Fuzzy Logic and Neural Networks
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Lecture – 02
Introduction to Fuzzy Sets (Contd.)

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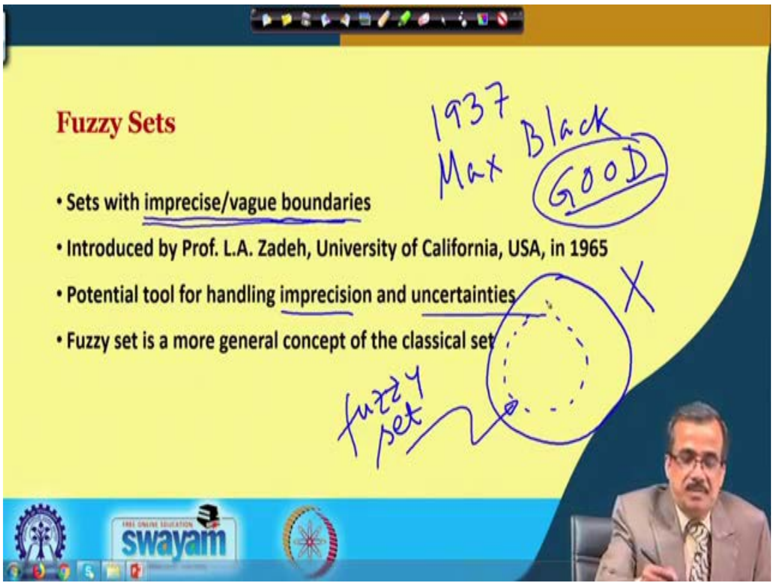

Now, as I told, Professor LA Zadeh of University of California, he actually argued that there are many uncertainties, which cannot be tackled using only the probability theory, which works based on the classical set or the crisp set. So, there are many uncertainties and if you want to tackle, if you want to handle those uncertainties, you will have to take the help of another concept, that is the concept of the fuzzy sets.

Now, if you see the literature, you will find that the concept of fuzzy set or if the similar concept of fuzzy set was proposed long back in the year 1937 by one American Philosopher, whose name is Max Black. So, Max black, an American Philosopher, introduced the concept of fuzzy sets and as usual, he was opposed by the traditional mathematician of USA and he is stopped, and then after a few years, in the year 1965, the concept of fuzzy set was reintroduced by Professor Zadeh.

Now, to define this particular concept of fuzzy set, let me try to take one example and this particular example, I have already taken, but I will slightly modify this particular example. Now, if you remember at the beginning, we talked about the universal set that is nothing, but the set of all technical universities in this particular world, and next, we try to find out the set of technical universities having at least 5 departments.

And, now I am just going to make it more complex, I am just going to find out a set of technical universities in the world having at least 5 good departments. The moment, you add this particular adjective good, the problem becomes very difficult, because, how to define this particular adjective "good" and this particular definition will vary from person to person and that is why, this particular problem is very complex and very difficult to answer.

Now, if I just draw in the form of say universal set at this particular fuzzy set, supposing that the universal set is nothing, but this. So, this is capital X, that is the set of all technical universities. In this particular word and as I told, the definition of this particular term that is good will vary from person to person.

That is why, you may not get a very precise subset and we may get a set that is called the fuzzy set, which is nothing, but sets with imprecise or vague boundaries and that is why, this particular fuzzy set, we try to draw with the help of dotted line, ok. So, this particular dotted line, this set is nothing, but the fuzzy set. If you remember, while drawing the crisp set, we use the solid lines, but for drawing this particular fuzzy set, we use the dotted line because, this particular definition of the subset will vary from person to person and there is no well-defined boundary for this particular set and that is why, this is known as the fuzzy set, that is set with imprecise or the vague boundaries.

Now, this particular sets, fuzzy sets are potential tools for handling imprecision and uncertainties and we can say that the fuzzy set is a more general concept of this particular classical set.

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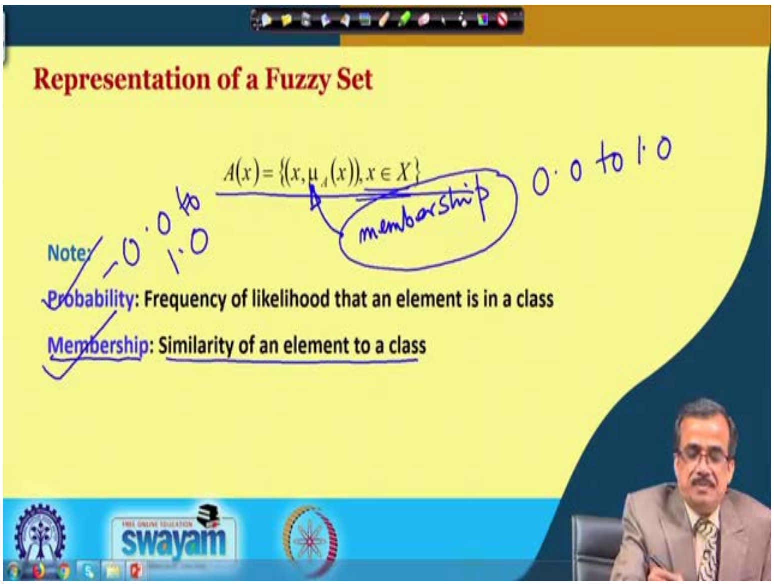

