Fuzzy Logic and Neural Networks
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Lecture – 06
Introduction to Fuzzy Sets (Contd.)

Now, we are going to discuss how to determine the composition of fuzzy relations.

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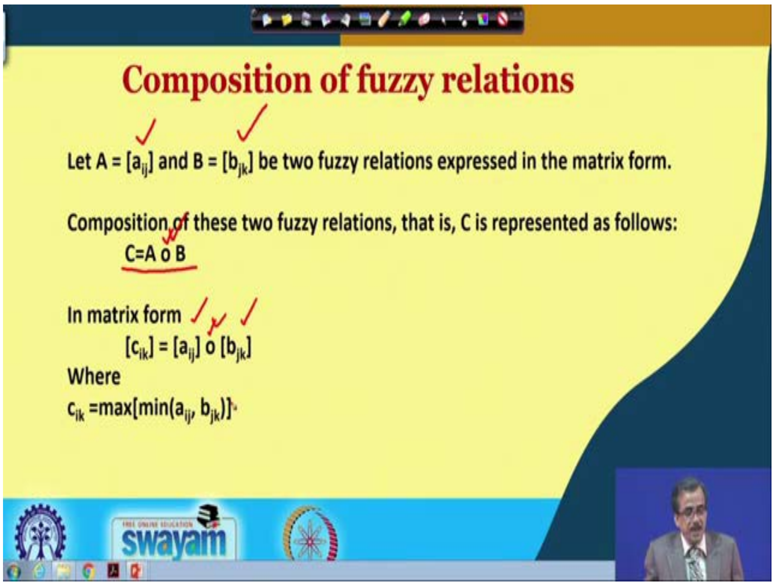

Now, supposing that I have got two fuzzy relations in the matrix form, for example, A is nothing but say a_ij and B is nothing but say b_jk. Now, how to find out the composition of these two fuzzy relations? Now, the composition that is denoted by C is nothing but A composition B, and this particular symbol indicates actually the composition.

So, C is nothing but A composition B. Now, in the matrix form, this can be written as, say I know a_ij, I know b_jk. So, a_ij composition b_jk is nothing but is your c_ik; and how to find out this particular c_ik? Now, c_ik is determined between the maximum of the minimum of this a_ij coma b_jk. Now, let us see with the help of one numerical example. So, how to find out this particular c or the composition of two fuzzy relations.

So, I am just going to take one numerical example.

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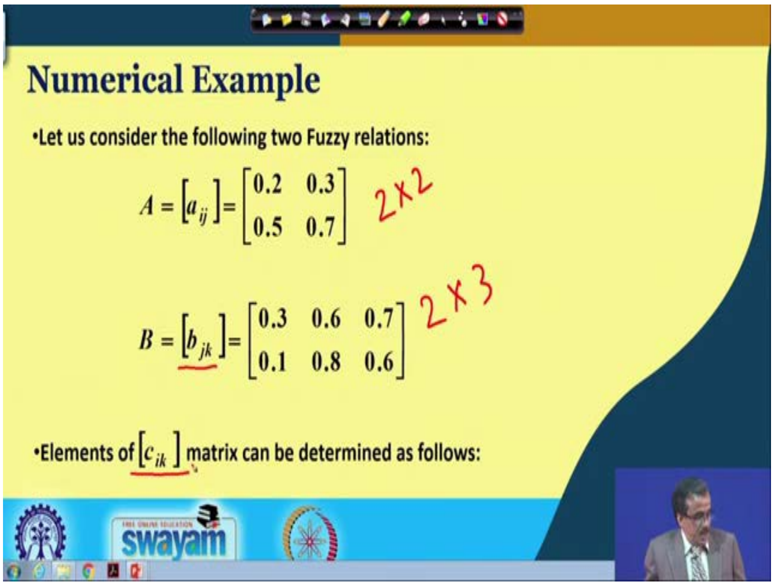

Now, supposing that I have got, one fuzzy relation, that is, A is a_ij in the matrix form and this is nothing but say in the 2×2 matrix like 0.2 0.3 0.5 0.7; the next is your the B is another fuzzy relation and in the matrix form. So, this is nothing but b_jk and supposing that I have got a 2×3 matrix and the elements are 0.3, 0.6, 0.7, 0.1, 0.8, 0.6.

Now, our aim is to find out the elements of these particular c_ik. Now, let us see how to find out the elements, that is the c_ik. Now, here, if I just try to find out the element-wise.

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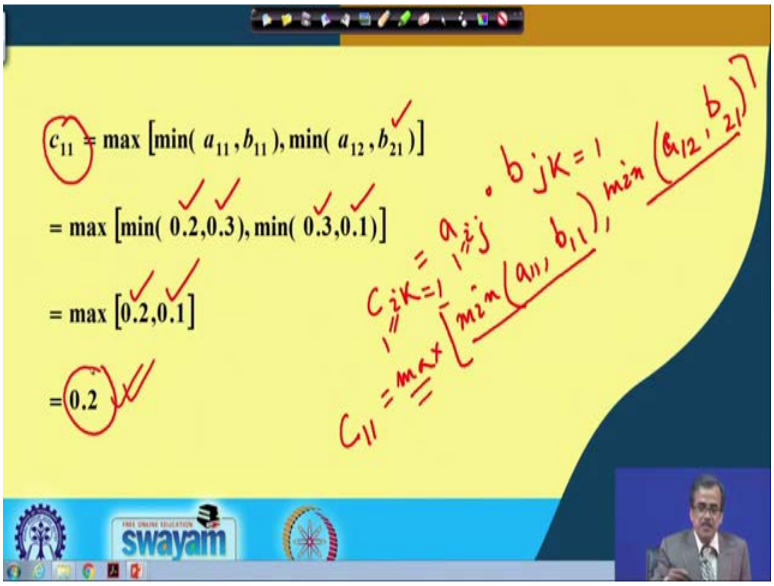

