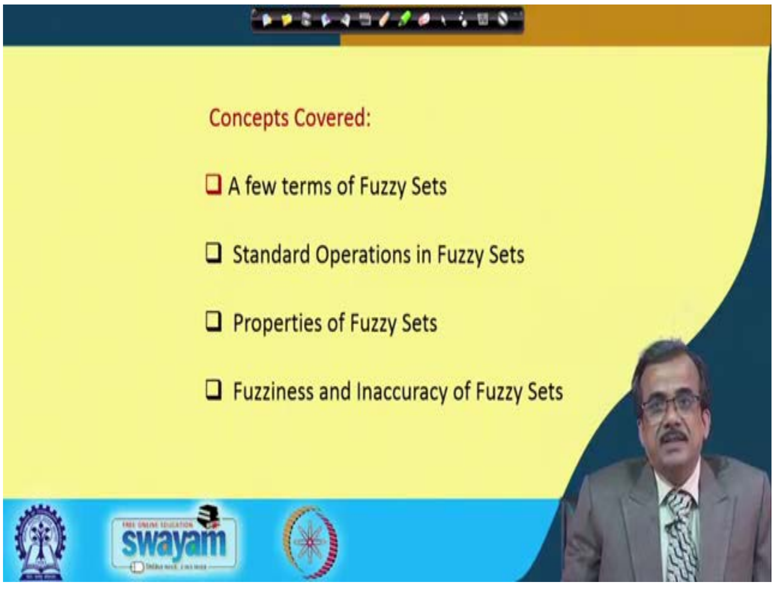
Fuzzy Logic and Neural Networks
Prof. Dilip Kumar Pratihar
Department of Mechanical Engineering
Indian Institute of Technology, Kharagpur

Lecture – 03
Introduction to Fuzzy Sets (Contd.)

We are discussing the grammar of fuzzy sets. So, we will be continuing with the topic introduction to fuzzy sets.

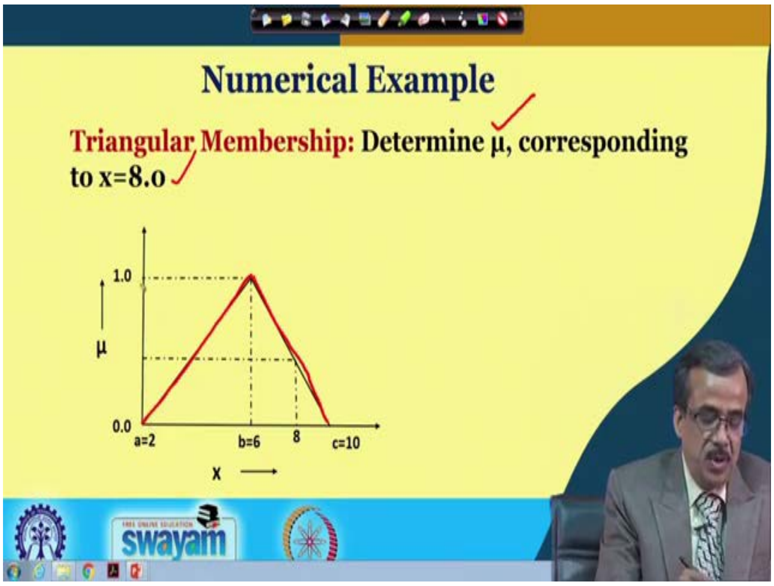
(Refer Slide Time: 00:27)



Now, at the beginning of this lecture. So, we are going to discuss how to solve some numerical examples related to determination of membership value for different types of membership function distribution and after that we are going to define a few terms related to fuzzy sets, some standard operations used in fuzzy sets will be discussed.

The properties of fuzzy sets will be explained and at the end. Two terms, namely fuzziness and inaccuracy of fuzzy sets will be defined and we will be solving some numerical examples like how to determine the fuzziness and inaccuracy of fuzzy sets.

(Refer Slide Time: 01:24)

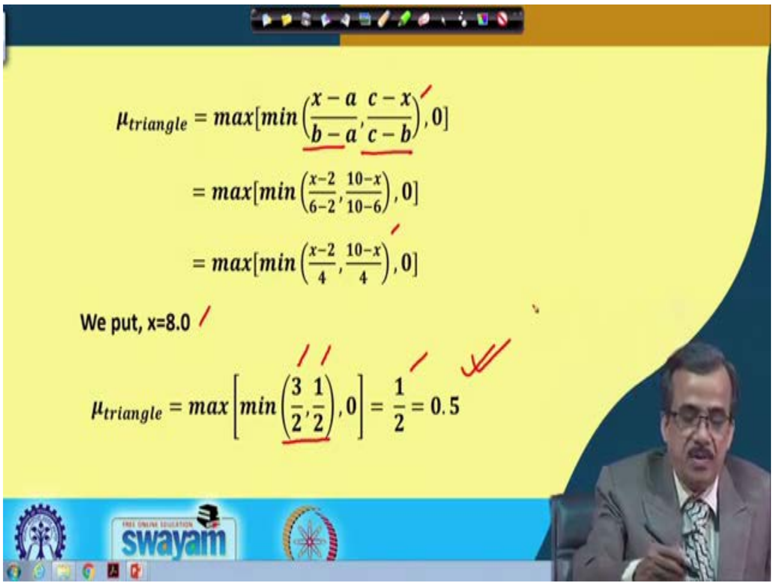


Now, to start with the numerical examples related to determination of the membership value, that is, μ, let me start here with a triangular membership function distribution and our aim is to determine the membership value, that is, μ corresponding to a particular value of the variable say x equals to 8.0.

