Fuzzy Logic and Neural Networks
Prof. Dilip Kumar Pratihar
Department of Mechanical Engineering
Indian Institute of Technology, Kharagpur

Lecture – 07
Applications to Fuzzy Sets

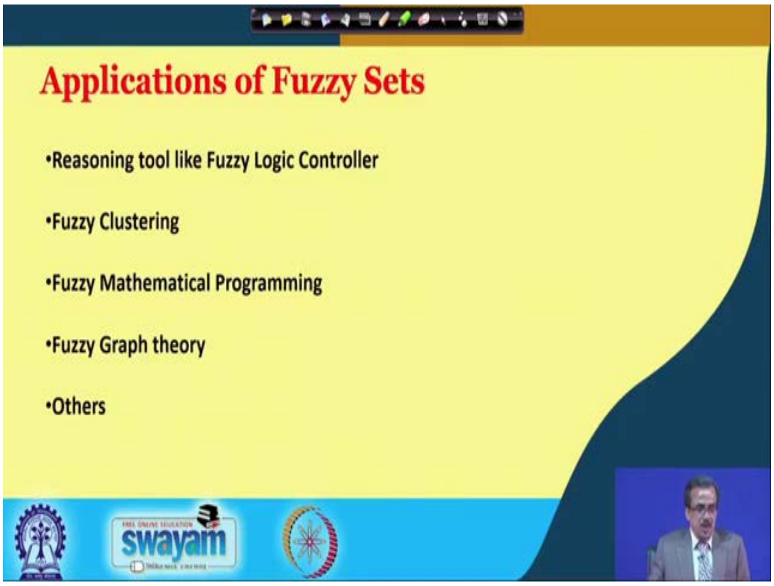
We are going to discuss how to use the concept of fuzzy sets to solve a variety of real-world problems. We have already explained the grammar of fuzzy sets and let us see, how to utilize that fuzzy set to solve a variety of problems.

(Refer Slide Time: 00:35)

Now, as we told that we are going to concentrate on application of fuzzy sets. Now, in this lecture actually, we are going to concentrate mainly on two applications: one is how to design and develop fuzzy reasoning tool in the form of fuzzy logic controller.

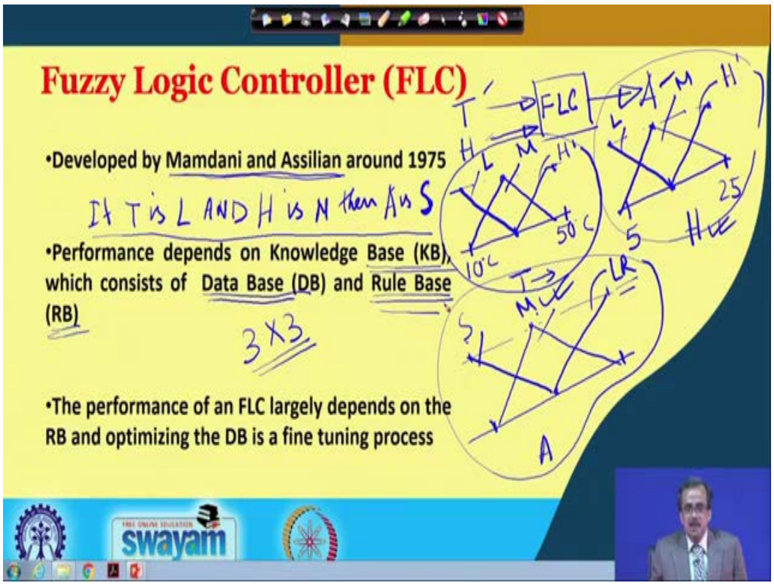
Now, if you see the literature, we have got two very popular approaches: one is called the Mamdani approach and another is called Takagi and Sugeno's approach. Now, both the approaches will be discussed, in detail, with suitable numerical examples, then we will concentrate on fuzzy clustering. Now, clustering is done based on the concept of similarity. Now, two similar points should belong to the same cluster and two dissimilar point should go to two different clusters. Now, let us see how to use the concept of fuzzy sets to design and develop the clustering tools.

(Refer Slide Time: 01:43)



Now, this I have already mentioned that the concept of fuzzy sets have been used to develop fuzzy reasoning tool like fuzzy logic controller, then fuzzy clustering and if you see the literature, the fuzzy sets have been used for fuzzy mathematical programming, then comes fuzzy graph theory, and others.