Now, the first thing is your c_11 that is the first row the first column element. Now, if you remember. So, this particular the relationship that is c_ik is nothing but a_ij composition that is your b_jk. Now, here actually what you will have to do is; so here to determine the c 11. So, I will have to put i equals to 1, I will have to put k equals to 1.

Now, here, I can put i equals to 1 and k equals to 1 and j will vary from 1 to 2. Now, if j varies from 1 to 2. So, very easily I will be getting C_11 is nothing but the maximum between the minimum between. So, i equals to 1 and let me put j equals to 1 here. So, 1 1 then comes your b_jk now j is equals to 1 and k is equal to 1. So, b_11 comma the minimum of a_1 now I will have to put j equals to 2. So, a_12 then comes your b_j equals to 2. So, it is 21. So, first you will have to find out the minimum between these two the minimum between these two and then I will have to find out the maximum between these two; that will be nothing but is your C_11.

Now, corresponding to this particular numerical example like a_11 that is a matrix first row first column and that is nothing but 0.2, then comes your b_11. So, the b matrix first row first column is 0.3. Next, comes the minimum between a_12 that is your first row second column and that is nothing but 0.3. Then, comes your b_21 that that is your second row first column and that is nothing but 0.1.

Now, if I compare 0.2 and 0.3. So, the minimum will be 0.2. Similarly, if I compare 0.3 and 0.1, the minimum will be 0.1 and the maximum between 0.2 and 0.1 is nothing but is your 0.2. So, I can find out; so this c_11 is nothing but 0.2 and by following the same procedure, I can also find out the other elements of the matrix.

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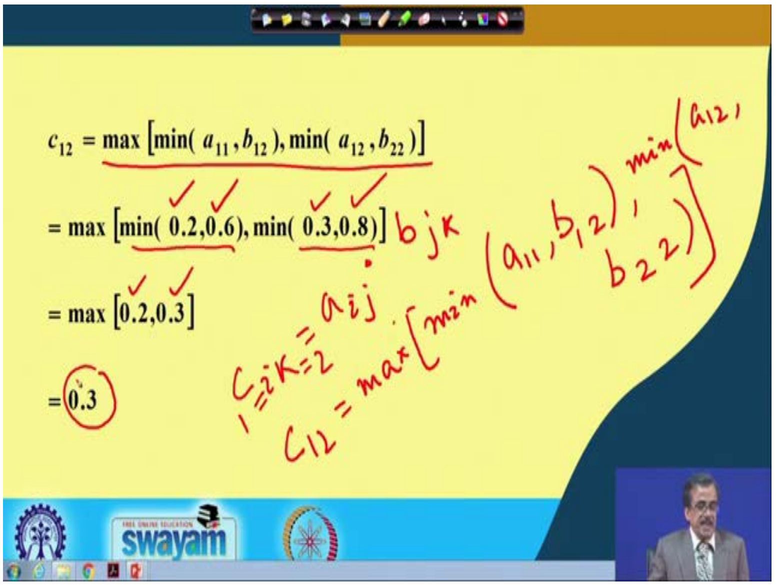

For example say c_12 and let me try to derive this particular thing once again. Now, this c_12 means what?

So, c_ik is nothing but a_ij then comes your b_jk. Now, the moment I am writing c_12. So, i equals to 1 and k is equals to 2 and once again, j will vary from 1 to 2. So, this C_12 is nothing but the maximum of the minimum between. So, i equals to 1, j equals to 1. So, a_11 then comes your b_jk, j equals to 1 and what is k? k is nothing but is your 2 minimum your next is what like a_12. So, i equals to 1 and what about j? j is equals to your 2. So, a_12 and next is your b_jk. So, j equals to 2 and what about your k? k is also 2.

So, this is the relationship which will be getting. Now, if you just put the elements the numerical values like a_11 first row first column is 0.2, b_12 that is your first row second column 0.6, next a_12 is 0.3 and b_22 is 0.8. So, the minimum between 0.2 and 0.6 is nothing but 0.2, similarly the minimum between 0.3 and 0.8 is nothing but 0.3 and the maximum between them 0.3.

So, we can find out what should be the numerical value for this particular your C_12.

