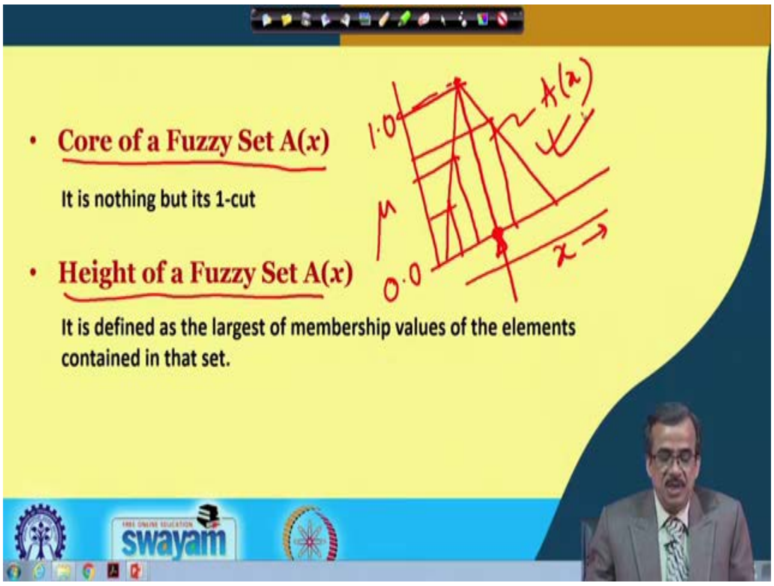
Fuzzy Logic and Neural Networks
Prof. Dilip Kumar Pratihar
Department of Mechanical Engineering
Indian Institute of Technology, Kharagpur

Lecture – 04
Introduction to Fuzzy Sets (Contd.)

(Refer Slide Time: 00:14)



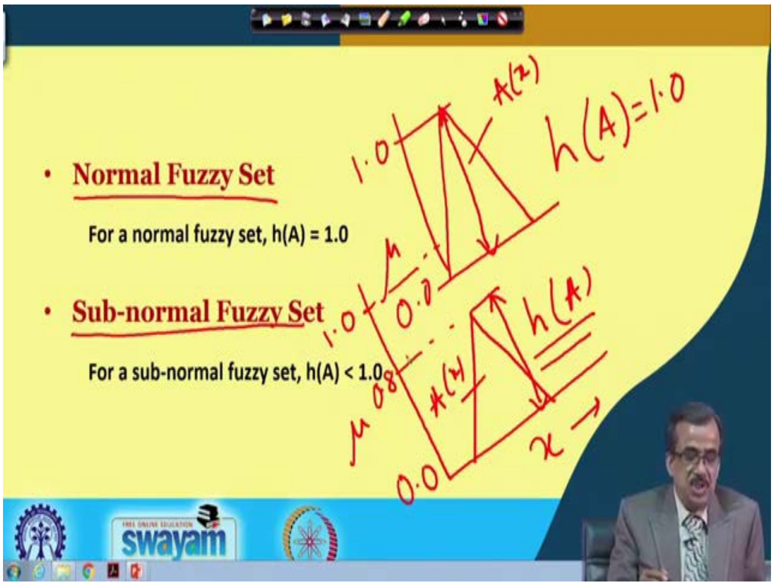
Now, we are going to define another term of a fuzzy set and that is known as the core of a fuzzy set. Now, let us see what do you mean by the core of a fuzzy set. The core of a fuzzy set is defined as your one-cut. Now, let me just try to draw one fuzzy set here, supposing that this is nothing, but a fuzzy set, that is, your A(x) and the μ is varying; so this is 0.0 and this is nothing, but 1.0 and this is the x direction.

Now, here, this is nothing, but a one-cut; one-cut means what? According to the definition. So, μ should be greater than or equal to 1, but it cannot be more than 1. So, it is exactly equal to 1. So, corresponding to this particular 1.0, that is μ equals to 1, I can find out what should be the corresponding value for this particular x, ok. So, this is actually the value for the x for which μ becomes equal to 1.0 and this is going to indicate the core of a fuzzy set. So, the core of a fuzzy set is nothing, but it is a one-cut. So, I hope, the meaning for this particular term is clear to you.

Now, I am just going to define another term, that is called the height of a fuzzy set. Now, how to define the height of a fuzzy set? Now, corresponding to the different values for this particular variable, that is, x, I can find out so what should be the value for this μ another value x. So, what should be the value of the mu for another value of x, what should be the value for this particular μ? So, corresponding to the different values for this particular x, so I can find out what should be the membership function values and out of all the membership function values, we try to find out the largest one.

So, the height of a fuzzy set is defined as the largest of all the membership function values corresponding to the different values of this particular element. Now, here, if I consider. So, here, the maximum value for the membership is nothing, but 1.0. So, height of this particular fuzzy set is nothing, but is your 1.0. So, corresponding to this particular fuzzy set, its height is nothing, but 1.0. So, this is the way actually, we define the height of a fuzzy set.