(Refer Slide Time: 02:05)



Now, here, I am just going to concentrate on how to design and develop fuzzy logic controller or fuzzy reasoning tool. Now, the first notable approach developed in this particular direction is that by Mamdani and Assilian.

Now, Mamdani and Assilian, in the year around 1974-75, developed one fuzzy reasoning tool or fuzzy logic controller. The purpose was how to model the input-output relationships of a process. Now, this Mamdani approach if you see the working principle, the performance of the fuzzy logic controller depends on the knowledge base and this knowledge base consists of data base as well as rule base.

Now, let me first define the concept of this data base and rule base, now, let me take a very simple example, very practical example, supposing that for this lecture room, I want to control the temperature and humidity, and I want to keep the temperature and humidity within a very reasonable range or very acceptable range. Now, let us see how to control with the help of one air conditioner, supposing that I have got one AC here in this particular room, and I want to control the temperature and humidity of this room.

Now, the first thing we will have to do is we will have to find out or we will have to identify, what are the variables the design variables. Now, the design variables could be something like this, for example, say the temperature of this particular room, humidity of this particular room, the temperature outside the room, humidity outside the room thermal conductivity of the wall, number of people sitting in this particular room. So, these are all the variables or these are actually the design variables.

Now, supposing that for simplicity let me consider that I will have to develop fuzzy logic-based expert system, which is going to control the valve opening of this particular air conditioner so, that the temperature and humidity remain within the comfortable zone. So, let me consider, for simplicity, that there are two inputs: one is the temperature T and another is humidity H, and the output is nothing, but the angle of valve opening for this particular air conditioner.

Now, let us see, how to define the data base and the rule base, if I want to design and develop one fuzzy reasoning based tool or fuzzy logic controller. So, here, the inputs are the temperature and humidity inside this particular room and output is nothing, but the angle of valve opening for this air conditioner. So, what we do is, we try to define some range for the temperature. Now, supposing that this is nothing, but the range for the temperature, this is the minimum temperature and this is the maximum temperature. Let me consider the minimum temperature is 10 degrees centigrade and the maximum temperature is 50 degrees centigrade, and what we do is. So, this whole range of temperature is expressed with the help of some linguistic terms and for simplicity, let me consider that I am using only three linguistic terms.

Now, supposing that the linguistic terms are as follows: for example, say I am just going to say, this is the low temperature, this is the membership function distribution or the fuzzy set for the low temperature denoted by L, this is the membership function distribution or the fuzzy set for the medium temperature M and this is say for the high temperature say that is your H, so H'. So, we have got low temperature denoted by L, the medium temperature denoted by M and high temperature denoted by say H'. So, this is nothing, but the membership function distribution for the temperature.

Similarly, for the humidity, what we do is, say we defined the minimum and the maximum value for this particular humidity in a particular scale and using some unit. So, let me assume, let me put some numerical value say, the minimum is 5 and say maximum is 25 in certain scale and using some units and once again supposing that I am just going to use say three linguistic terms.

So, if I use three linguistic term, one is say L that is low value of humidity, another is the medium value of humidity that is denoted by M and another could be your the high value of humidity and that is denoted by your H prime. So, the whole range of humidity that is expressed with the help of three linguistic term, one is low, another is medium and another is high.

So, L, M and H'. So, this is nothing, but the humidity, now this is what we mean by the data base for the temperature and this is nothing, but the database for your the humidity. Now, similarly, I can also construct one database for the output that is nothing, but the angle of valve opening. So, this is the angle of valve opening say denoted by A, say I know the minimum value, I know the maximum value and once again I use say three linguistic terms: say one is say small denoted by S, another is say the medium say denoted by M and another could be the high that is the high angle or the large angle say denoted by say LR, ok.

