzzification, whether it is center of sums method or centroid method or mean of maxima method, we are going to optimize. We are going to determine the knowledge base of the fuzzy reasoning tool.

And, in fact, whether we are following a mean of maxima or center of sums method or centroid method, that is not the thing. We will see the performance like the whether the fuzzy reasoning tool or the fuzzy logic controller is going to or it is able to determine the output for a set of inputs accurately or not. So, that is actually the performance of this fuzzy reasoning tool and we are interested to get very accurate modeling for this particular input-output relationships of a process. Now, how to ensure that? That I am going to discuss in the next lecture.

Thank you.