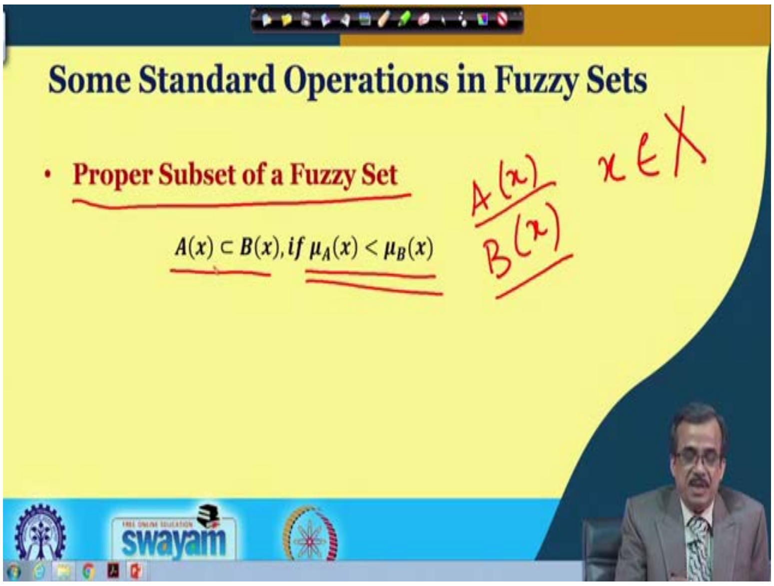
(Refer Slide Time: 03:09)



Now, I am just going to define another term, that is called the normal fuzzy set. Now, a fuzzy set is called normal, if its height is found to be equal to 1.0, now let me draw one membership function distribution and this is nothing, but a fuzzy set. So, this is A(x) is the fuzzy set and μ is say 0.0 and here, it is 1.0. Now, what is the height here? The height is nothing, but the height of this particular fuzzy set and here, the height of the fuzzy set is nothing, but 1.0 and this is a normal fuzzy set. Now, there is a another concept that is called the sub-normal fuzzy set.

Now, let me take another example, now supposing that I am drawing a fuzzy set here and this is the μ, say μ is equal to 0.0 and here, I have got 1.0. Now, supposing that I have got a fuzzy set something like this and if I can say and this is the x. Now, if I try to find out the height, this is nothing, but the height of this particular fuzzy set, if this is your A (x), ok. Now, here so this particular height is less than 1.0 because a 1.0 is here and this particular height could be around say 0.8 or 0.7 something like this, ok. Now, here the height of the fuzzy set is less than 1 and this is what you mean by the sub-normal fuzzy set.

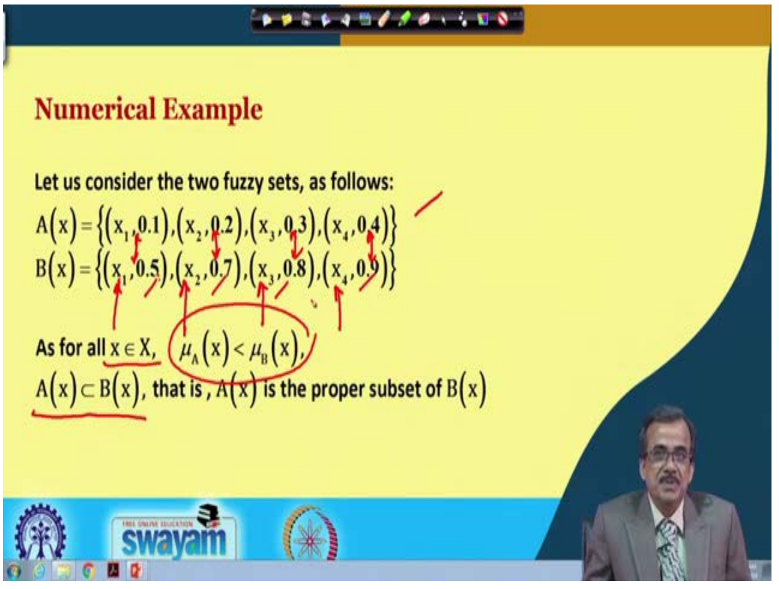
(Refer Slide Time: 04:57)



So, we have defined the normal fuzzy sets and the sub-normal fuzzy sets. So, we have defined a few terms related to the fuzzy sets and now, we are going to concentrate on some standard operations, which are very frequently used in fuzzy sets and let me try with the first operation, that is called the proper subset of a fuzzy set. Now, how to define the proper subset of a fuzzy set? Supposing that I have got two fuzzy sets say A(x) and another is your B(x) and these are defined in the universe of discourse or say x belongs to capital X.

Now, I am just going to compare these two fuzzy sets. And, I am just going to say how to carry out this particular operation, that is called the proper subset of a fuzzy set or how to declare that this particular fuzzy set is a proper subset of another fuzzy set. Now, here the set A(x) will be called the proper subset of B(x), if $\mu_A(x)$ becomes a less than $\mu_B(x)$. So, if this particular condition holds good, we declare that A(x) is a proper subset of this particular B(x). Now, I am just going to take the help of one numerical example.

