Fuzzy Logic and Neural Networks
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Lecture – 05
Introduction to Fuzzy Sets (Contd.)

We are discussing some standard operations used in fuzzy sets. We have already discussed a few and now, we will be discussing a few more.

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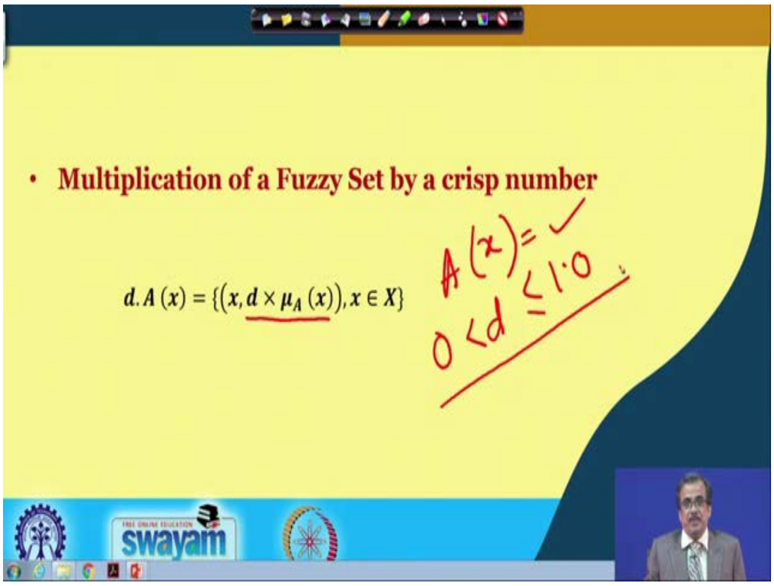

Now, let me start with the concept of multiplication of a fuzzy set by a crisp number. Now, here so, what we are going to do is, we are going to multiply a fuzzy set by a crisp number, as follows:

Say A (x) is a fuzzy set; now this particular fuzzy set, I am just going to multiply by a crisp number, that is, d. Now, if you see how to write down this? So, d multiplied by A (x) is nothing, but x such that you have got the membership function value which is nothing, but d multiplied by $\mu_A(x)$, x belongs to capital X.

Now, here, the range for this particular d, generally we consider the d is greater than 0 and less than equals to 1.0. Now, my question is, why do you need this? We need this particular concept, because if you want to optimize the shape and size of the membership function distribution, we will have to consider this particular concept of multiplication of a fuzzy set by a crisp number.

Now, let me take one numerical example, the things will be made more clear.

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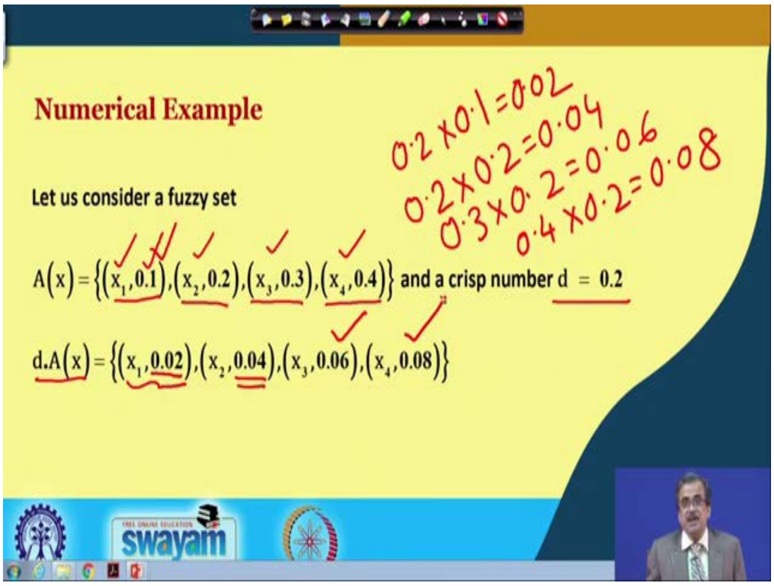

Now, supposing that I have got a fuzzy set A (x), which is nothing, but x_1 comma 0.1, x_2 comma 0.2, x_3 comma 0.3, x_4 comma 0.4. So, this is a fuzzy set, a discrete fuzzy set having 4 elements x_1, x_2, x_3 and x_4, and supposing that the crisp number d is set equal to 0.2.

Now, if d is equal to 0.2 now, d multiplied by A (x). So, this particular thing will become equal to x_1 comma, d is multiplied by this particular mu value. So, d is 0.2, multiplied by 0.1 and this is nothing, but 0.02. So, corresponding to this particular x_1; this will be d multiplied by μ and this will become equal to 0.02.

Now, following the same procedure I can also find out what will happen to corresponding to this particular x_2. Now, here corresponding to x_2 the membership function value is 0.2 multiplied by the d is 0.2. So, this is nothing, but 0.04. So, corresponding to this x_2 we will have 0.04, similarly corresponding to x_3 we will have 0.3 multiplied by 0.2 and that is nothing, but 0.06.

Then corresponding to x_4 we will have 0.4 multiplied by 0.2 and this is nothing, but 0.08. So, corresponding to your x 4; so we will be getting 0.08. Now, let us try to understand the physical significance of this particular multiplication. Now, supposing that I am just going to draw the discrete fuzzy set, that is, A (x).