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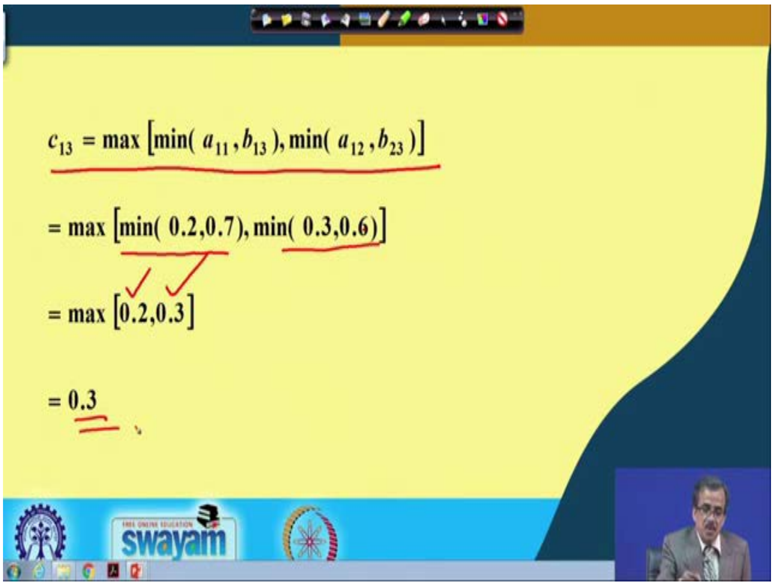

Now, by following the same procedure; so I can find out what is your c_13; and c_13 is nothing but the maximum or the minimum between a_11, b-13 comma the minimum between a_12 comma b_23; and if you just substitute the numerical values here. So, this will become the maximum between the minimum between 0.2 and 0.7 and the minimum between 0.3 and 0.6. So, here I will be getting 0.2. I will be getting 0.3 and the maximum between them is the 0.3. So, c_13 is nothing but is your 0.3.

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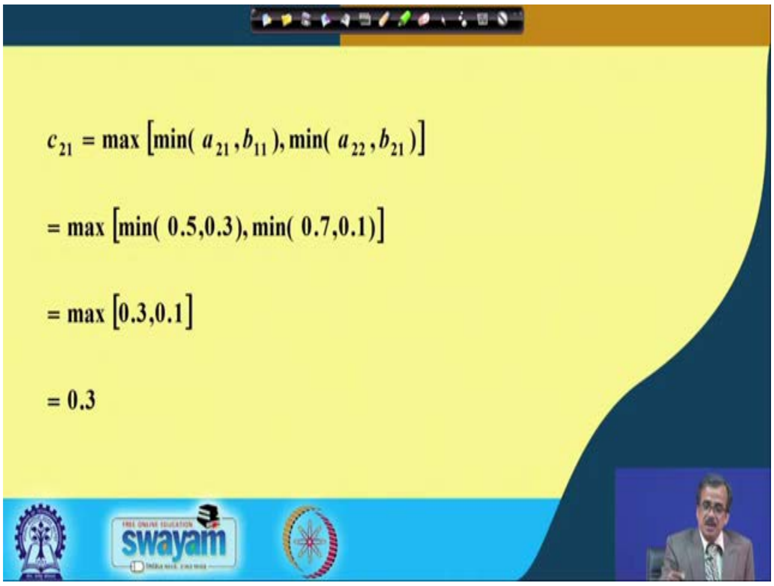

Now, following the same procedure. So, I can also find out your c_21 using this particular expression, and if you insert the numerical values I will be getting this, and I can find out the maximum between 0.3 and 0.1 and you will be getting 0.3.

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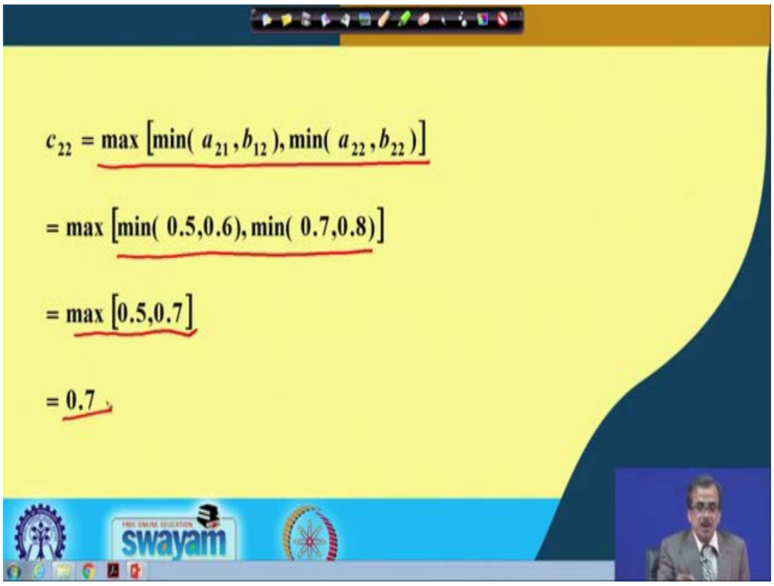

Now, by following the same procedure: so you can also find out the next element that is your c_22, which is nothing but this. And, we substitute the numerical values will be getting like this, then maximum between 0.5 and 0.7 and I will be getting your the 0.7.

Now the next is your c_23.