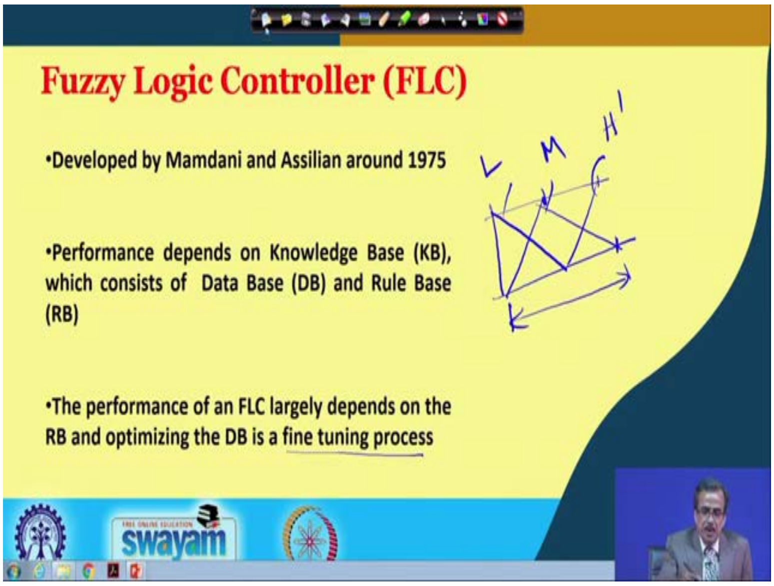
So, this particular range for this angle of the valve opening is divided into or that is expressed using three linguistic terms, one is small, another is your medium M and another is your large, that is denoted by LR. So, this is what we mean by the data base of some variables say inputs or the output variables. Now, using the concept of this particular database, now I can define the rules. Now, here, there are three linguistic terms for temperature and three linguistic term for this humidity. So, 3 multiplied by 3, there will be actually the 9 possible rules; that means, 9 possible conditions for the input variables.

Now, out of those 9, if I just write only 1, supposing that the rule could be something like this. So, if your temperature is low and the humidity that is H. So, H is your say medium then the angle of valve opening that is denoted by A is nothing, but say small. So, this is one rule and similarly, as I told, there could be 3 multiplied by 3, a maximum of 9 rules, this is what we mean by the rule base.

So, the rule base consists of 9 rules, a maximum of 9 rules and out of the 9 rules, here, I have just written a particular one. For example, let me repeat, if temperature is low and humidity is medium then the angle of valve opening is small. So, this is a particular rule. So, this is the way actually, we define the database and rule base of this particular fuzzy reasoning tool and knowledge base consist of both the database as well as the rule base.

Now, here, I am just going to make one comment that the performance of an FLC largely depends on the rule base and optimizing the database is a fine tuning process. Now, here the presence or absence of a rule that is going to dictate the output of that particular fuzzy reasoning tool in a very large way. On the other hand, if I see the database optimization that is nothing, but the fine tuning process, now let us see, what do you do during this particular database optimization.

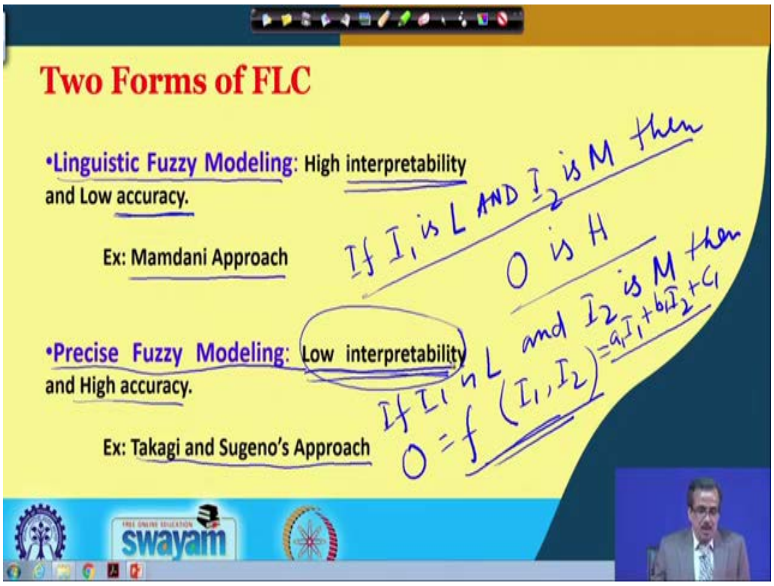
(Refer Slide Time: 11:57)



Now, if I consider, the database is nothing, but this type of your triangle. So, if I consider this type of triangular distribution for this particular database. So, this is low, this is medium, at this is the high. So, what does it mean? During the optimization, we try to say, either it increases or decreases. So, this particular base of the triangle; that means, I will be getting some sort of flatter triangle or I will be getting some sort of the smaller or the steeper triangle sort of thing.

Now, during the optimization actually we can vary the width or half base-width of this particular triangle and by doing that, we are simply doing some sort of fine tuning. So, this is the way actually, we do some sort of fine tuning just to improve the performance of this fuzzy reasoning tool.

(Refer Slide Time: 12:55)



Now, if you see the literature. So, this fuzzy reasoning tool or fuzzy logic controller has been divided into two groups. Now, one is called your the linguistic fuzzy modeling and another is called the precise fuzzy modeling.