Now, here, I am just going to consider one triangular membership function distribution. So, this is nothing, but the triangular membership function distribution and if you see now, here, I have written a equals to 2, b equals to 6 and c equals to 10. Now, using these a, b and c, mathematically I can define. So, this particular triangular membership function distribution, I have already discussed in the last lecture. Now, along this axis, it is the variation of μ, and μ varies from 0.0 to 1.0.

Now, here let us see how to determine the value of μ corresponding to a particular value of x and here, we have assumed x equals to 8.0.

(Refer Slide Time: 02:50)

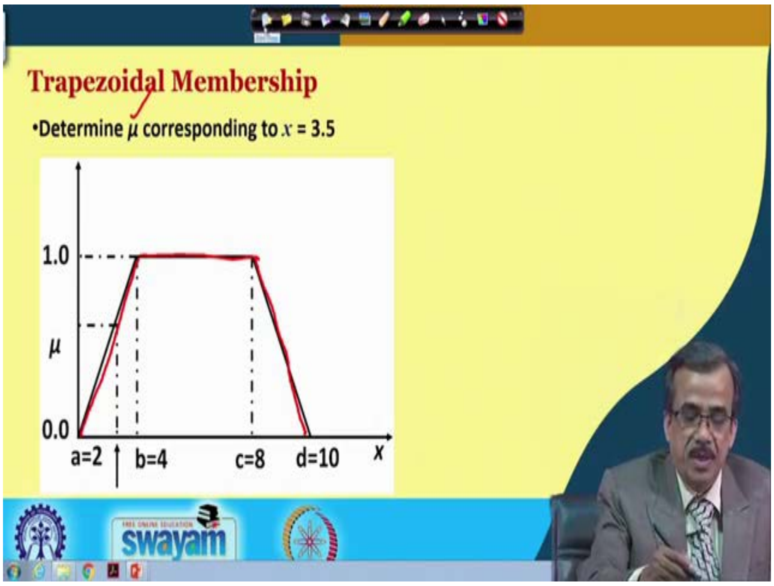


Now, this $\mu_{triangle}$ is nothing, but maximum of the minimum between x minus a divided by b minus a comma c minus x divided by c minus b. So, what you will have to do is, you will have to find out the minimum between these two, and after that you will have to compare this particular minimum and 0, and find out the maximum. Now, here, if I just insert the values for this a, b and c. So, by substituting the values for a, b and c we get the maximum between the minimum of x minus 2 divided by 6 minus 2, comma 10 minus x divided by 10 minus 6, comma 0. Here, a equals to 2, b equals to 6 and c equals to 10.

Now, if you simplify. So, this can be written as maximum of the minimum between x minus 2 divided by 4 comma 10 minus x divided by 4 comma 0. Now, if you put x equals to 8, then we will be getting $\mu_{triangle}$ equals to maximum of the minimum between. So, x equals to 8 if I put. So, this will become 6 by 4 and that is nothing, but 3 by 2 and here, if I put x equals to 8, this will become 2 by 4 is nothing, but 1 by 2. So, what you can do is. So, we can find out the minimum between 3 by 2 comma 1 by 2 and the minimum is your half and now, we will try to find out the maximum between half and 0 and that is nothing, but half. So, this $\mu_{triangle}$ is coming to be equal to 0.5.

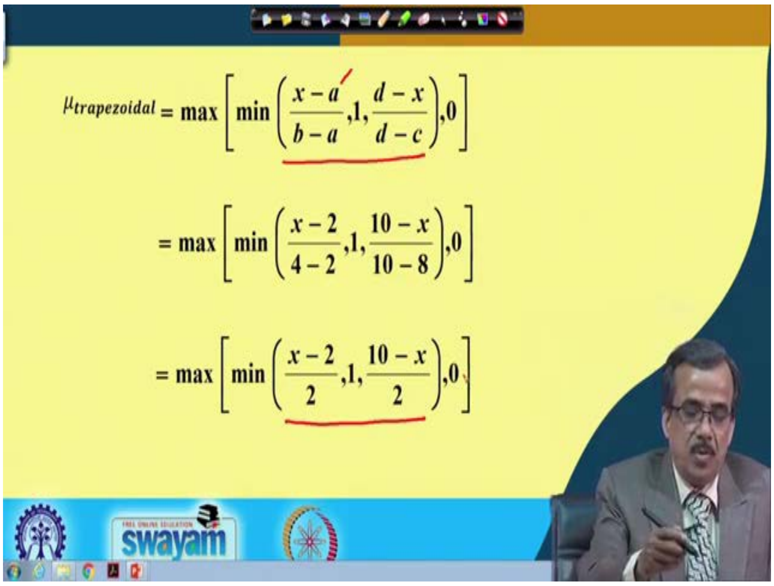
So, this is the way corresponding to a particular value of x, we can find out, what should be the value for the membership for the triangular membership function distribution.

(Refer Slide Time: 05:02)



Now, we are going to discuss what happens in case of trapezoidal membership function distribution. So, once again our aim is to determine the value of this particular μ, that is the membership function value corresponding to x equals to 3.5. Now, this is nothing, but the trapezoidal membership function distribution. So, this is the trapezoidal membership function distribution. So, here a is kept equal to 2, b is kept equal to 4, c is kept equal to 8 and d is equal to 10. Now, using this information of a, b, c and d, mathematically you can express $\mu_{trapeziodal}$.