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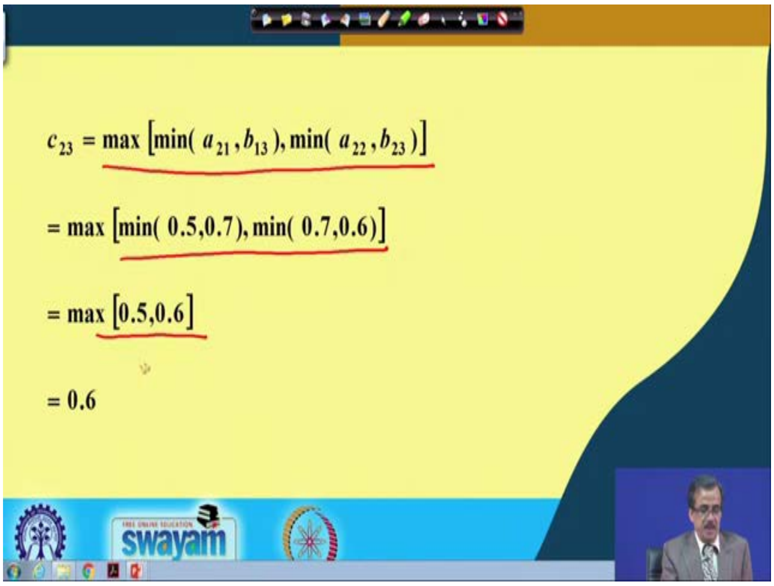

So, I can find out what is this c_23 this is nothing but this particular expression substitute the numerical values will be getting this, find out the maximum between 0.5 and 0.6.

So, you will be getting your c_23 as 0.6.

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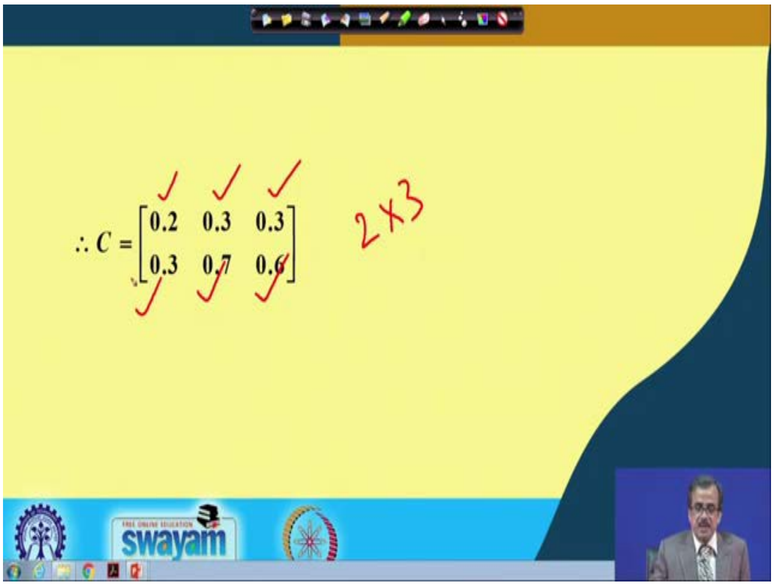

Then, we can write down the c matrix. So, all the elements, I can write out and it is nothing but actually 2×3 matrix, and these are the element c_11 c_12 c_13 c_21 c_22 c_23. So, you will be getting this particular matrix. So, this is the way actually we can find out the composition of the two fuzzy relations.

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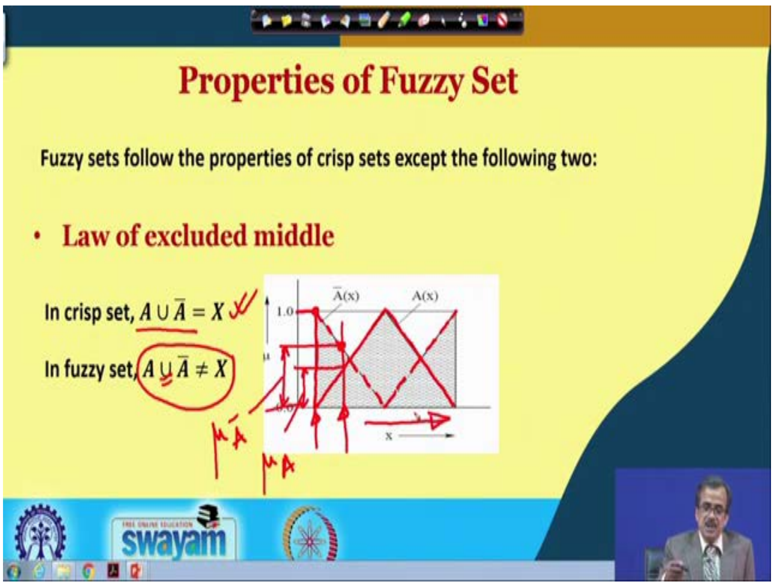

Now, I am just going to start with another topic, that is the properties of fuzzy sets. Now, if you remember while discussing the concept of crisp set, we have already discussed about ten properties. Now, these ten properties if you look into. Now, this fuzzy set can follow the first 8 properties out of the 10, but the last two properties of the crisp set are not followed by the fuzzy sets and these are nothing but the law of excluded middle; and then comes law of contradiction.

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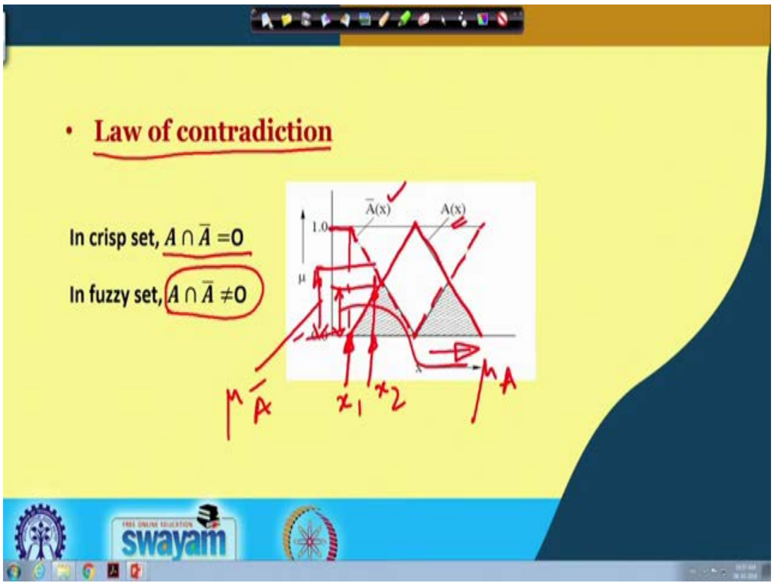