So, linguistic fuzzy modeling and this precise fuzzy modeling, now let me try to explain what we mean by the linguistic fuzzy modeling. So, by linguistic fuzzy modeling we mean those fuzzy modeling where we have got high interpretability, but low accuracy. Now, let us try to understand, what do you mean by this interpretability of a rule. Now, let me just write down one hypothetical rule. So, if I_1 that is the first input is say low and the I_2 that is the second input if it is medium, then the output O is some sort of say high. So, if this is a rule, the moment I read this particular rule, I will be able to understand what is the control action?

So, if I read, if I_1 is low AND I_2 is M, then output is high. So, immediately some control action is coming to my mind; that means, its interpretability is very good; that means, your understandability of the meaning of this particular output for a set of input is very high and that is what, we mean by interpretability of a particular rule for the fuzzy listening tool. Then, comes the accuracy; accuracy is nothing, but the precision; that means, your the accuracy in prediction of the output for a set of inputs. Now, the example for this linguistic fuzzy modelling is nothing, but the Mamdani approach, which will be discussed in much more details.

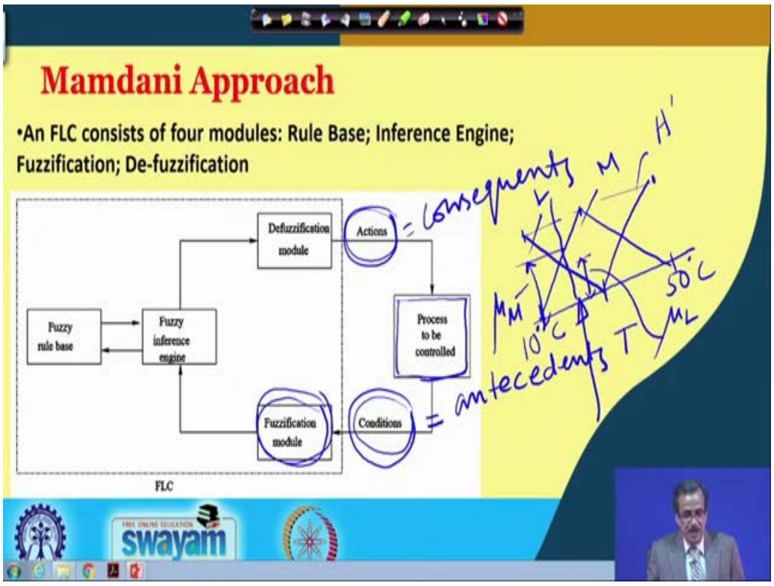
Now, then comes the precise fuzzy modeling and here, we get low interpretability, but we will be getting actually the high accuracy or the precision. Now, the example is nothing, but the Takagi and Sugeno's approach of fuzzy reasoning tool. Now, in Takagi and Sugeno's approach, the way we write down the rule is as follows like if I_1 is low and (this is not the operator end and this is the conjunction and) and I_2 is your medium, then output is expressed as a function of the input parameters, that is your I_1 and I_2.

Now, this particular output could be either the linear function of the input parameters or it could be non-linear function. And, the coefficients of this particular function will be determined with the help of some optimizers, with the help of some training scenarios, and that is why, we can give a guarantee of high accuracy. But, interpretability is low in the sense, if I just read this particular rule, no control action is coming to my mind directly.

So, if I read, if I_1 is low and I_2 is medium, then output is a particular function of this particular your I_1 and I_2 and let me write that. So, this is nothing, but $a_1 I_1 + b_1 I_2 + c_1$ and if I just write this particular output something like this, no control action is directly coming to my mind and that is why, the interpretability of this type of fuzzy reasoning tool is less.

Now, I am just going to discuss this Mamdani approach in much more details.

(Refer Slide Time: 17:09)



Now, this schematic view shows actually the working principle of the Mamdani approach of fuzzy reasoning tool. Now, if we want to implement the Mamdani approach of this fuzzy reasoning tool the first thing we will have to do is, we will have to concentrate on the process to be controlled and we will have to identify what are the condition variables and what are the action variables; that means, what are the inputs and what are the outputs.

Now, let me once again take the same example like the temperature and humidity control of this particular room with the help of one air conditioner. And, here the inputs are, as I mentioned, the temperature and humidity inside the room temperature and humidity outside the room, then comes the thermal conductivity of the wall and the number of people sitting inside this particular room. So, these are all condition variables and what is the output or the action? The output is nothing, but the angle of valve opening, so that we can keep the temperature and humidity of this particular room in a very comfortable zone.