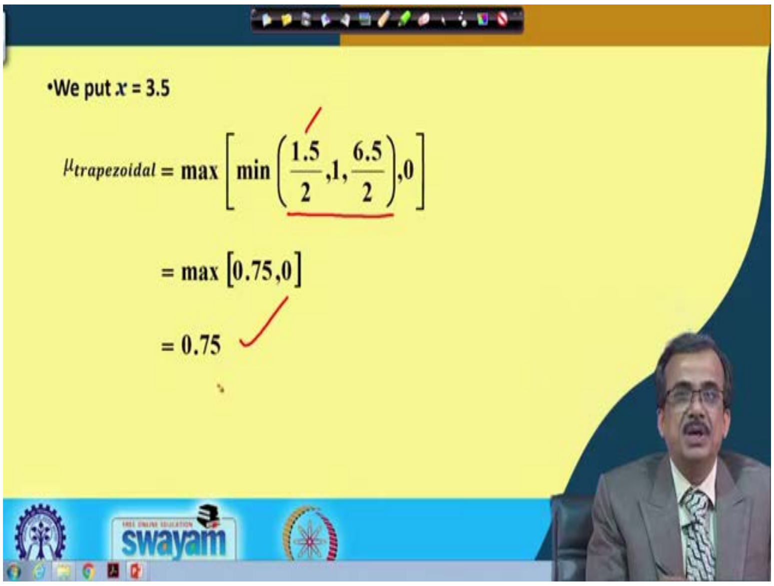
(Refer Slide Time: 05:53)



So, if you see that $\mu_{trapeziodal}$, that particular mathematical expression, this will become something like this. So, $\mu_{trapeziodal}$ is nothing, but the maximum between two. Now, the first is actually, we will have to find out the minimum among your x minus a divided by b minus a comma 1 comma d minus x divided by d minus c. So, we will have to find out the minimum among these three and then, will have to compare with 0 and will have to find out the maximum.

Now, if we substitute the values for these a, b, c and d, we will be getting the maximum between the minimum among x minus 2 divided by 4 minus 2 comma 1 comma10 minus x divided by 10 minus 8 comma 0. Now so, if you simplify this, it will become the minimum among x minus 2 divided by 2 comma 1, 10 minus x divided by 2 and we compare the minimum among these three and this 0, and we will try to find out the maximum and whatever you get. So, let me just try to see.

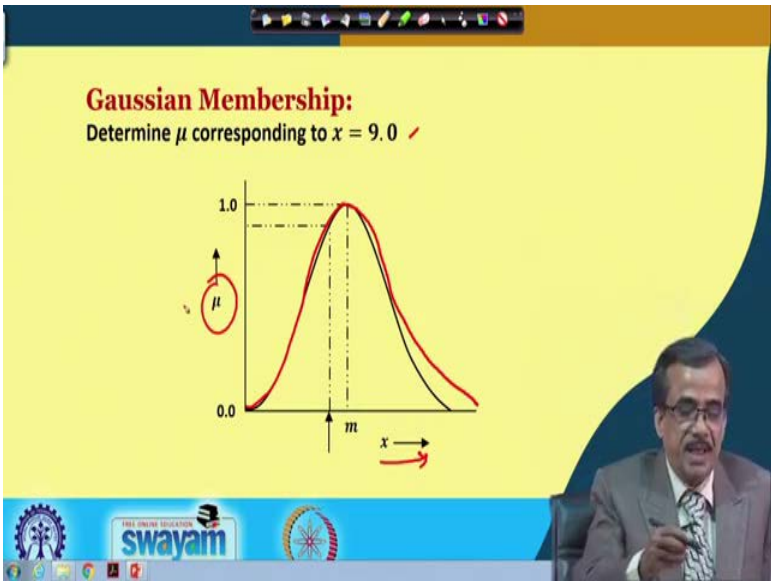
(Refer Slide Time: 07:18)



If we put x equals to 3.5, we will be getting $\mu_{trapeziodal}$ is nothing, but the maximum between these two, and before that, we will have to find out the minimum among these three, that is your that is your 1.5 divided by 2 comma 1 comma 6.5 divided by 2.

So, the minimum among these three is nothing, but 0.75 that is your 1.5 divided by 2 and now, we will have to compare 0.75 and 0 and will have to find out the maximum and that is nothing, but 0.75. So, you can find out the value for this $\mu_{trapeziodal}$ and that is becoming equal to 0.75.

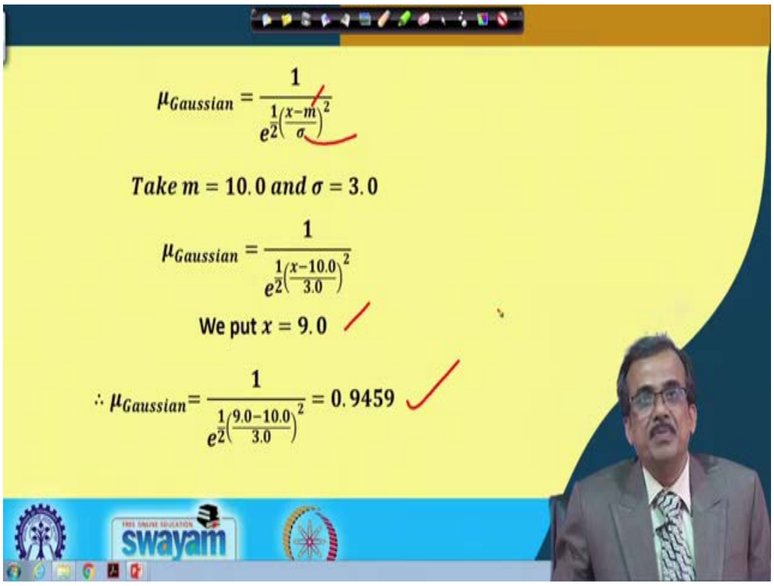
(Refer Slide Time: 08:12)



Now, then we are going to concentrate on the Gaussian membership function distribution and our aim is to determine membership value. So, this particular μ corresponding to your x equals to 9.0. Now this is nothing, but the Gaussian membership function distribution.