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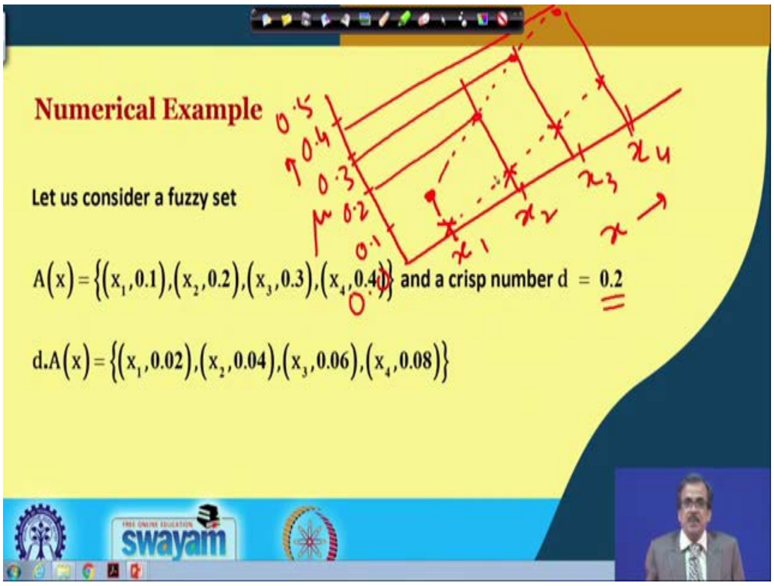

So, let me just draw it here and element-wise, I am just going to write down say the membership value. So, this is mu and mu varies from say 0 to 1 and here, let me write down say 0 to only say 0.5 and we have got the values up to 1.0.

So, this is corresponding to say x_1, this is x_2, this is x_3 and this is your x_4. So, this is the direction of x, now if I consider the original fuzzy set, that is, A (x). So, corresponding to your x_1, I have got 0.1 supposing that. So, this is nothing, but 0.1 this is 0.3, this is 0.3 and this is your 0.4. Now, corresponding to x_1; so, I have got say 0.1. So, I am here, then corresponding to x_2. So, I have got 0.2; so I am here.

Now, corresponding to x_3 say I have got 0.3 say might be I am here, now this is say 0.3 and corresponding to x_4. So, I have got say 0.4. So, might be I am here; now if this is the situation. So, this is the distribution of your A (x) that is the fuzzy set A x, now if I just multiply by d, where d is equals to 0.2. So, corresponding to x_1; so I will be getting 0.02. So, might be I am here, then corresponding to x_2. So, I will be getting 0.04. So, might be I am here.

Corresponding to x_3. So, I will be getting 0.06. So, might be I am here and corresponding to your x_4. So, I have got 0.08; So I could be here. Now, the original fuzzy set, the discrete fuzzy set is something like this. So, as it is discrete, I am not going to draw in continuous curve and if I just multiply by d. So, I will be getting this type of nature; that means, there is a chance of variation of the particular distribution.

Now, remember one thing. So, this particular concept might be required if you want to optimize the shape and size of this particular membership function distribution. Because, what I you do is, initially we assume some membership function distribution but we do not know whether that is the correct one, and with the help of some optimizer and with the help of some training scenarios, what we do is, we try to optimize the shape of that particular member function distribution. And, this is actually the physical significance of your multiplication of a fuzzy set by some crisp number. So, I think this particular idea is clear to you.

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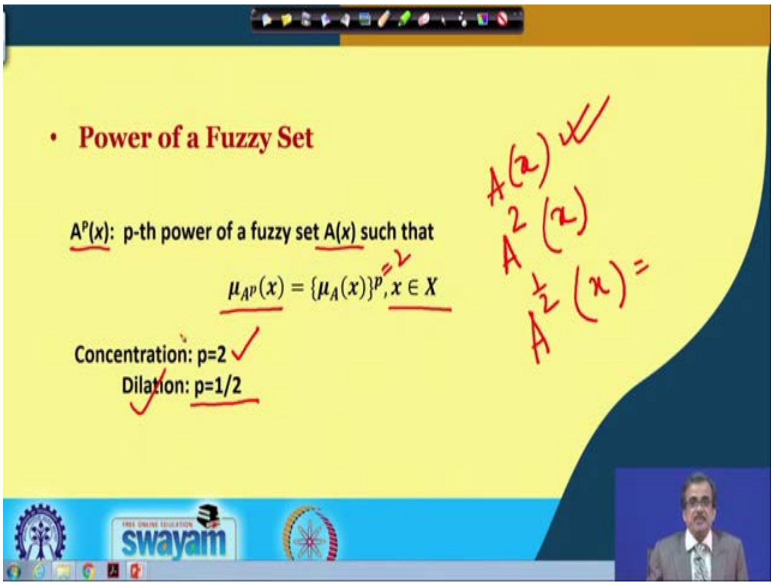

Now, I am just going to consider the power of a fuzzy set. Now the power of a fuzzy set that is defined as your $A^p(x)$ and this indicates actually the p-th power of the fuzzy set A (x).

Now, the moment we are considering the p-th power of the fuzzy set A (x). So, it will have the membership value that is denoted by $\mu_{A^p}(x)$ and $\mu_{A^p}(x) = \{\mu_A(x)\}^p$. Now, p is actually the power and where small x belongs to the universe of discourse, that is your capital X. Now, if I consider, say a particular value for this p; supposing that p is set equal to 2. Now, this is known as the concentration of the fuzzy set. So, if I put p is equals to 2, this is known as the concentration of a fuzzy set.

For example, say I have got a fuzzy set A (x), now if I say, can you please find out the concentration of this particular fuzzy set? So, what we will have to do is, you will have to find out A (x) raise to the power 2. So, this is nothing, but the concentration of a fuzzy set. On the other hand, if I set p is equal to say half, that is called the dilation of a fuzzy set. So, for the fuzzy set A (x), if I tell you that can you please find out the dilation of a fuzzy set and that is nothing, but is your A (x) raise to the power half. So, this is nothing, but the dilation of a fuzzy set.

Now, I am just going to consider some numerical example just to show, how does it work.