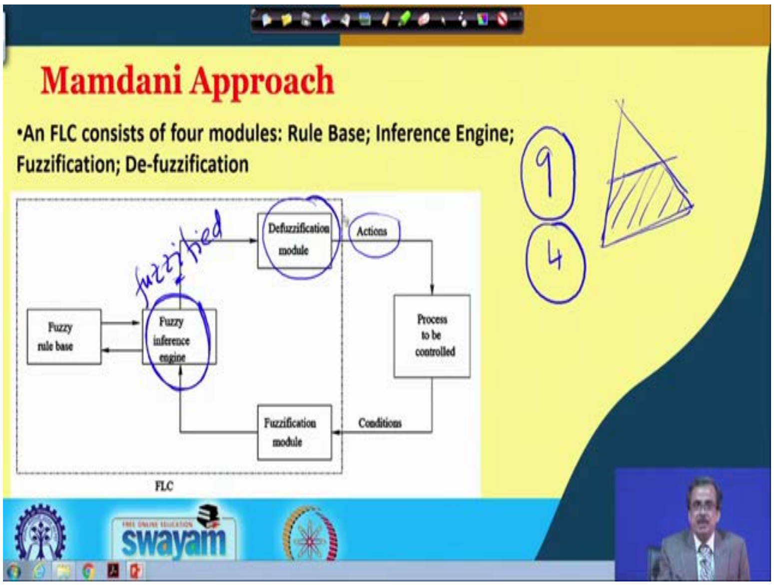
So, the first task is, we will have to identify the inputs and the outputs; that means your condition variables and action variables. Now, these condition variables are also known as antecedents. So, this is known as antecedents and the action variables are known as your consequents. So the antecedents and consequents, we will have to identify first and once we got these particular antecedents, that is, condition variables, we go for the fuzzification module.

So, by fuzzification module, actually we mean that corresponding to this set of inputs, we try to find out what should be the membership function values. Now, let me take a very simple example of temperature. Now, if I consider there are three linguistic terms to represent the temperature. So, this is the low temperature, this is actually the medium temperature and this is, say, the high temperature is H'.

Now, here, this is the temperature. So, this is 10 degree centigrade and this is say 50 degree centigrade. Supposing that the room temperature is around say around 20 degree or something. So, might be I am here; so might be I am here. So, if it is so, so what you can do is, you will have to find out corresponding to this particular temperature, what is the membership function value for the low and what is the membership function value for the medium.

So, this is the membership function distribution corresponding the low temperature and this is the membership function distribution corresponding to the medium temperature. Now, this particular task of determining the membership function value is nothing, but the fuzzification. Now, once that particular fuzzification has been done, next we concentrate on the fuzzy inference.

(Refer Slide Time: 20:45)



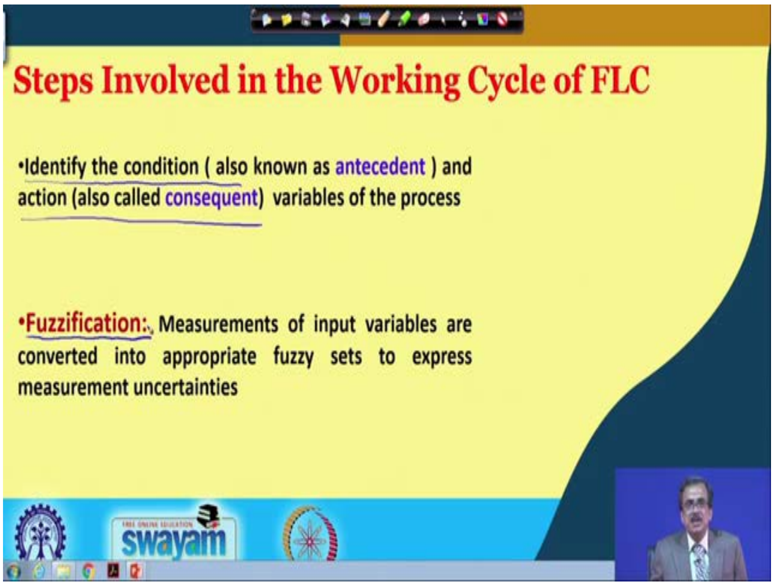
Now, let us try to understand the utility of this fuzzy inference engine. Now, let me take a very simple example supposing that the same example of the expert system or fuzzy logic base-based expert system, to control temperature and humidity. Now, as I told that we consider a maximum of 9 rules, which are present in this particular the rule base. Now, supposing that for a set of inputs, at a time, there could be only four fired rules. Now, out of these 9 rules, which four will be fired that is decided by this particular fuzzy inference engine.