So, if you see, this is the Gaussian membership function distribution. So, this is x direction and this is the μ and we know the mathematical expression for this particular Gaussian membership function distribution.

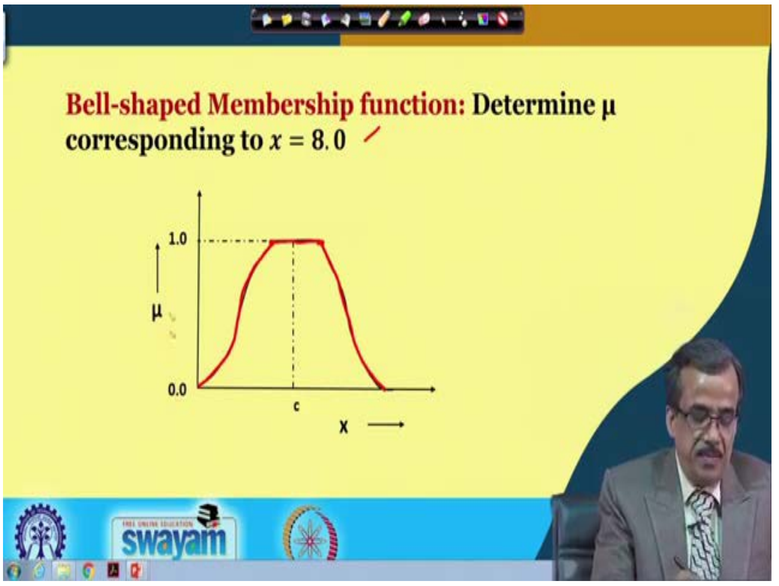
(Refer Slide Time: 08:44)



As the mathematical expression for this is nothing, but $\mu_{Gaussian}$ is 1 divided by e raised to the power half into x minus m divided by sigma square and m is nothing, but your the mean of the Gaussian distribution and sigma denotes the standard deviation. Now we are going to substitute the values for your m equals to 10.0 and the standard deviation is equal to 3.0.

So, we will be getting this particular expression that is mu Gaussian is nothing, but 1 divided by e raised to the power half x minus 10 divided by 3 square, and now we put x equals to 9.0 then $\mu_{Gaussian}$ is nothing, but 1 divided by e raised to the power half. So, x equals to 9.0 minus 10.0 divided by 3.0 square and if you simplify you will be getting 0.9459. So, we can find out, what should be the value for this membership function corresponding to the Gaussian distribution.

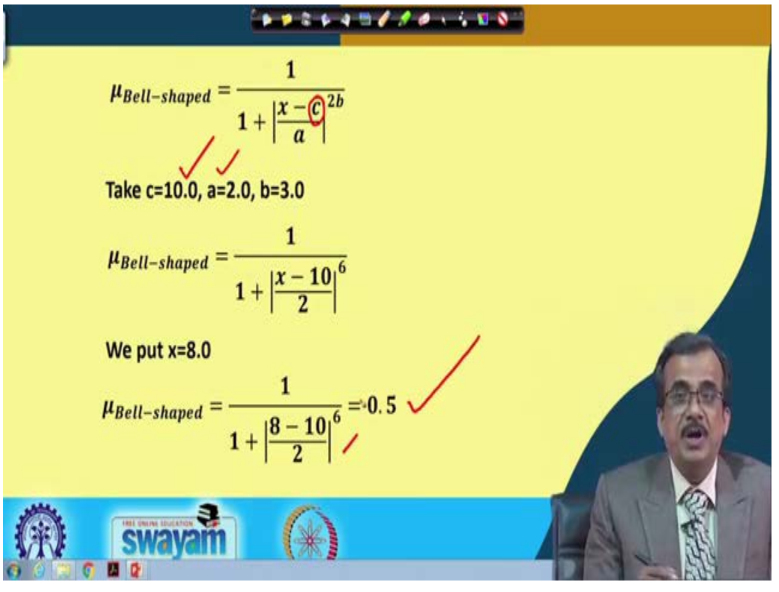
(Refer Slide Time: 10:02)



So, this is how to tackle the Gaussian distribution, Now, we are going to concentrate on another membership function distribution, that is called bell-shaped membership function a distribution and our aim is to determine the value for μ corresponding to say x equals to 8.0.

Now, let us see how to do it and this is nothing, but is your bell-shaped membership function distribution. So, this is the distribution and μ varies from 0 to 1. So, let us try to find out the value for μ corresponding to x equal to 8.0.

(Refer Slide Time: 10:44)