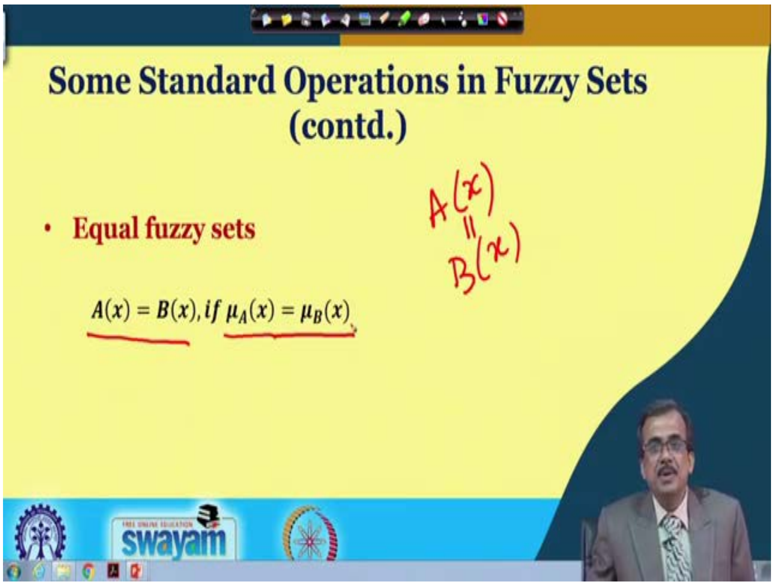
(Refer Slide Time: 06:32)



Now, supposing that I have got two fuzzy sets, two discrete fuzzy sets: one is A(x) is nothing, but x_1 comma 0.1, x_2 comma 0.2, x_3 comma 0.3, x_4 comma 0.4 and another that is B(x) x_1 comma 0.5, x_2 comma 0.7, x_3 comma 0.8, and x_4 comma 0.9. Now, I am just going to compare. Now, for all x belonging to capital X, there is the universe of discourse; now if I compare the element-wise, their μ values for example, corresponding to x_1. So, if I compare these two corresponding to x_2, if I compare these two then corresponding to x_3, if I compare these two then corresponding to x_4, if I compare these two.

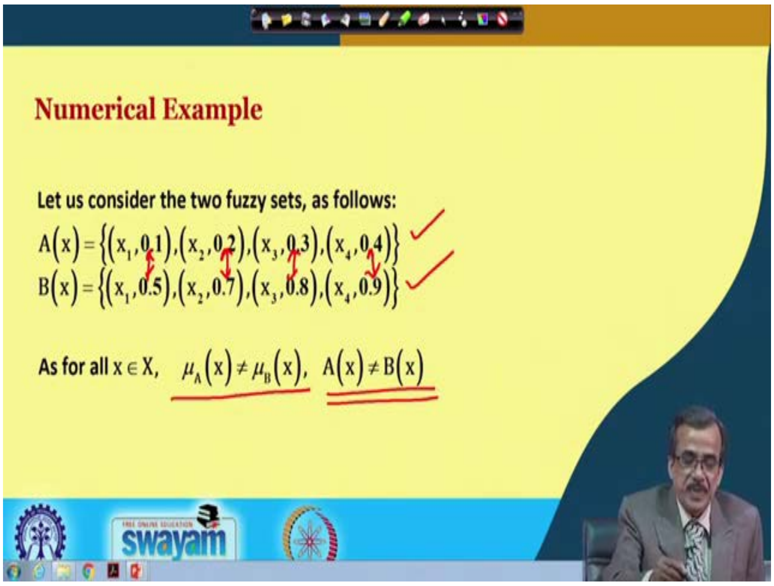
So, we can observe that $\mu_A(x)$ is less than $\mu_B(x)$ because 0.1 is less than 0.5, 0.2 is less than 0.7, and so on. So, we can declare that this particular A(x) is a proper subset of your B(x) and because this particular condition holds good. So, this is the way, actually we can compare and declare whether a particular set is a proper subset of another set or not.

(Refer Slide Time: 08:21)



So, the next operation is actually how to declare that two fuzzy sets are equal. Now, let me take the same example, say I have got two fuzzy sets A(x) and B(x) defined in the same universe of discourse. Now, we call like A(x) is equal to B(x), if and only if $\mu_A(x)$ is found to be equal to your $\mu_B(x)$.

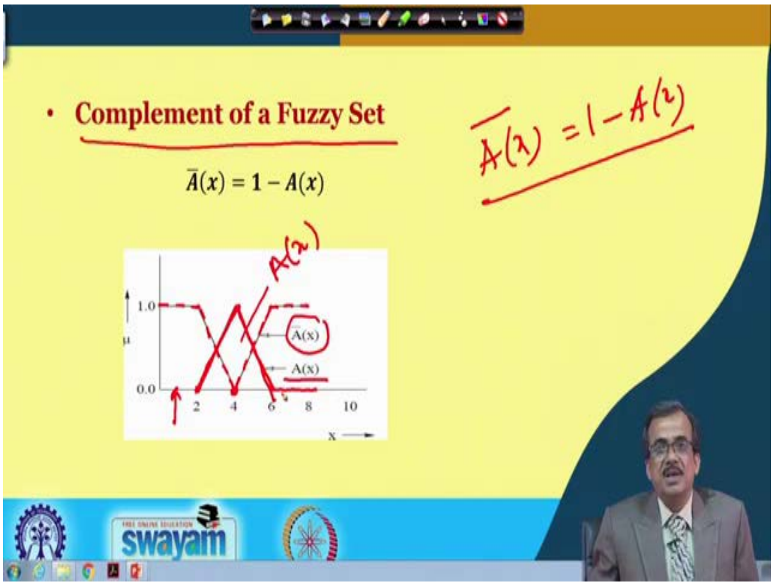
(Refer Slide Time: 08:54)