Now, if you see this law of contradiction, if you see the law of excluded middle first. Now, as I told that these two properties are not followed by the fuzzy set, one is law of excluded middle and the law of contradiction. Now, according to the law of excluded middle in the crisp set. $A \cap \bar{A}$ that is the intersection of two fuzzy sets like A and its complement is null set. The union of two fuzzy sets that is A and its complement is nothing but the universal set.

But, in fuzzy sets we will find that A union A bar is not equals to X. So, this particular violation we are getting in the fuzzy set now. So once again, let me repeat according to the law of excluded middle in crisp set. So, A union A bar is X that is universe of discourse, but in fuzzy sets A union A bar is not equal to the universe of discourse. Now, let us try to explain now, supposing that this particular triangle is going to represent the fuzzy set A (x). Now, if this is A (x). So, we have already discussed now, how to find outs its complement supposing that this is denoted by the dotted line. So, its complement is denoted by this, this I have already discussed.

Now, if you want to find out the union. So, what you will have to do is. So, you will have to concentrate here corresponding to this value of x and according to your A (x). So, the μ is nothing but 0, but corresponding to $\bar{A}(x)$ that is the compliment. So, its your membership function value is your 1.0 and this is the union. So, we will have to consider the maximum; that means, corresponding to this value of x. So, I will have to consider this as the μ. Next, the moment I am here; so this particular value of say x. So, now, this is actually your μ corresponding to your the fuzzy set A (x) and this is nothing but the μ corresponding to this particular $\bar{A}$.

So, this is nothing but μ corresponding to A and this is nothing but μ corresponding to $\bar{A}$. Now, these two μs, we will have to compare and we will have to consider the maximum. So, I am here; now if I follow this principle and if we increase the value of x. So, there is a possibility that I will be getting actually this union, which is nothing but this.

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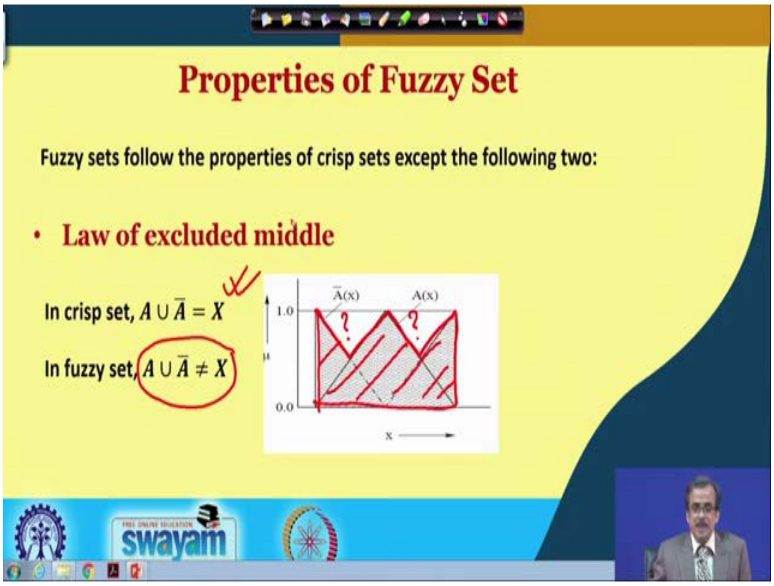

So, this type of thing I will be getting as the union of you fuzzy set A (x) and its complement. So, this is the thing actually, we will be getting, ok. So, this shaded portion actually indicates your union of a fuzzy set and its complement, but you see here, I have got one white portion here, I have got one white portion which has not been attended.

So, we cannot say that in fuzzy sets, say $A \cup \bar{A}$ is equal to the whole thing, that is capital X. So, this particular condition holds good for the fuzzy set, but according to the crisp set this should not be the condition. So, there is a violation of the fuzzy set, that violence is in terms of law of excluded middle. That means your fuzzy set does not follow the law of excluded middle. The reason, I am just going to tell you after sometime and let me explain this another law, which is not followed by this particular fuzzy set and that is known as law of contradiction.

Now, according to this law of contradiction, in crisp set like $A \cap \bar{A}$ is nothing but the null set, but in fuzzy set A intersection A bar is not equal to 0. So, there is a violation of this law of contradiction by fuzzy sets. Now, let us try to explain here now supposing that, this is nothing but the membership function distribution of a fuzzy set A (x) and its complement is nothing but this dotted line.

So, this is your $\bar{A}(x)$, now I try to find out their intersection. So, we will start from here, now corresponding to this value of to x say x_1 according to this A (x). So, I have got the μ value, which is equal to 0 and corresponding to this value of x_1, the μ value corresponding this $\bar{A}(x)$ is nothing but 1 and we will have to consider the minimum. So, I will have to consider this particular 0. The next is, I consider another value of x say x_2 and this indicates the μ corresponding to your A that is nothing but μ corresponding to A, and μ corresponding to your $\bar{A}$ is nothing but μ corresponding to your $\bar{A}$.