So, depending on the set of inputs, which four out of these maximum 9 rules will be fired, that is decided by the fuzzy inference engine. Now, once you have got those five fired rules, now for each of these particular fired rules, I will be able to find out, what should be the output and that I am going to discuss in much more details and once you have got that particular output for each of the fired rules, I will be able to find out one output and that is nothing, but the fuzzified fired output.

So, here, I will be getting the fuzzified output, now this particular fuzzified output will be nothing, but an area, which I will be discussing much more details after some time. Now, this output is nothing, but an area, for example, say if it is triangle. So, there could be a possibility that I will be getting this type of truncated triangle. So, this truncated triangle shown as the shaded area is nothing, but the output of a particular rule, but here there, is no such crisp value.

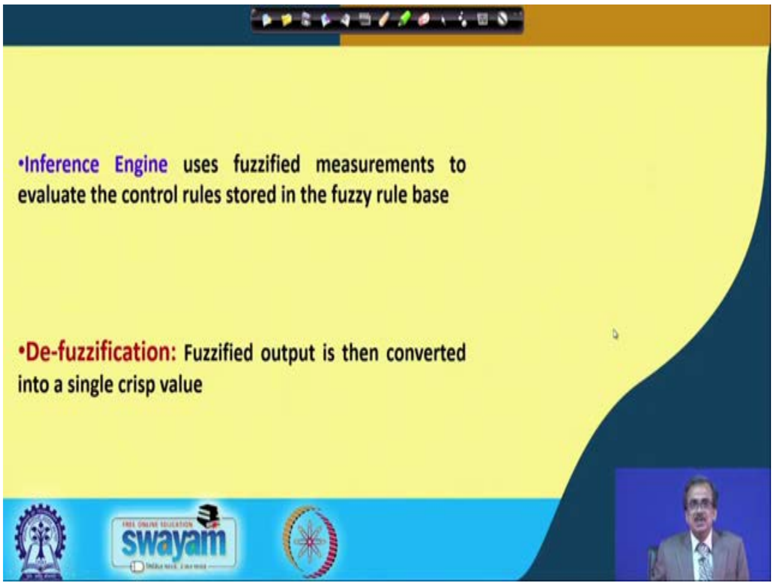
So, what I need is, a crisp value corresponding to this particular area and how to find out that particular crisp value? To find out the crisp value from this particular fuzzified area, we take the help of some defuzzification module. Now, a few defuzzification modules are available in the literature and we will be discussing all such things in much more details with the help of some numerical examples. And, once you have got the crisp output, that is nothing, but the actions to be taken for the same example like here the output will be the angle of the valve opening; whether it is clockwise or anti clockwise and by how much, I will have to rotate and what should be the valve opening, whether it is plus 10 degree or minus 10 degree clockwise or anti-clockwise. So, that I will have to decide, as the action variable that is nothing, but the output of this fuzzy reasoning tool.

(Refer Slide Time: 23:59)



Now, here, whatever we discussed, all such things I have written it. Now, the first task, let me repeat is actually, I will have to identify the condition and the action variables. So, the first task will be your identification of the condition and the action variables, that is nothing, but the antecedents and consequents, and once you have got that, now for a set of inputs, we will try to find out the membership values and that is nothing, but the task of fuzzification and that is the fuzzification module.

(Refer Slide Time: 24:41)



And, once you have got that now, next we go for the inference engine, I have already discussed the purpose of inference engine, out of all the possible rules depending on the set of inputs, which rules will be getting fired, so that will be decided by this particular the inference engine.

And, once you have got that particular fired rule, for each of the fired rules will be getting the output and then, how to combine that I am going to discuss in details and once you have got that particular fuzzified output, then how to determine the crisp value corresponding to that particular fuzzified output, so that we get some crisp value corresponding to the output and we can control that particular process. Now, let me repeat once again the purpose of designing or developing.

So, this fuzzy reasoning tool or fuzzy logic controller is to find out the relationship between the inputs and outputs of an engineering process. So, the moment we supply a set of inputs, I should be able to determine the set of outputs. So, that we can control that particular process very accurately. Now, how to design and develop I am just going to discuss in much more details with the help of some numerical examples.

Thank you.