Now, what will have to do is element-wise, we will have to compare, now element-wise if we compare, the two fuzzy sets for example, say one is nothing, but is your A(x) is x_1, 0.1; x_2, 0.2; x_3, 0.3; x_4, 0.4 and B(x) is defined something like this. So, element-wise will have to compare the μ values. Now, if I compare then 0.1 and 0.5 they are not equal, 0.2 and 0.7 they are not equal, and so on. So, my decision is $\mu_A(x)$ is not equal to your $\mu_B(x)$; that means, your A(x) is not equal to the B(x).

So, the fuzzy set A(x) is not equal to fuzzy set B(x). So, this is the way actually, we can compare and declare whether the two fuzzy sets are equal or not.

(Refer Slide Time: 09:53)

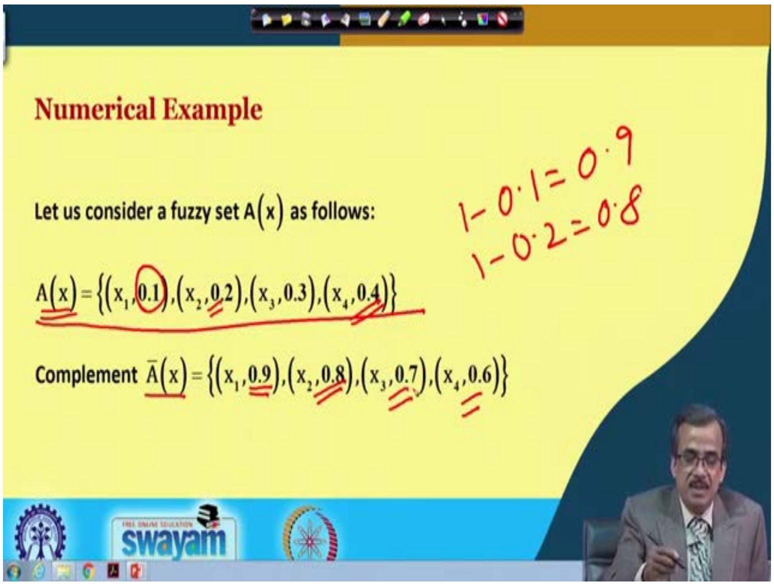


Now, I am just going to discuss another operation, which is nothing, but the complement of a fuzzy set. Now, how to determine the complement of a fuzzy set, by definition the complement of a particular fuzzy set A(x) is nothing, but is your $\bar{A}(x)$ and that is nothing, but 1 minus your A(x). So, this is the definition of the complement of a fuzzy set.

Now, supposing that I have got the fuzzy set like this, that is A(x). So, this is nothing, but the fuzzy set and its complement is nothing, but this. How to find out the complement? It is very simple, now here, this is nothing, but your A(x). Now, corresponding to this value of x, what is the value of μ? μ is equals to 0, corresponding to this, what is the value of mu is equals to 0. So, 1 minus 0 is nothing, but 1. So, I will be getting up to this, I will be getting this type of compliment, then from here to here, this μ is going to increase from 0 to 1; that means, your A(x) bar, its complement, is going to decrease starting from 1 up to 0.

Now, you concentrate here, corresponding to this value of x. So, this is the value of μ corresponding to A(x) and that is nothing, but 1. So, 1 minus 1 is 0. So, this will be the μ corresponding to its complement and starting from here, the value of μ is decreasing. So, if I concentrate on its complement, the value for the μ will be increasing from 0 to 1 and after that, the value for the μ is kept equal to 0. So, 1 minus 0 is nothing, but 1. So, the dotted is going to indicate actually the complement of the fuzzy set.