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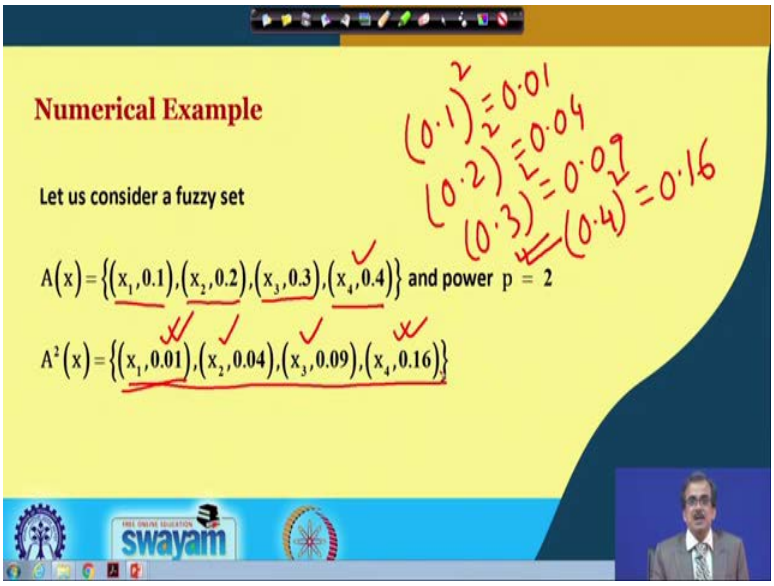

And, after that, I will try to find out what should be the physical significance of this particular concept of power of a fuzzy set. Now, let us consider the same discrete fuzzy set denoted by A (x) is equal to x_1 comma 0.1, x_2 comma 0.2, x_3 comma 0.3 and x_4 comma 0.4 and let me consider the power is equal to 2. So, this is nothing, but the concentration of a fuzzy set. Now, this $A^2(x)$ is nothing by x_1. So, what we will have to do is we will have to find out 0.1 raise to the power 2, square of that and that is nothing, but 0.01.

So, corresponding to x_1, we will have to write down 0.01, similarly corresponding to x_2. What we will have to do is, we will have to write 0.2 square that is nothing, but 0.04. Then, corresponding to x_3, it will be 0.3 square that is your 0.09 and corresponding to x 4 actually what we will have is, 0.4 raise to the power 2 and that is nothing, but is your 0.16. So, this is the way, actually we can find out the concentration of the fuzzy set.

Now, I am just going to discuss its physical significance. Now, let us try to find out the philosophy behind this particular concept.

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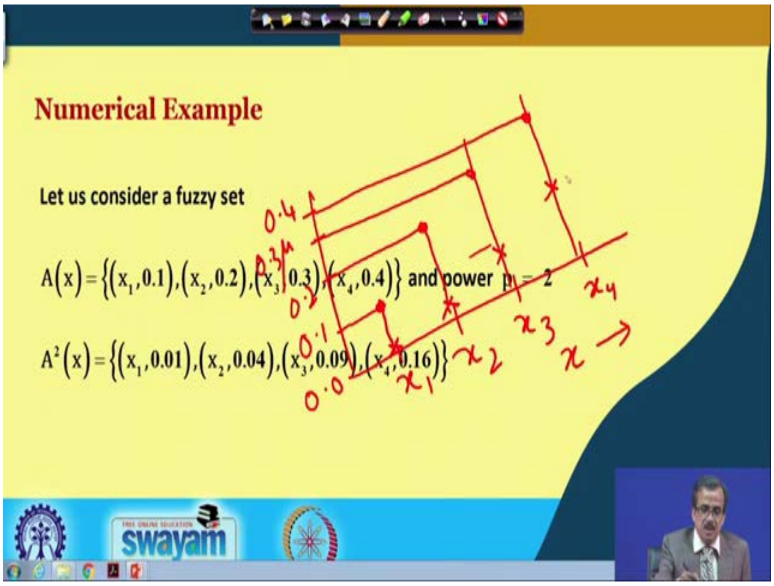

Now, once again let me draw a fuzzy set like if this is your μ, and let me consider, this is 0.0, this is 0.4, say this could be 0.1, 0.2 and this is your 0.3 and along this, we have got x, and supposing that, this is x_1, this is x_2, this is your x_3 and this is x_4. Now corresponding to x_1; so, I have got 0.1, similarly corresponding to x_2. So, I have got 0.2 and corresponding to x_3 say I have got 0.3 and corresponding to this x_4 say I have got 0.4.

So, this is actually the fuzzy set A (x), now we will try to find out what should be its concentration. Now, concentration corresponding to your x_1 is 0.01. So, very near to 0 corresponding to x_2 it is 0.04. So, I could be here; corresponding to x_3 it is 0.09. So, I could be here and corresponding to x_4. So, this is nothing, but 0.16; so, this is 0.1. So, might be I am here.

Now, this is actually the concentration of this particular fuzzy set and once again, the purpose is actually to find out the optimal distribution of this particular fuzzy set. Now, once again, we can take the help of one optimizer and with the help of some training scenarios or the training data, we can find out what should be the optimal distribution of the membership function of that particular fuzzy set.

So, this is actually the physical significance of this particular concept of the power of a fuzzy set.

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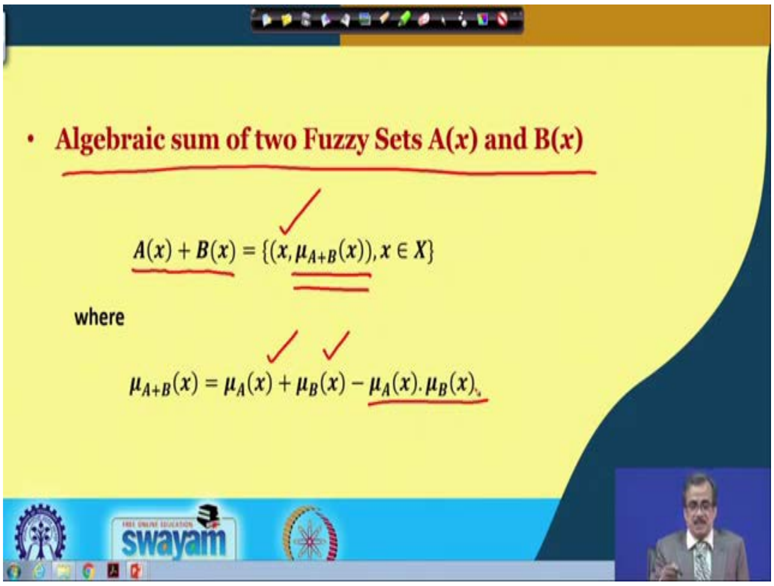

Now, I am just going to find out how to find out your the algebraic sum of two fuzzy sets say I have got two fuzzy sets A (x) and B (x) defined in the same universe of discourse. So, their algebraic sum is nothing, but $A(x)+B(x)$ is nothing, but x, such that it has got the members value $\mu_{A+B}(x)$. So, this will be the membership value and x belongs to capital X, where $\mu_{A+B}(x)$ is nothing, but $\mu_A(x)+\mu_B(x)-\mu_A(x).\mu_B(x)$.