Now, we compare μ_A with your $\bar{\mu}_A$ and this μ_A is found to be minimum; so corresponding to this x_2. So, we consider up to this and by following the same principle. So, if I just proceed with different values of x. So, I will be getting actually one area and this particular area is the intersection area. So, this type of intersection area I will be getting and that is why A intersection your A bar is not equals to 0 in fuzzy set.

Now, these two rules are not followed by the fuzzy set. Now, I am just going to tell you the reason behind this particular fact. Now, let us try to understand what is the reason that these two rules are not followed by the fuzzy sets? Now, to understand that particular reason, let me assume that, let me just draw it here the same picture.

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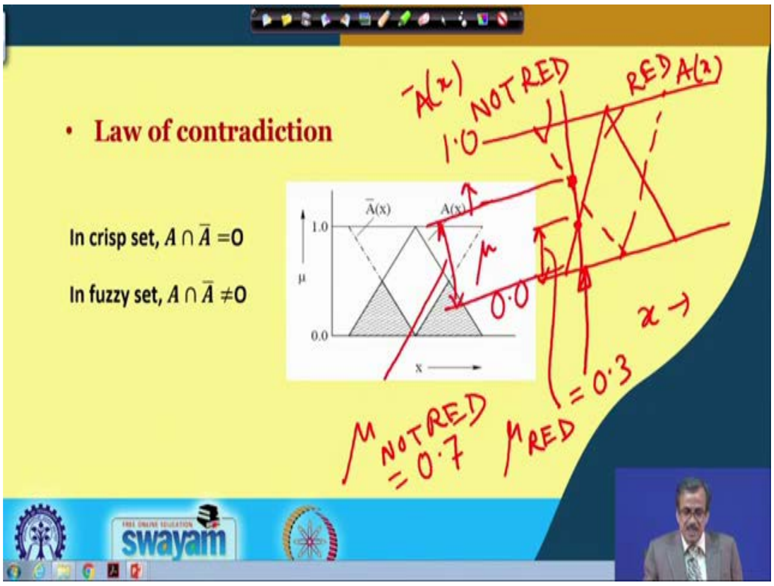

Now, supposing that I have got a fuzzy set for say the red color. Now this is for the red color and this is nothing but μ, μ equals to 0.0, 1.0, as we have discussed here. So, here corresponding to this thing, what I will be getting here is this is nothing but its compliment.

So, this is the compliment, say it is not red. So, not red is actually the complement of the red. So, if the red is nothing but is your A (x). So, not red is nothing but is your $\bar{A}(x)$ and here. So, this is x now, supposing that I am considering a particular color. So, that color is nothing but this, and I am just going to give a statement whether it is red or not red or if it is red with what membership function value.

Now, what you do is, corresponding to this value of x or the color, you we try to find out the μ. Now, this particular value of mu is nothing but that it is red with the membership function value say μ_{red}, and it is the same color it is not red with another membership function value. And, this particular value is nothing but is your; so μ_{notred}. So, this is nothing but is your μ_{notred} .

So, corresponding to this, I will be getting this is nothing but μ_{notred} and here corresponding to the red color, I will be getting your the μ_{red}. Now, let us take very hypothetical situation supposing that this μ_{red} is nothing but 0.3, now, what will happen to the value of your μ_{notred}, if it is a triangular distribution? So, this will become your 0.7, now let me give the statement. So, the same color is considered red with some membership function value, and it is also considered as not red with another membership function value and let me put it in another way.

That means, an element that is the red color belongs to its fuzzy set as well as its complement with different values of membership and this is actually the reason, why this particular fuzzy set is not going to follow these two rules. So, let me repeat the reasons behind violation of these two rules by the fuzzy set, which are as follows. Here, in fuzzy sets and element belongs to a particular fuzzy set as well as its complement with different values of membership and that is why, we are getting this type of violation of the two rules.

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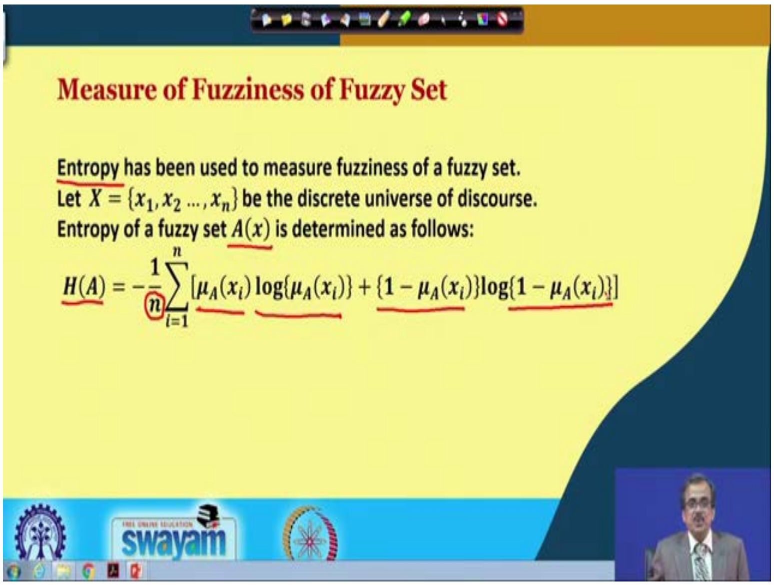