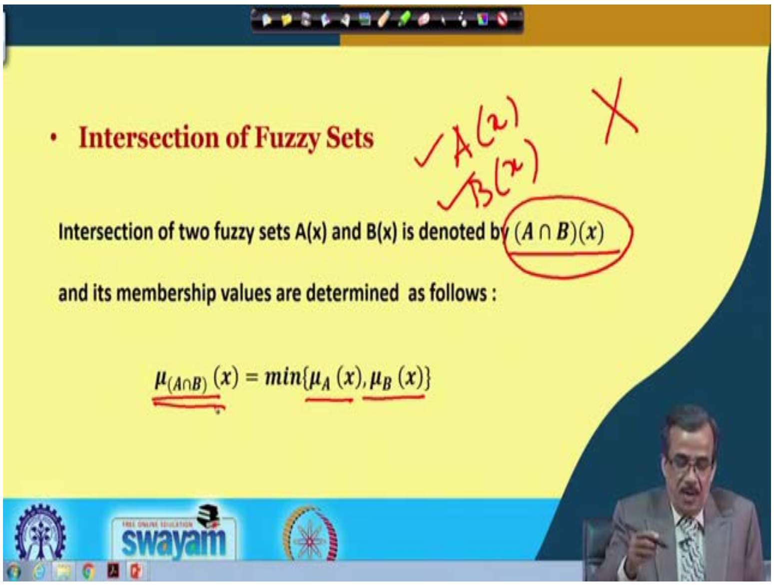
(Refer Slide Time: 12:17)



Now, I am just going to solve one numerical example just to show you like how to find out the complement of a fuzzy set.

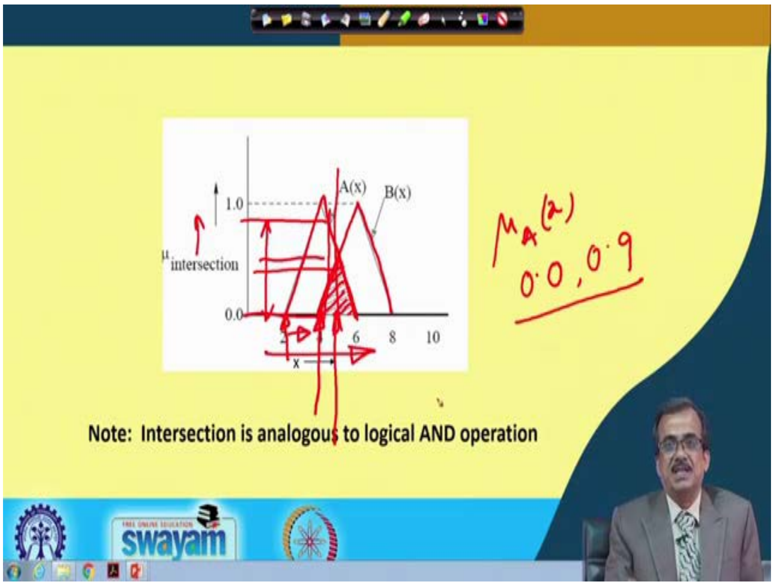
Now, supposing that I have got a discrete fuzzy set something like this. So, A(x) is nothing, but this, that is, x_1 0.1 comma x_2 0.2 comma x_3 0.3 comma x_4 0.4 is nothing, but is your A(x). Now, its complement $\bar{A}(x)$, so element-wise I will have to find out 1 minus that μ. So, here, μ is 0.1. So, 1 minus 0.1 is nothing, but 0.9. So, for its complement corresponding to x_1, the μ value will be 0.9, corresponding to x_2 it will be 1 minus 0.2 and that is nothing, but 0.8. So, I will be getting this then corresponding to x_3, I will be getting 0.7, corresponding to x_4. So, I will be getting 0.6. So, following this particular principle; we can find out the complement of the fuzzy set.

(Refer Slide Time: 13:37)



Now, I am just going to discuss the concept of intersection of two fuzzy sets, now once again let me repeat that I have got two fuzzy sets A(x) and B(x) defined in the same universe of discourse, that is capital X. Now, how to find out the intersection of these two fuzzy sets, now intersection of these two fuzzy sets is denoted by this particular symbol. So, A intersection B(x) is nothing, but intersection of the two fuzzy sets A(x) and B(x). Now, to define this particular intersection, so what we do is, we try to find out what should be the μ value and we will have to compare, in fact, the μ values. So, mu A intersection B(x) is nothing, but the minimum between $\mu_A(x)$ and $\mu_B(x)$. So, the two μ values will have to compared and we will have to find out its minimum and that is going to indicate the intersection of the two fuzzy sets.

(Refer Slide Time: 14:57)