Now, let us see, let us try to solve one numerical example corresponding to this.

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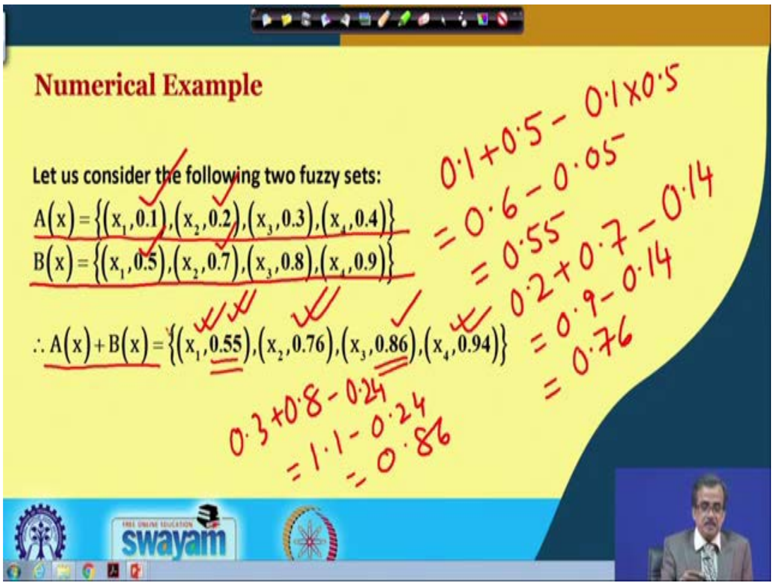

Now, supposing that I have got two fuzzy sets now one is your A (x) is nothing, but this is a discrete fuzzy set and I have got another discrete fuzzy set which is nothing, but B (x) and B (x) is nothing, but your x_1 comma 0.5, x_2 comma 0.7, x_3 comma 0.8 and x_4 comma 0.9 and A (x), I have already use several times the same fuzzy set.

Now, how to find out. So, this A (x) + B (x)? Now, as I told that to find out the membership value corresponding to x_1. So, what we will have to do is, we will have to add. So, this particular membership function value is added to this particular membership function value and then, we will have to subtract these multiplied form.

So, what you can do is. So, 0.1 plus 0.5 minus 0.1 multiplied by 0.5. So, that will be actually the μ corresponding to this particular the x_1. So, this is nothing, but 0.6 x multiplied by 0.05 and this is nothing, but 0.55. So, corresponding to x_1; so we have got the membership function value 0.55. Similarly corresponding to this x_2; so, I have got 0.2 here, 0.7 here. So, 0.2 plus 0.7 minus 0.2 multiplied by 0.7; so, it is 0.14. So, this is nothing, but 0.9 minus 0.1 4 and this will become equal to your 0.76.

So, corresponding to this x_2. So, I will be getting 0.76; similarly corresponding to x_3, it will become 0.3 plus 0.8minus 0.24. So, this is nothing, but 1.1 minus 0.24 and this is nothing, but 0.86. So, you will be getting 0.86, similarly by following the same method corresponding to x_4. So, I will be getting 0.94. So, this is the way actually we can find out. So, A (x) + B (x).

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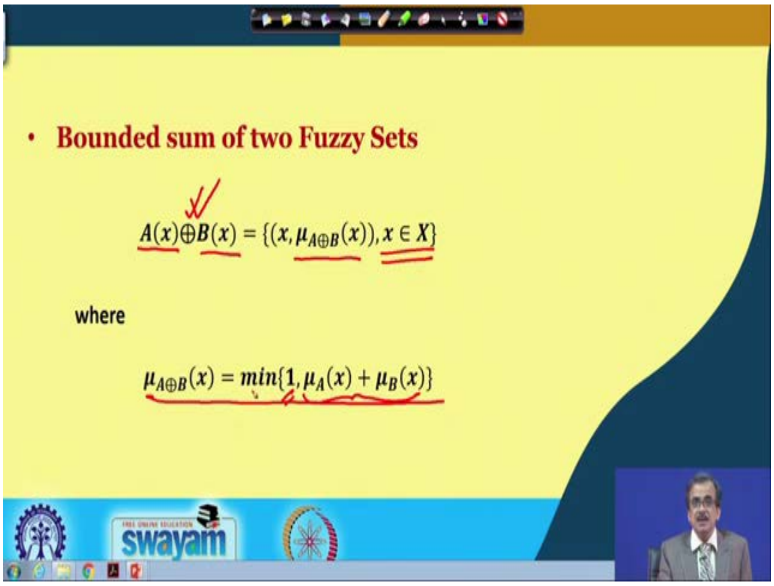

Now, the next operation is known as the bounded sum up of fuzzy sets.

So, once again, I have got two fuzzy sets like your A (x) and B (x), they are defined in the same universe of discourse that x belongs to your capital X and I will have to find out A x bounded sum. So, this is the symbol for the bounded sum. So, A (x) bounded sum B (x) and this is nothing, but x such that it has got the membership value, which is nothing, but $\mu_{A\oplus B}(x)$ and how to determine? So, this particular $\mu_{A\oplus B}(x)$. So, this is the way actually we can find out. So, this is nothing, but the minimum between two quantities one is 1 and another is your $\mu_A(x)+\mu_B(x)$. So, what I do is we simply add the two μ values and we compare with one, and we will be taking the minimum.

Now, let us see how does it work, and let me solve one numerical example here for this bounded sum.