Now, I am just going to discuss, in fact, I am just going to define two terms: one is called the measure of fuzziness. So, how to mathematically find out the fuzziness of a fuzzy set? Now, to define this particular fuzziness of a fuzzy set, we use a particular term that is called the entropy. So, the term: entropy is used just to define the fuzziness of a fuzzy set. Now supposing that I have got a discrete fuzzy set, say A (x) and I know element-wise. So, its membership function values, how to find out the entropy of this particular fuzzy set, which is denoted by capital H (A). Now, $H(A) = -\frac{1}{n}\sum_{i=1}^{n}[\mu_A(x_i)\log\{\mu_A(x_i)\} + \{1-\mu_A(x_i)\}\log\{1-\mu_A(x_i)\}]$. So, using this particular the expression; so I am just going to find what should be the entropy of a fuzzy set and that is nothing but the measure of fuzziness of a fuzzy set.

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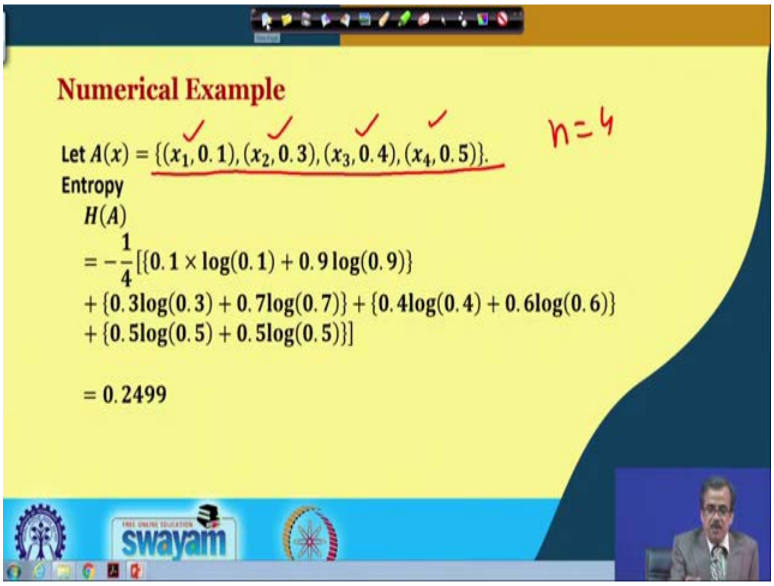

Now, here, I am just going to take one numerical example, supposing that I have got a discrete fuzzy set A (x) like this and its entropy, according to the rule which I have already discussed is nothing but minus 1 divided by n. Now, small n is equal to 4 here, because I have got four such elements x_1 x_2 x_3 and x_4. So, this particular mu if you see this particular formula once again; so I can see. So, this is actually the formula: $H(A) = -\frac{1}{n}\sum_{i=1}^{n}[\mu_A(x_i)\log\{\mu_A(x_i)\} + \{1-\mu_A(x_i)\}\log\{1-\mu_A(x_i)\}]$; now if you see this particular formula. So, I very easily I can find out corresponding to this x 1 I have got 0.1 log of 0.1, then comes your 1 minus 0.1 is 0.9 and log of 1 minus 0.1, (that is, 0.9).

So, corresponding to this particular x_1, I can find out this part, then corresponding to x_2, I can find out your this particular component, corresponding to x_3, we have got this and corresponding to x_4 we have got this and now, if we calculate. So, we will be getting actually one numerical value and that is nothing but 0.2499 and that is entropy of this particular fuzzy set and that is nothing but the fuzziness of the fuzzy set in the numerical term.

So, this is the way actually we can find out the fuzziness of a fuzzy set using the concept of entropy.

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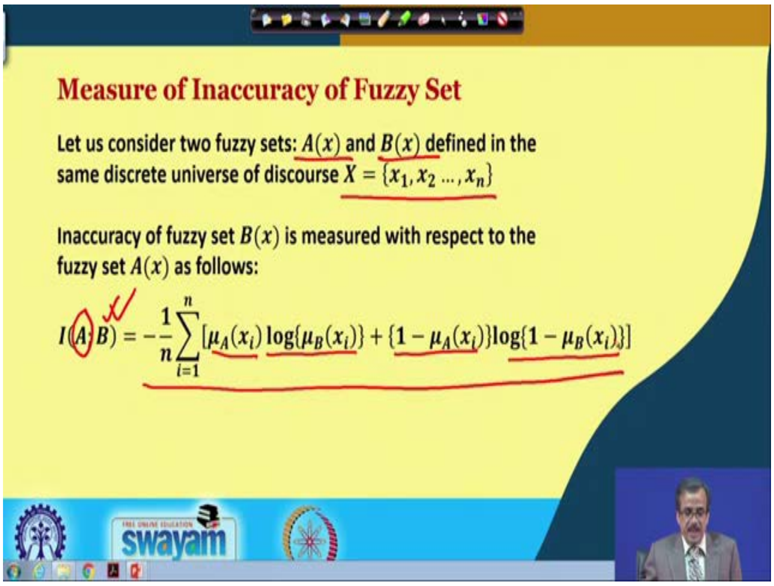

Now, we are going to discuss like how to measure the in accuracy of a fuzzy set. Now, suppose if that I have got two fuzzy sets: A (x) and B (x) defined in the same universe of discourse and I want to find out the inaccuracy of fuzzy set with respect to fuzzy set A. So, I am just going to find out the inaccuracy of the fuzzy set B with respect to the fuzzy set A and mathematically this can be expressed as follows: $I(A;B) = -\frac{1}{n}\sum_{i=1}^{n}[\mu_A(x_i)\log\{\mu_B(x_i)\} + \{1-\mu_A(x_i)\}\log\{1-\mu_B(x_i)\}]$. So, this is the way actually, we can find out the inaccuracy of a fuzzy set with respect to another fuzzy set.