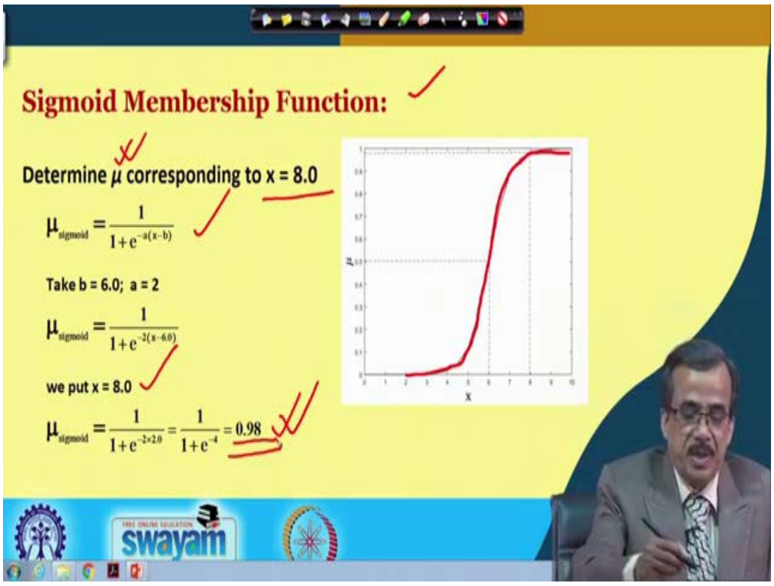


Now, this is the mathematical expression, which I have already discussed that $\mu_{Bell-shaped} = \frac{1}{1+|\frac{x-c}{a}|^{2b}}$ and this I have already discussed that the meaning for this a, b and c. Now, here, we take c equals to 10.0 and that is actually nothing, but the centre of this distribution, a indicates the spread of this distribution and let be consider, a is equal to your say 2.0 and b is nothing, but a 3.0, we have assumed.

Now, we put c is equal to 10, a is equal to 2, b is equal to 3.0. So, we will be able to find out. So, this $\mu_{Bell-shaped}$ is nothing, but 1 divided by say 1 plus the mod value of 8 minus 10 divided by 2 raised to the power 6. Now 8 minus 10 is minus 2 divided by 2 is equal to minus 1, the mod value of that is nothing, but plus 1, it is raised to the power of 6. So, this will become equal to 1. So, 1 divided by 2 and this is going to give rise your 0.5.

So, this $\mu_{Bell-shaped}$ is equal to 0.5, corresponding to x equals to 8.0. Now, this is the way actually, we should be able to find out.

(Refer Slide Time: 12:37)

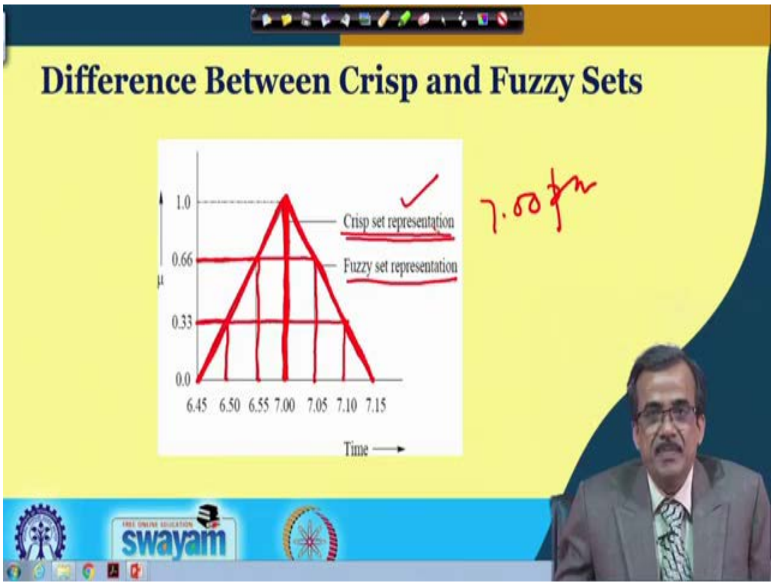


The value for the membership function distribution. Now, I am just going to concentrate on another very popular membership function distribution that is known as sigmoid membership function distribution. Now, this sigmoid membership function distribution is mathematically expressed as follows. So, $\mu_{sigmoid} = \frac{1}{1+e^{-a(x-b)}}$. Now, our aim is to determine the value for this μ corresponding to x equal to 8.0 now.

So, this is actually the distribution for this sigmoid membership function. Now, corresponding to your b equals to 6.0 and a equals to 2, I will be getting the expression for $\mu_{sigmoid}$, that is nothing, but 1 divided by 1 plus e raised to the power minus a, (a equals to 2) multiplied by x minus b, b is equal to 6. Now, here, if you substitute the value for x that is your x equals to 8.0, you will be getting $\mu_{sigmoid}$ is nothing, but 1 divided by e raised to the power minus 2 multiplied by 2.0. So, this is nothing, but 1 divided by 1 plus e raised to the power minus 4 and that is nothing, but 0.98, corresponding to x equals to 8.0.

So, we are able to find out the value for this particular μ for the sigmoid membership function distribution and that is coming to be equal to 0.98. So, this is the way actually we can determine the membership function value for various distributions used for the representing the fuzzy sets.

(Refer Slide Time: 14:44)



So, till now actually we have discussed, we have defined, the concept of fuzzy sets and the concept of your classical sets. Now, to recapitulate the definitions and the difference between the classical set that is crisp set and the fuzzy set. Now, let me take another example, now this particular example actually is going to tell us, what is the difference between the fuzzy sets and the crisp sets more clearly.

Now, this particular example is something like this supposing that I am just going to invite my friends for today's dinner party and the time has been given as 7 pm for the dinner. Supposing that, I have invited a large number of friends. Now, some of my friends will be coming might be at 6.50 pm, some of them may come at 6.55 pm, some will be coming exactly at 7.00 pm, some may also come at say 5 minutes past 7 or some other may come say 10 minutes past 7, and so on. Although I have invited them and I have requested them to come exactly at 7 pm. So, some of them may come before 7 and some other people may come after 7.