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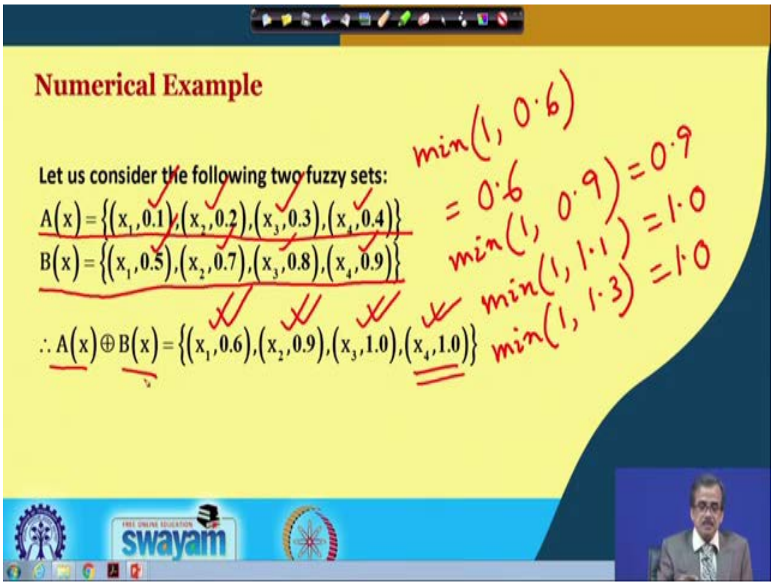

Now, supposing that A (x) is a fuzzy set, the discrete fuzzy set like this, and B (x) is another discrete fuzzy set like this. So, what we will have to do is. So, you will have to find out the minimum between one and what we will have to do is, you will have to add. So, this particular μ value and that particular the μ value; so, 0.1 plus 0.5

So, it is 0.6 and we will have to find out the minimum and that is nothing, but 0.6. So, corresponding to x_1. So, I will have to write down 0.6; similarly corresponding to x_2. So, it is nothing, but the minimum between 1 and 0.2 plus 0.7 is your 0.9 and the minimum is your 0.9. So, corresponding to x_2; so will have to write down the membership value as 0.9. Now following the same procedure for this particular x_3. So, this will become minimum between 1 comma; so 0.3 plus 0 0.8. So, this is nothing, but 1.1 and minimum is nothing, but 1.0.

So, here, we will have to write down 1.0. Similarly, corresponding to this x_4. So, this is nothing, but the minimum between 1, and 0.3 sorry 0.4 plus 0.9. So, this is nothing, but 1.3 and we will have to find out the minimum that is your 1.0. So, corresponding to this particular x_4. So, we are writing 1.0. So, this is the way actually we can find out the bounded sum of your A (x) and B (x).

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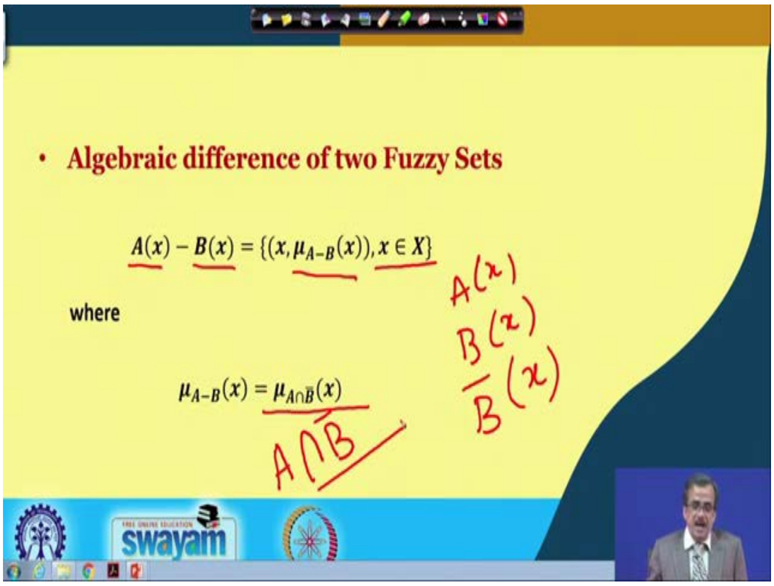

Now, let us see how to carry out the algebraic difference between the two fuzzy sets. Now supposing that I have got your the two fuzzy sets like A (x) and B (x) and they are defined in the same universe of discourse, that is the x belongs to capital X. So, how to find out the algebraic difference that is your A (x) minus B (x)? Now, $A(x) - B(x)$ is nothing, but x, such that it has got the membership function value that is your $\mu_{A-B}(x)$.

Now, how to define this $\mu_{A-B}(x)$? The $\mu_{A-B}(x)$ that is defined as $\mu_{A \cap \bar{B}}(x)$. So, what will have to do is. So, I have got two fuzzy sets A x and B x. So, we have got A (x) and B (x). So, what we will have to do is, will have to find out the complement of these particular B and that is nothing, but is your $\bar{B}(x)$. So, this is the complement of B (x) and after that we will have to use the concept of intersection that is your $A \cap \bar{B}$. Now, let us say how does it work and just to saw. So, I am just going to solve one numerical example.

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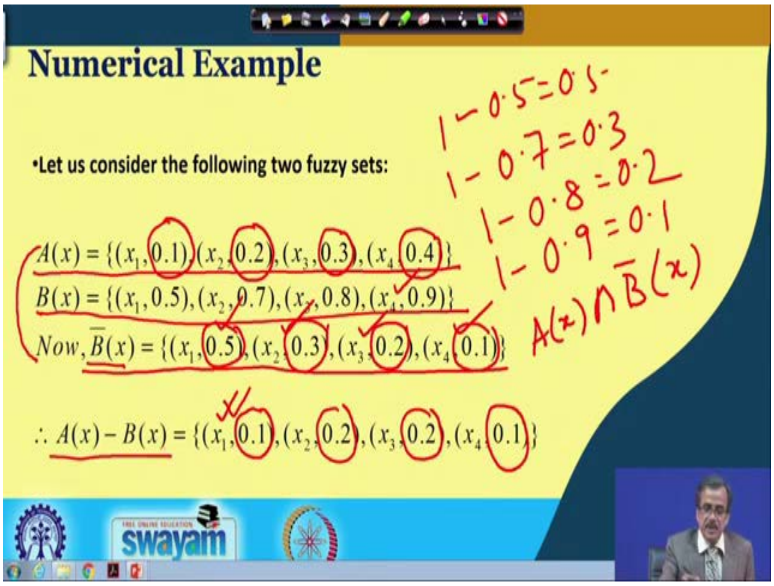

Now, supposing that your A (x) is nothing, but a discrete fuzzy sets something like this, and B (x) is a another discrete fuzzy set something like this.