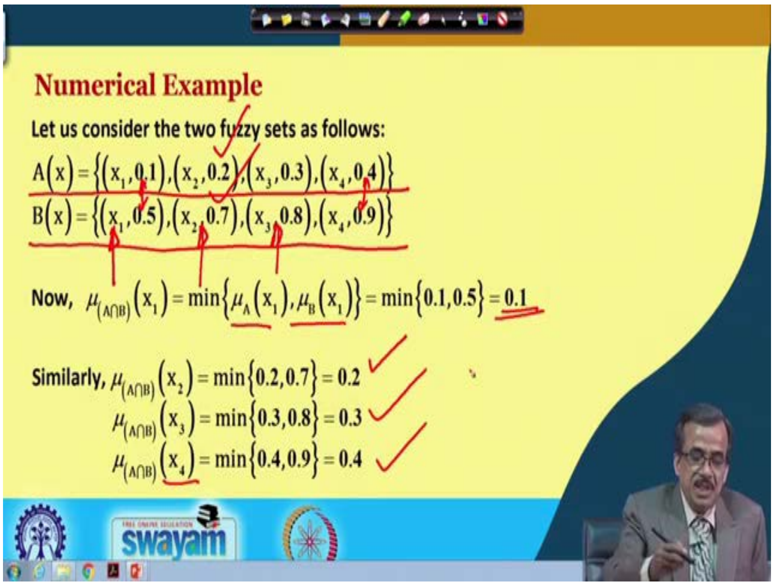


Now, I am just going to take actually one numerical example just to show you like how to determine the intersection of two fuzzy sets, but before that, let me try to find out this. Now, as I told that this is nothing, but A(x) one fuzzy set and this is another fuzzy set, that is B(x) defined on the same universe of discourse and this is actually the direction of x and this is the μ, now the moment I am here. So, corresponding to this A(x), I have got $\mu_A(x)$ and that value is what? That is your 0 here, but $\mu_B(x)$, the B(x) has not yet been started. So, I am just going to proceed in this particular direction. The moment, I come here, so I have got the membership function value corresponding to this particular A(x). So, this is nothing, but the membership function value corresponding to this A(x) and corresponding to this B(x) the membership function value is equal to 0.0 and we try to consider the minimum.

So, the minimum between 0.0 and a particular value, which is very near to say 0.9. So, the minimum is 0. So, I am just going to consider that μ corresponding to this is nothing, but 0 its intersection is nothing, but 0, the moment I consider another value for this particular x. So, corresponding to A(x), I will be getting some μ corresponding to the B(x). I will be getting another μ you compare and you consider the minimum. So, ultimately, I will be getting this type of the area which is the common to both the fuzzy sets.

So, by intersection actually, what we mean is, we try to mean the common area between the two fuzzy sets. So, this is actually the common between the two fuzzy sets and this is nothing, but the intersection of the two fuzzy sets and this is similar to the logical AND operation. So, by AND operation actually, we always try to consider the minimum and this is also known as the mean operation or the mean operator. So, AND is nothing, but the mean operator and that is nothing, but the intersection, the concept of intersection of two fuzzy sets.

(Refer Slide Time: 17:31)

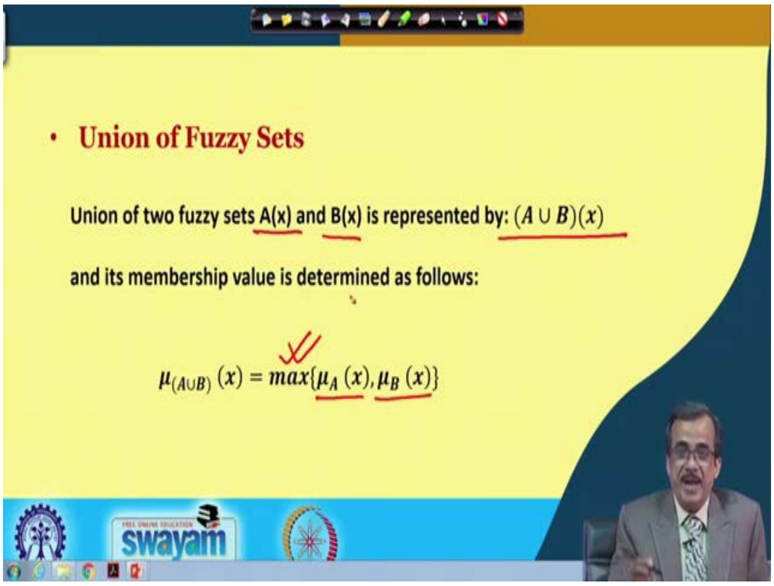


Now, I am just going to solve one numerical example, now supposing that I have got two fuzzy sets, two discrete fuzzy sets one is A(x) is nothing, but x_1 comma 0.1 comma x_2 comma 0.2 comma x_3 comma 0.3 comma x_4 comma 0.4. So, this is nothing, but the fuzzy set A(x) and I have got another fuzzy set which is nothing, but the B(x). Now, what we do is, we concentrate element-wise first you try to concentrate on this particular the x_1. So, $\mu_{(A\cap B)}(x_1)$ is nothing, but the minimum between $\mu_A(x_1)$ and $\mu_B(x_1)$.

So, what you do is, these two values, we try to compare, that is the minimum between 0.1 and 0.5 and the minimum is nothing, but 0.1. Now, next, we try to concentrate on x_2. So, the $\mu_{(A\cap B)}(x_2)$ is nothing, but the minimum between 0.2 and 0.7 and we will be getting 0.2, next we try to concentrate on x_3. So, $\mu_{(A\cap B)}(x_3)$ is nothing, but the minimum between 0.3 and 0.8 and that is nothing, but 0.3, next we try to find out the