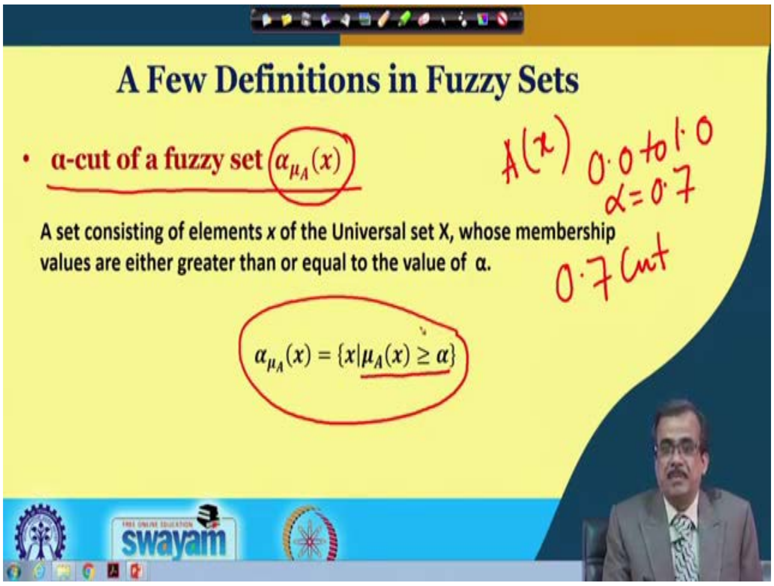
Now, if you see the concept of the classical set or the crisp set, this particular distribution indicates the crisp set representation for 7 pm. So, this particular thing once again let me tell you, if this is the crisp set presentation for your 7.00pm. Now the fuzzy set representation for 7 pm is something like this. So, this is actually the fuzzy set representation for 7 pm; that means, if some of my friend is coming at 6.50 pm will assume that he or she or they have come at 7 pm with the membership function value say 0.33. Similarly, if a friend comes at 6.55 pm, it is assumed that he has come for the dinner which is scheduled at 7 pm with the membership function value of 0.66, the same situation occurs like, if we comes at 7.05 pm; that means, 5 minutes past 7.00pm.

Then, we will assume that he has attended that particular party and he has come at 7 pm with the membership function value of 0.66 and if he comes at 7.10pm, we will assume that he has come at 7 pm with the membership function value of 0.33. Now, here, let us try to understand.

Now, the 7 pm, as I told, has been expressed like this. On the other hand, the 7 pm in fuzzy set has been expressed by this particular triangle, ok. Now, let me try to find out the difference between the classical set and the fuzzy set. So, this is nothing, but the crisp set representation for 7 pm and this is nothing, but the fuzzy set representation for the 7 pm. Now, my query is, which one is more practical? Now, we know it is bit difficult to join the party exactly at 7 pm.

So, this particular distribution for 7 pm that is the crisp set representation is bit difficult to achieve, whereas the fuzzy set representation for the 7 pm, it is little bit easier to achieve and that is why, the fuzzy set representation for the 7 pm is a more practical way of representing the time, that is your 7 pm. So, let me conclude that fuzzy set representation could be more practical compared to your the crisp set representation for the 7 pm. So, I hope, you have understood the difference between the representation of crisp set and this particular the fuzzy set.

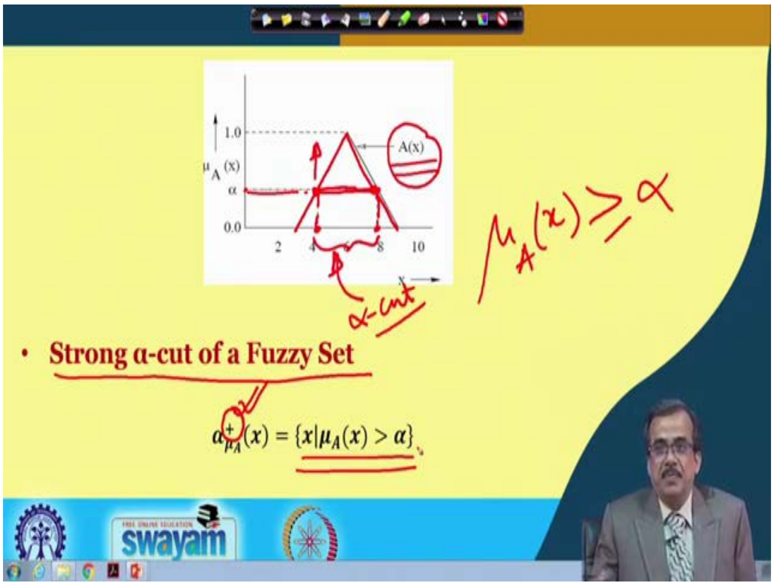
(Refer Slide Time: 19:34)



Now, I am just going to concentrate on a few terms, which are very frequently used in fuzzy sets and I am just going to define those terms and I am just going to solve a few numerical examples also. Now, the first term, which I am going to define related to fuzzy sets is very popular, that is known as the $\alpha - cut$ of a fuzzy set. Now, supposing that I have got a fuzzy set, which is denoted by A(x). Now, it is $\alpha - cut$ is represented by this alpha mu A x. So, this is nothing, but the alpha cut of a fuzzy set.

Now, the value for this particular alpha will vary from 0.0 to 1.0. Now, if I insert a particular value for this particular alpha, for example, say alpha equals to say 0.7. So, truly speaking, I am just going to find out, what should be the 0.7-cut of the fuzzy set. Now, let us see how to define this $\alpha - cut$ of a fuzzy set, which is represented by $\alpha_{\mu_A}(x) = \{x \mid \mu_A(x) \geq \alpha\}$. So, this is actually the definition for this particular the $\alpha - cut$.