So, what I do is, we try to find out its complement that is $\bar{B}(x)$ is nothing, but x_1 comma 1 minus 0.5 that is 0.5. So, I will be getting 0.5 here. Then, x_2 corresponding to this x_2 and this will be your1 minus 0.7 that is nothing, but is your 0.3, then corresponding to this particular x_3. So, this will become 1 minus 0.8. So, this is 0.2. So, I will be getting this as compliment, then corresponding to this x_4. So, this is nothing, but1 minus 0.9 and this is nothing, but 0.1. So, I will be getting this.

So, this is nothing, but the complement of B. Now, actually what we do is, we try to find out the intersection of this particular A (x) with your $\bar{B}$. Now, if I try to find out the intersection of A (x) and this particular your B bar. So, A (x) intersection. So, B bar I am just going to find out now, element-wise. So, I will have to find out what should be the mu now corresponding to this x 1. So, what I will have to do is, I will have to compare. So, this 0.1 with your 0.5 and will have to consider the minimum and that is nothing, but 0.1.

Then corresponding to x_2. So, I have got 0.2 and I have to 0.3 here. So, minimum of that is 0.2, then corresponding to x_3. So, I have got 0.3 here and 0.2 here and the minimum is 0.2 then comes your corresponding to x_4. So, I will have to compare 0.4

and 0.1 and the minimum is your 0.1. So, this is the way actually we can find out what should be your A x minus B x. So, this is the way we can find out the difference.

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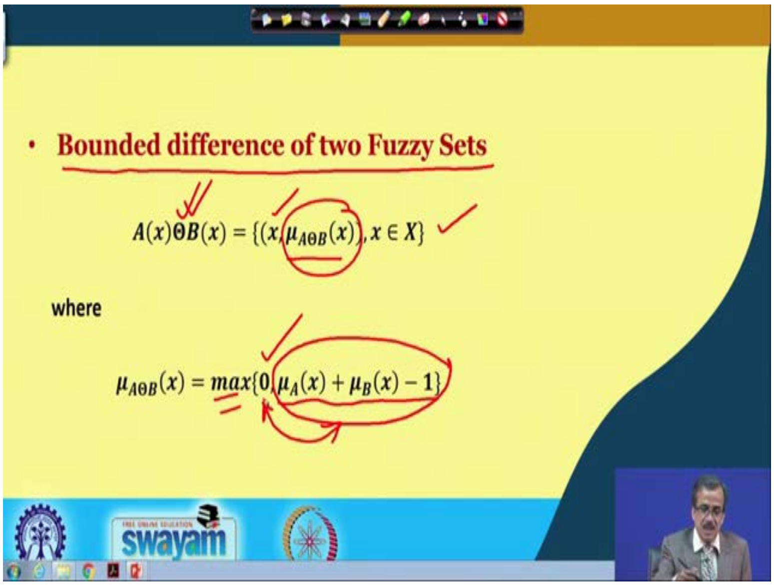

Now, then comes your the bounded difference between the two fuzzy sets. Now let us see how to find out the bounded difference of two fuzzy sets. Now, the two fuzzy sets are nothing, but A (x) and B (x) and we try to find out the bounded difference. Now, this particular symbol is actually the symbol for the bounded difference and this is nothing, but x, such that it has got the membership function value which is nothing, but a bounded difference, x belongs to capital X, now how to define this particular membership?

Now, this mu_(A bounded difference B) is nothing, but the maximum between two quantities one is 0 and another is your $\mu_A(x) + \mu_B(x) - 1$. So, what we will have to do is, will have to add $\mu_A(x)$ and $\mu_B(x)$ and subtract 1. So, this particular quantity we will have to determine and will have to compare with 0 and we will have to find out the maximum. Now, let us see, how does it work, let us solve one numerical example.

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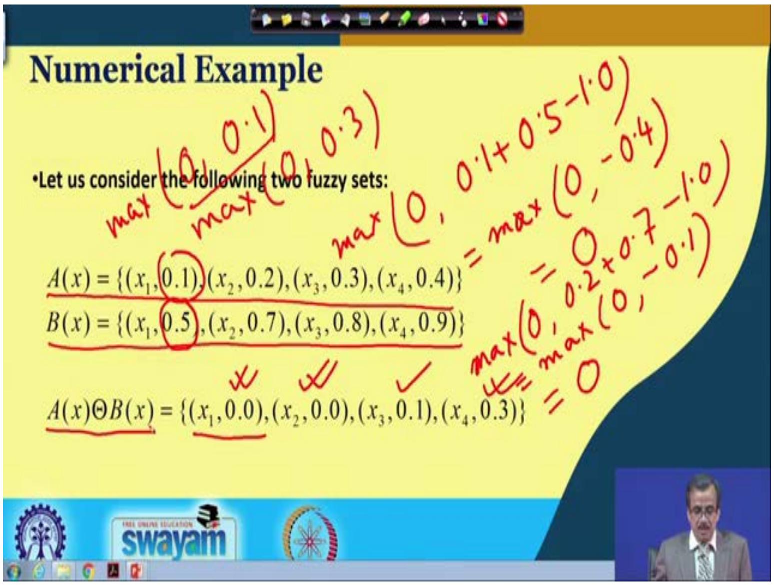

Now, if you just solve the numerical example. So, you can find out say I have got two fuzzy sets here, I have got two fuzzy sets here one is nothing, but is your A (x) another is your B (x) and we will have to find out the maximum between 0 and so, this particular μ that is your 0.1 plus 0.5 minus 1.0 and we will have to find out the maximum of this.