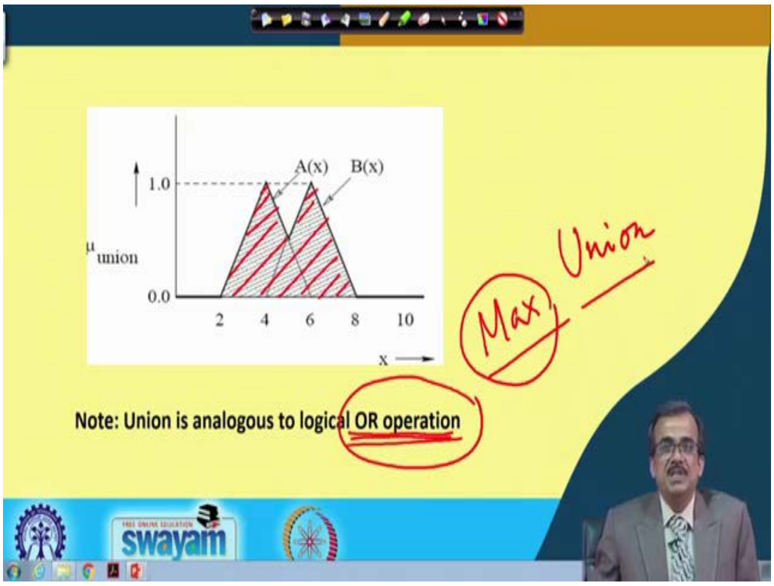
$\mu_{(A \cap B)}(x_4)$ and that is nothing, but the minimum between 0.4 and 0.9 and you will be getting 0.4. So, this is the way actually, we can find out your intersection.

(Refer Slide Time: 19:36)



Then, comes the concept of the union of two fuzzy sets. Now, let me once again consider the two fuzzy sets like A(x) and B(x) define in the same universe of discourse and their union is represented by this particular symbol, that is, $(A \bigcup B)(x)$, such that its membership function value, that is, $\mu_{(A \bigcup B)}(x)$ is nothing, but the maximum between $\mu_A(x)$ comma $\mu_B(x)$. So, what we do is, we try to compare the membership function values element-wise and we are going to consider the maximum.

(Refer Slide Time: 20:27)

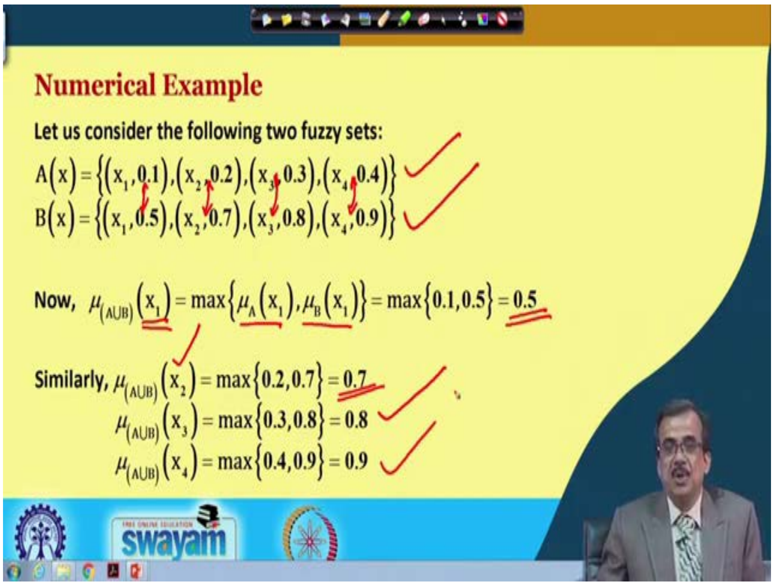


So, I am just going to take some example. Now, here you can see that I have got one membership function distribution and one fuzzy sets like A(x) and another fuzzy sets like B(x) defined in the same universe of discourse.

Now, I am just varying the value of x, the moment I am here, I have got the μ corresponding to your say A(x) is nothing, but 0 but corresponding to the B(x), suppose that this is absent. So, we consider this thing as the maximum, now the moment I am here, corresponding to this A(x) might be this is the membership function value, say 1.0 and corresponding to B(x) the membership function value is 0.0 and if I just compare the maximum will be 1.0. So, I will have to consider up to this. Next, the moment whenever I am here, so corresponding to this particular A(x), I will be getting some μ value and corresponding to the B(x), I will be getting some μ value, and we will have to consider the maximum.

Now, if I follow this particular method, then there is a possibility that I will be getting this shaded portion as actually the union of these two fuzzy sets. So, by union, we mean actually the AND operator and this OR, sorry I am sorry. So, this is the OR operator and this OR operator is nothing, but is actually the max operator. So, we try to find out the maximum between the two μ values and that will give you the concept of union and that is nothing, but is your OR operator. So, OR operator is nothing, but the max operator and that is nothing, but the union of the two fuzzy sets.