(Refer Slide Time: 21:10)

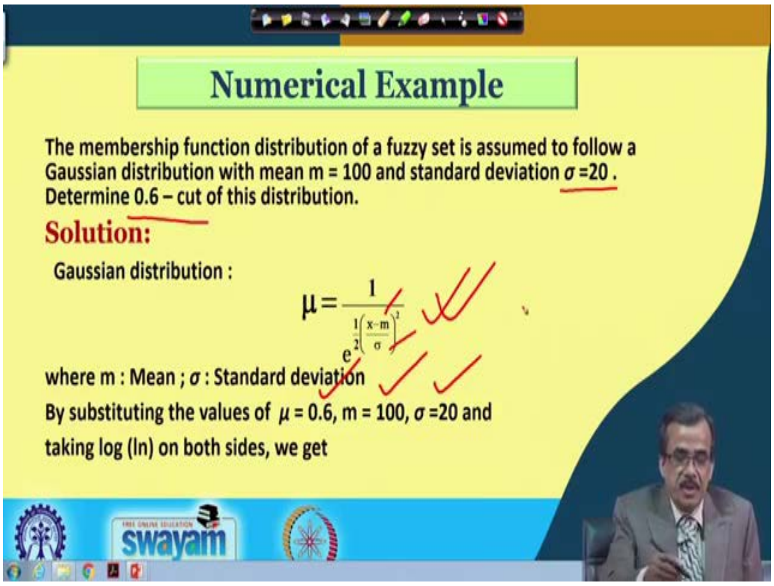


Now, I am just going to take one example just to tell you, what do we mean by the definition of alpha cut. Now, let me assume that this particular triangular membership function distribution is going to represent a fuzzy set and that is nothing, but A(x). So, A(x) is going to represent a fuzzy set and it is $\alpha - cut$, I am just going to define. Now, supposing that the α is here say 0.4 or 0.45 something like this. Now, corresponding to this particular α, you draw one line here. So, this is the line and it is going to intersect at these particular points.

Now, corresponding to this particular point, you try to find out the value for this particular x and according to the definition of this particular $\alpha - cut$, this $\mu_A(x)$ should be either greater than or equal to α. So, this is the value of α. So, more than α means, I am here, ok. So, either alpha or more than alpha and if I just follow that, I will be able to find out the subset of this particular fuzzy set and that is nothing, but this. So, this is known as the $\alpha - cut$ of this particular the fuzzy set. So, this is the way actually we can define the $\alpha - cut$ of that particular fuzzy set.

Now, another term I am just going to define here and that is known as the strong alpha cut of a fuzzy set. Now, by a strong $\alpha - cut$ of a fuzzy set, we mean it is $\alpha^{+}{}_{\mu_A}(x)$. So, just to indicate the strong $\alpha - cut$, we take the help of this particular symbol of plus and by definition this is nothing, but x such that $\mu_A(x) \geq \alpha$. So, this is actually the definition of your the strong $\alpha - cut$ of a fuzzy set.

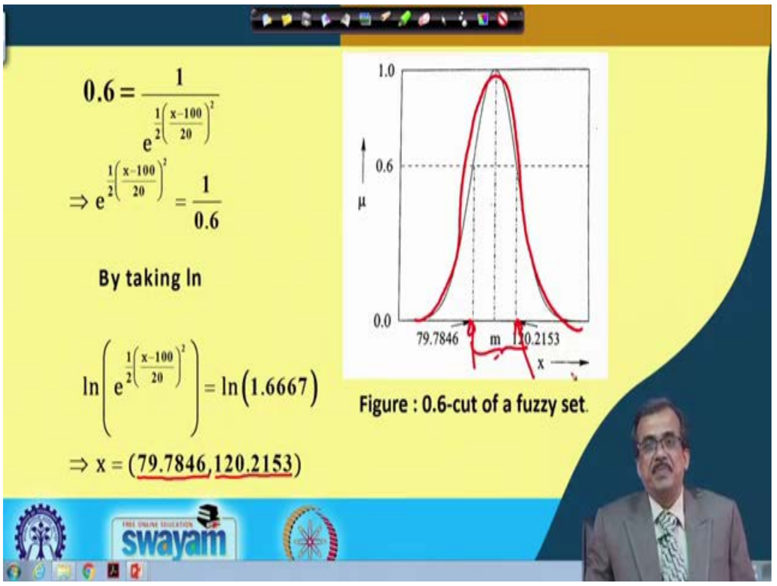
(Refer Slide Time: 23:31).



Now, we are going to solve one numerical example, just to make it more clear.

Now, the statement of the numerical example is as follows: the membership function distribution of a fuzzy set is assumed to follow a Gaussian distribution with mean m equals to 100 and standard deviation, that is, your sigma is equal to 20. Now, determine 0.6-cut of this particular distribution. So, I have got a membership function distribution, which is nothing, but Gaussian and whose mathematical expression is nothing, but $\mu = \frac{1}{e^{0.5(\frac{x-m}{\sigma})^2}}$. So, this is nothing, but the membership function distribution for this particular Gaussian distribution. Now, where m is nothing, but the mean and sigma is the standard deviation. So, what we do is, we substitute the values for μ. So, μ is equal to 0.6, the mean m equals to 100 and standard deviation sigma equals to 20, ok. Now, let us try to find out like what should be the values of x for this μ?