And, this is nothing, but your the maximum between 0 and here it is 0.6 minus 1. So, it is your minus 0.4 and the maximum between 0 and minus 0.4 is nothing, but 0. So, corresponding to this particular x_1; so I will have to write down 0.0, then corresponding to x_2. So, what we will have to do is; so you will have to find out the maximum between 0 comma 0.2, plus 0.7 minus 1.0. So, this is nothing, but the maximum between your 0.9 minus 1.0. So, this is nothing, but minus 0.1 and the maximum value is 0.

So, corresponding to x_2; so there will be 0.0, then corresponding to x_3; so what we will have is your the maximum between 0 then 0.3 plus 0.8 is a 1.1 minus 1; so, this is nothing, but 0.1 and the maximum between 0 and 0.1 is nothing, but 0.1 and corresponding to your x_4. So, this will be your maximum between 0 comma 0.4 plus 0.9 is a 1.3 minus 1.0. So, this is nothing, but 0.3 and the maximum between 0 and 0.3 is nothing, but is your 0.3.

So, we can find out this particular your A (x) bounded difference your the B (x).

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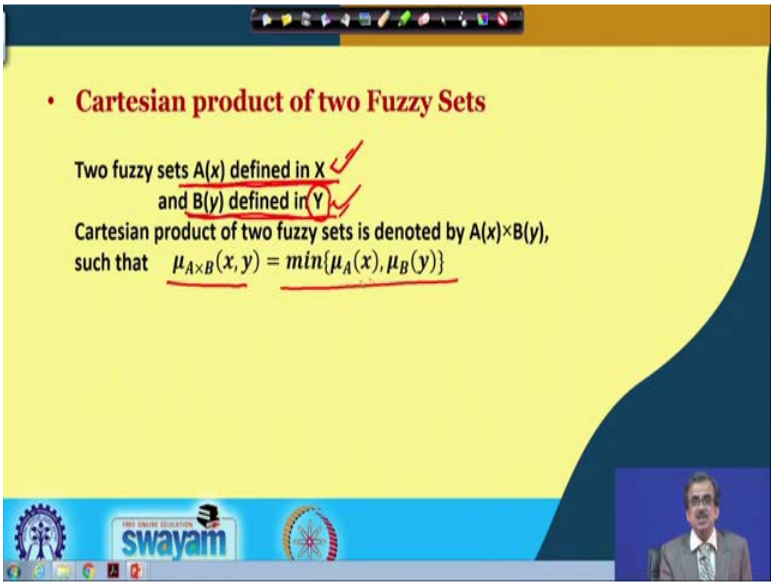

Now, let us see how to find out the Cartesian product of two fuzzy sets. Now, let us consider that I have got two fuzzy sets, say A (x) that is defined in the universe of discourse capital X, and I have got another the fuzzy set that is B (y) that is defined is another universe of discourse. Now, this A (x) might be the set of temperature in a month for the first few days. So, that is nothing, but the A (x) and B (y) could be the values or the collection of values of say the humidity of a particular place and where y is nothing, but the collection of all humidity values.

So, I have got two universe of discourses, that is the X, another is Y; one could be temperature another could be humidity and A (x) is defined in X and B (y) is defined in Y. So, how to find out the Cartesian product of these two fuzzy sets? The Cartesian product of these two fuzzy sets is defined as $\mu_{A\times B}(x,y)$ is nothing, but the minimum between $\mu_A(x)$, $\mu_B(y)$. So, what we will have to do is. So, I will have to compare. So, these two μ values and will have to consider the minimum.

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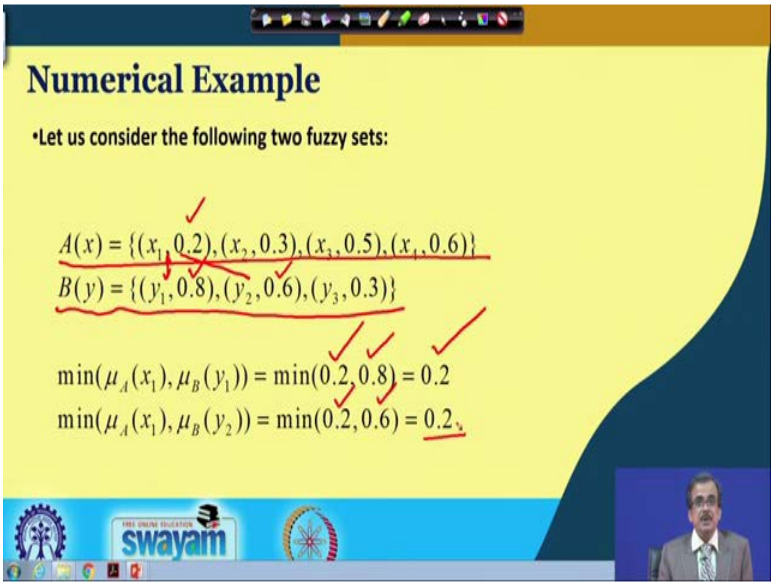

Now, let us see, how does it work. Now, supposing that say I have got two fuzzy sets one is your A (x) and I have got another fuzzy set in another universe of discourse that is say B (y). So, how to find out their Cartesian product? So, what we will have to do is. So, we will have to find out the minimum of $\mu_A(x_1), \mu_B(y_1)$. So, I will have to compare. So, that is nothing, but the minimum. So, $\mu_A(x_1)$ is 0.2 and $\mu_B(y_1)$ is 0.8. So, we compare 0.2 and 0.8 and we try to find out the minimum.

Similarly, we will have to compare. So, $\mu_A(x_1), \mu_B(y_2)$; that means, I will have to compare your the 0.2 and 0.6, and their minimum value is nothing, but 0.2.