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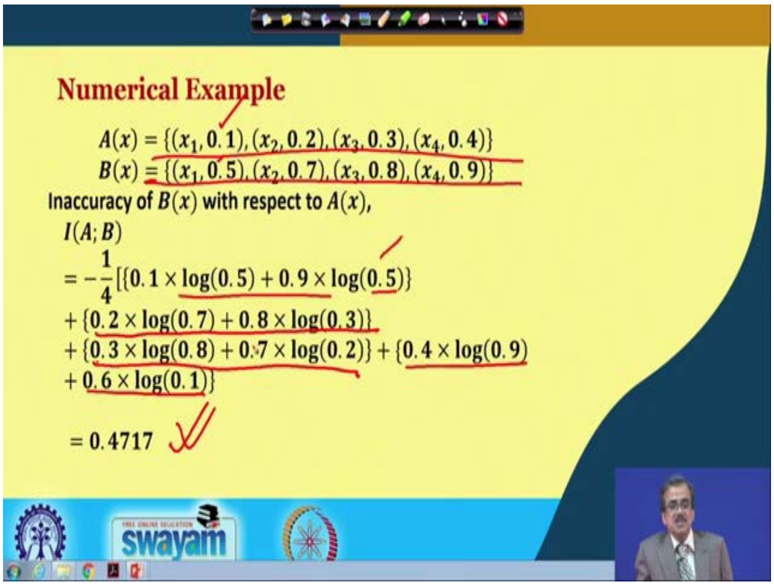

Now, here if you see; so I have got two fuzzy sets here.

So, A (x) is nothing but this and B (x) is nothing but this, now in accuracy of B (x) with respect to A (x) is determined as $I(A;B)$ is nothing but minus 1 by 4 0.1 log of 0.5, then comes your 1 minus 0.1 is 0.9 and log of 1 minus 0.5 is 0.5. This is actually corresponding to x_1 similarly corresponding to x_2 0.2 and log of 0.7, then comes your 1 minus 0.2 is 0.8 and 1 minus 0.7 is your 0.3. So, corresponding to x_2 we can find out this then corresponding to x_3 we can find out this and corresponding to your x_4.

We will be able to calculate this, and ultimately if we calculate you will be getting 0.4717 and this is nothing but in accuracy of fuzzy set B with respect to your fuzzy set A. Now, this is the way actually, we can calculate what should be the inaccuracy of a fuzzy set with respect to another fuzzy set.

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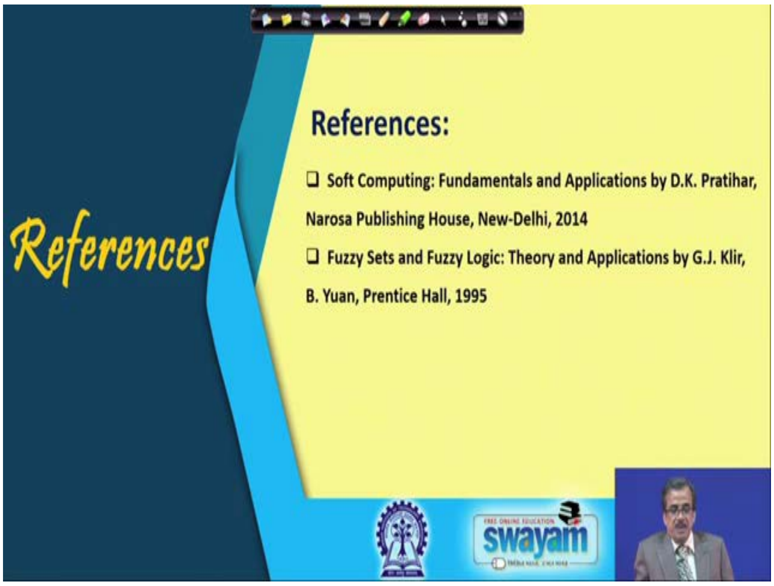

Now, the reference for this the discussion, which I have already discussed the text book you can see the Soft Computing Fundamentals and Applications written by me and published by Narosa Publishing House. And, the reference book you can see Fuzzy Sets and Fuzzy Logic Theory, and Applications written by George Klir & others, which has published in the year 1995.

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Now, just to summarize quickly like till now, we have discussed a few terms related to fuzzy sets, some standard operations in fuzzy sets have been discussed in details and after that, the properties of fuzzy sets we have discussed. And, we have already mentioned that out of the ten properties of the crisp set, the first eight properties are followed by the fuzzy sets, but the last two properties are not followed by the fuzzy sets. And, at the end, we have defined mathematically, how to determine the fuzziness of a fuzzy set and inaccuracy of a fuzzy set with respect to another fuzzy set.

Thank you.