(Refer Slide Time: 24:59)

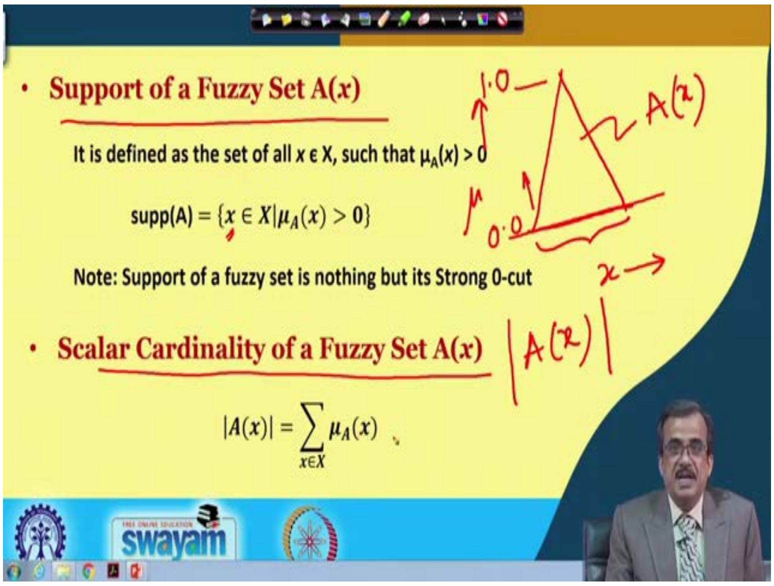


Now, what I do is, substitute the values for the μ, m and σ, and this is nothing, but is your the Gaussian membership function distribution. So, this is the Gaussian membership function distribution with mean equals to m.

Now if you substitute the values. So, this will become μ equals to 0.6 is equal to 1 divided by e raised to the power half multiplied by x minus 100 divided by 20 square and if you simplify. So, this will become e raised to the power half x minus 100 divided by 20 square and that is equals to 1 divided by 0.6. Now, we can take log on both the sides. So, ln (that is log base e) e raised to the power half x minus 100 divided by 20 square is nothing, but ln 1.6667 and if you simplify, you will be getting the values for x like 79.7846 and 120.2153.

Now, if I just plot. So, on this plot, if I just indicate the values, this is nothing, but 79.7846 and this is nothing, but 120.2153 and here. So, you will be getting one range for this particular x. So, this is nothing, but is your 0.6-cut of this Gaussian distribution. So, this is the way, actually you will be able to find out the $\alpha - cut$ of that particular fuzzy set.

(Refer Slide Time: 26:51)

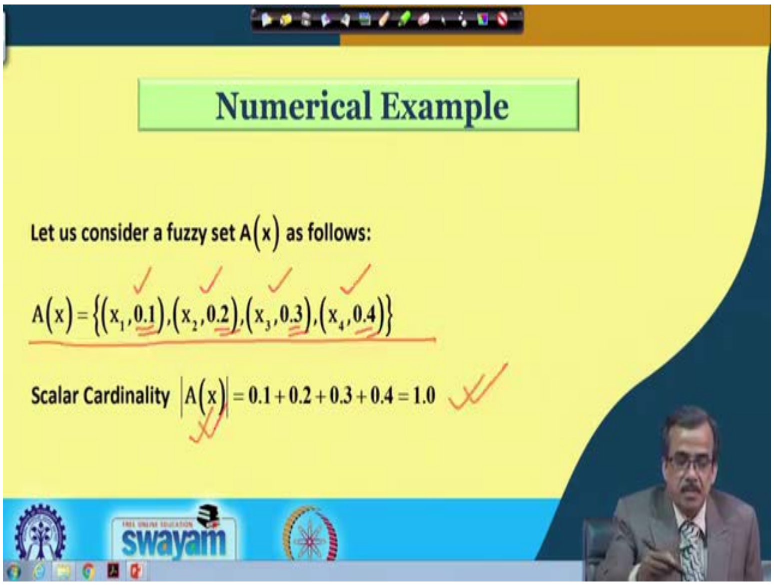


Now, we are going to define another term and that is known as support of a fuzzy set. Now let us see, what do we mean by the support of a fuzzy set. Now, by definition, support of a fuzzy set x is nothing, but say x belongs to the universe of discourse that is capital X, such that mu_A(x) is greater than 0. So, this is what we mean by actually the support of a fuzzy set.

So, mu_A(x) is greater than 0, if I consider. Now, let me try to consider say, this is a particular fuzzy set, say the triangular membership function distribution, if I consider. So, this is nothing, but A(x). So, μ is actually along this direction. So, this is 0.0 and corresponding to this is 1.0. Now, let me try to read the definition once again, support of a fuzzy set A(x) is nothing, but x belongs to capital X, that is the universe of discourse, such that $\mu_A(x)$ is greater than 0. So, corresponding to this, μ is equal to 0.0 and if I consider that μ is greater than 0. So, as if I am just going to touch, I am just going to indicate the limiting value for this particular x and this is going to indicate the support of a fuzzy set and here, you see x belongs to capital X.

So, this is nothing but x. So, x belongs to capital X, that is universe of discourse and this particular condition holds good, that is, $\mu_A(x)$ is greater than 0. So, this is going to indicate the support of a fuzzy set. Now, the next is the scalar cardinality of a fuzzy set. So, by scalar cardinality, we mean and that mod value of A(x). So, A(x) is nothing, but the fuzzy sets and it is scalar cardinality is denoted by mod value of A(x). So, this particular symbol and that is nothing, but is your summation $\mu_A(x)$, x belongs to capital X. So, this is nothing, but the collection of all membership function values.

(Refer Slide Time: 29:26)



Now, I am just going to solve one numerical example, just to find out the scalar cardinality of a particular fuzzy set.

Now, let me assume that say this is nothing, but a fuzzy sets having say the discrete values corresponding to the four values of the elements. So, A(x) is nothing, but x_1 comma 0.1, x_2 comma 0.2, x_3 comma 0.3, x_4 comma 0.4. So, this is nothing, but a discrete fuzzy set. Now, a scalar cardinality is denoted by this particular symbol and that is nothing, but the sum of all the μ values; that means, we have got 0.1 + 0.2 + 0.3 + 0.4 and this is nothing, but 1.0. So, 1.0 is coming as scalar cardinality of this particular fuzzy set A(x).

Thank you.