(Refer Slide Time: 22:42)

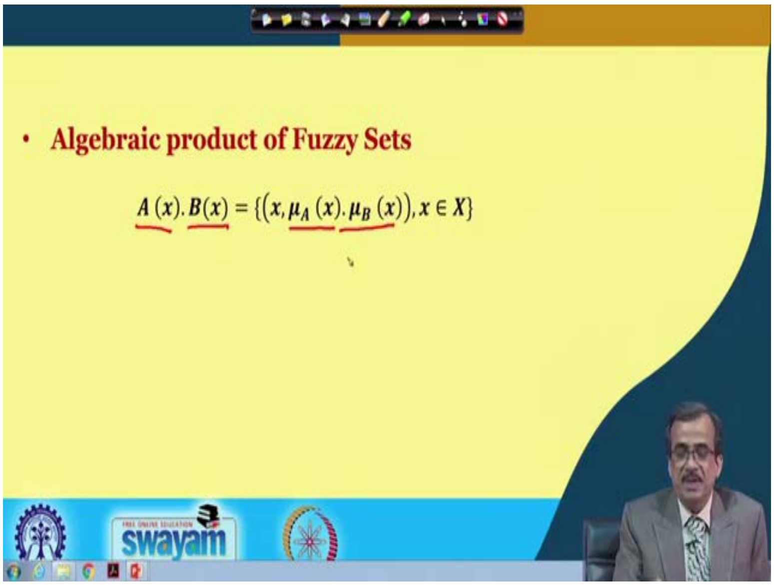


Now, I am just going to just solve one numerical example just to give you the concept of this particular union of two fuzzy sets.

Now, let me consider once again that I have got two fuzzy sets like one is A(x) and another is B(x) and these particular fuzzy sets are nothing, but the discrete fuzzy sets. So, element-wise we have got the membership function values, that is the μ values, now if I want to find out that $\mu_{(A \cup B)}(x_1)$; that means, corresponding to this particular x_1. So, I am just going to compare these two μ values, that if your $\mu_A(x_1)$ and $\mu_B(x_1)$ and we try to find out the maximum and; that means, we try to find out the maximum between 0.1 and 0.5 and the maximum value will be 0.5. Similarly corresponding to this particular x_2, we are trying to compare the two μ values.

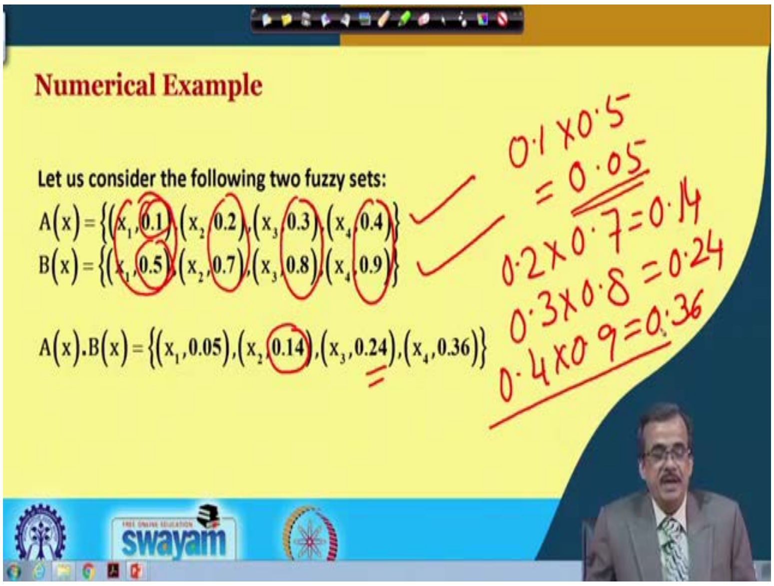
So, $\mu_{(A \cup B)}(x_2)$ is nothing, but the maximum between 0.2 and 0.7 and the maximum value is 0.7 then corresponding to x_3. So, $\mu_{(A \cup B)}(x_3)$ is nothing, but the maximum between 0.3 and 0.8. So, I will be getting 0.8, then corresponding to this particular x_4. So, $\mu_{(A \cup B)}(x_4)$ is nothing, but the maximum between your 0.4 and 0.9 and this is nothing, but 0.9. So, I can find out the union of these two fuzzy sets.

(Refer Slide Time: 24:40)



Then, comes the concept of algebraic products of two fuzzy sets, now supposing that I have got say two fuzzy sets, one is A(x) another is B(x). So, by algebraic product actually, we mean another set whose membership function values will be nothing, but a $\mu_A(x)$ multiplied by your $\mu_B(x)$. So, this is actually what you mean by algebraic product of two fuzzy sets, now I am just going to take one numerical example just to make it clear.

(Refer Slide Time: 25:18)



Now, supposing that I have got two fuzzy sets one is your A(x) another is your B (x). So, what I do is, element-wise, the μ values we simply multiply.

So, what I do is corresponding to this particular your x_1. So, what I do is your so, we multiply 0.1 multiplied by 0.5. So, this is nothing, but 0.05. So, this $A(x).B(x)$ is nothing, but x_1 comma 0.05.

Similarly, corresponding to this x_2, I will have to multiply 0.2 by 0.7 so, this will become 0.14. So, I will be getting 0.1 4, then corresponding to this particular x_3. So, I will be getting 0.3 multiplied by 0.8 and this is nothing, but is your 0.24, so you will be getting 0.24. Then, corresponding to this particular x_4, I will be getting 0.4 multiplied by 0.9, so I will be getting 0.36. So, by following this particular method, you can find out the product of two fuzzy sets.

Thank you.