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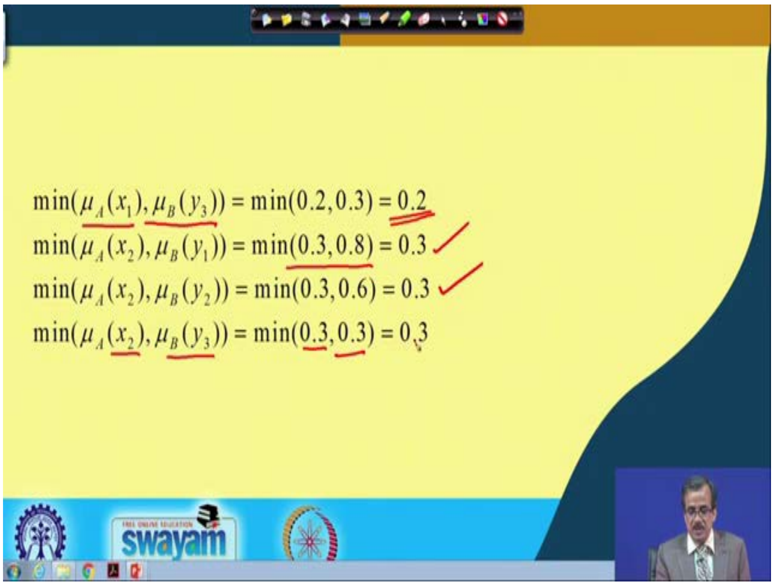

And, by following the same procedure so, I will have to find out the other elements that is your the minimum between $\mu_A(x_1),\mu_B(y_3)$ that is the minimum between 0.2 and 0.3 and that is nothing, but 0.2; next is the minimum between $\mu_A(x_2),\mu_B(y_1)$ that is nothing, but the minimum between 0.2 and 0.8 and that is nothing, but is your 0.3.

Next is the minimum between $\mu_A(x_2),\mu_B(y_2)$ and that is nothing, but the minimum between 0.3 and 0.6 and that is nothing, but 0.3 the next is the minimum between $\mu_A(x_2)$ and $\mu_B(y_3)$ and that is what the minimum between 0.3 and 0.3 and the minimum is your the 0.3.

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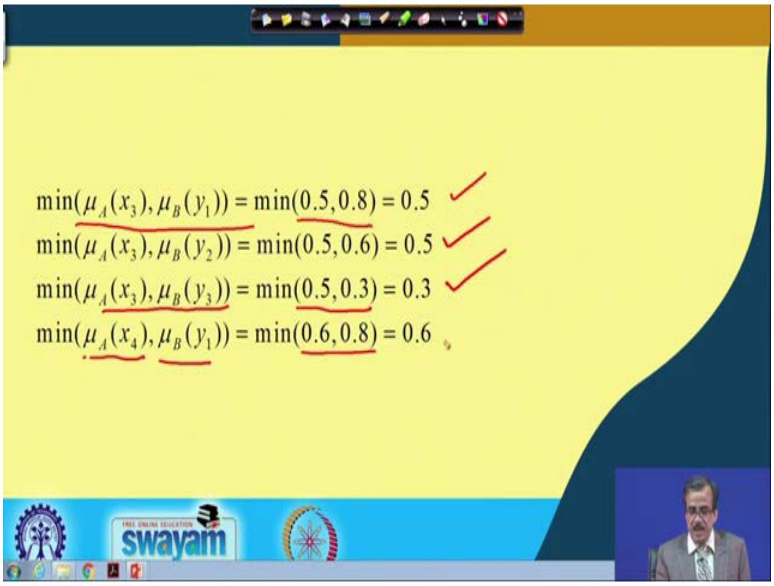

The next, we try to find out the minimum between $\mu_A(x_3)\mu_B(y_1)$ and that is nothing, but the minimum between 0.5 and 0.8 and that is 0.5.

Next is the minimum between $\mu_A(x_3)\mu_B(y_2)$ following the same procedure I will be getting 0.5 then comes your minimum between $\mu_A(x_3)$ and $\mu_B(y_3)$ and that is nothing, but the minimum between 0.5 and 0.3. So, I will be getting 0.3 here. Next is the minimum between $\mu_A(x_4)$ and $\mu_B(y_1)$ and that is nothing, but minimum between 0.6 and 0.8 and you will be getting 0.6.

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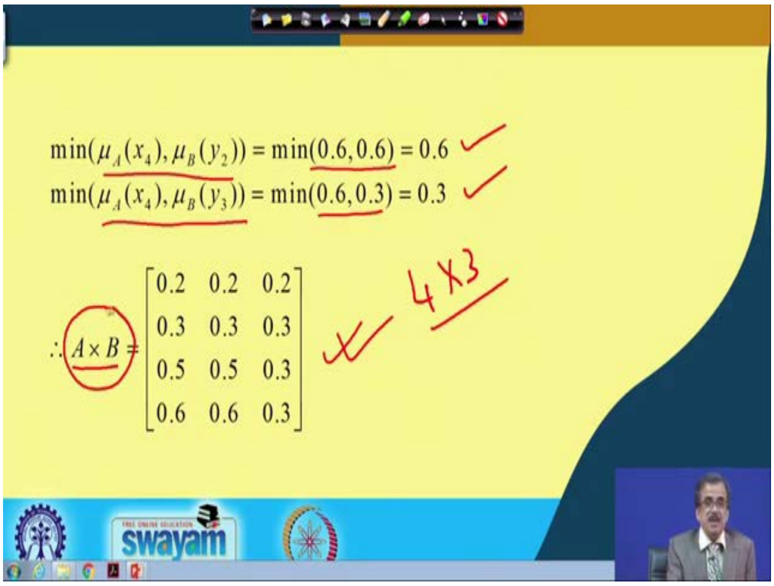

Now, here, this is the product and to find out the products, we need to find out some other elements like your the minimum between $\mu_A(x_4)\mu_B(y_2)$ is nothing, but the minimum between 0.6 and 0.6 it is 0.6.

Then comes your the minimum between $\mu_A(x_4)$, $\mu_B(y_3)$ that is nothing, but is your the minimum between 0.6 and 0.3 and will be getting 0.3. So, in the matrix form; all the elements have already been determined. So, this I can write down in the matrix form and this will look this. So, there will be your 4 rows and 3 columns. So, this is nothing, but a 4×3 matrix. So, this is actually the product of A and B.

Thank you.