Now, let us try to see, how to represent the particular fuzzy set. Now, this I have already discussed that to represent the fuzzy set, we take the value of membership and this membership is nothing, but the degree of belongingness and that is defined by, that is denoted by this particular μ. So, actually μ denotes this membership function value. Now the fuzzy set is defined as follows.

So, it is nothing, but x comma mu (x) and this particular small x belongs to the universal X, that is, your capital X; that means, if you want to represent the fuzzy set, we will have to take the help of this membership function value, which varies from 0 to 1, and if you remember the probability value that will also vary from 0 to 1, but truly speaking, the concept of probability and the concept of membership are not exactly the same.

So, by probability, we mean it is the frequency of likelihood that an element is in a class; that means, your probability is related to the frequency data. On the other hand, membership is nothing, but the similarity of an element to a particular class. For example, if I take the probability of getting apple, so that will vary from 0 to 1, and what is the guarantee that the apple is red? It has got some membership function value lying between 0 and 1. Now, if I compare these two uncertainties, one is related to the availability of the apple and another is related to the colour of this particular apple. So, these two uncertainties are not exactly the same and there is a difference.

Now, the availability of the apple that will be handled by probability, but the guarantee weather this particular apple is red. So, that will be indicated by the membership function value for example, with membership function value of 0.9 and some other people will say that this particular apple is red with membership function value of 0.4, and so on. So, this is how to represent the fuzzy set.

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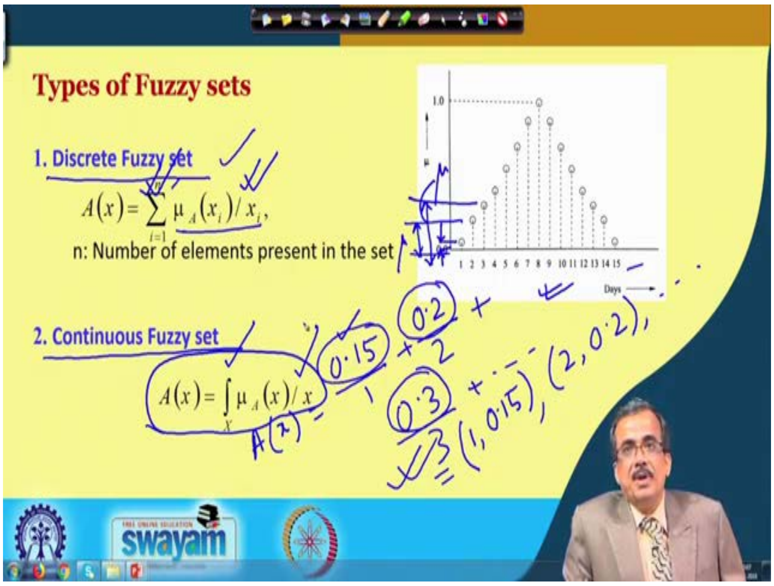

Now, if you see the literature. So, we have got two types of fuzzy sets, now one is called actually the discrete fuzzy set and another is called the continuous fuzzy set. Now, let me define the concept of this particular discrete fuzzy set first. Now, the discreet fuzzy set is defined as $A(x)=\sum_{i=1}^{n}\mu_A(x_i)/x_i$. Now, remember that this particular symbol does not indicate a division and this particular symbol does not indicate actually summation in that sense, it indicates actually the collection of data. The collection of this membership function value and here, the small n indicates the number of elements presents in that particular set. Now, I am just going to define the concept of this discrete fuzzy set with the help of this example. So, in this particular example, this figure is going to indicate supposing that the temperature of a particular place during the first 15 days of a month. Now, supposing that I have declared the temperature of city say B during the first 15 days of a month is moderate.

Now, on the first day, it has got a temperature value, now that is called moderate with this much of membership function value, on the second day it has got another temperature value and that is also called moderate with this much of membership function value μ, and μ varies from 0 to 1.

Similarly, on the third day is called the moderate temperature with this much of membership function value, that is μ, similarly, for all 15 days, I can represent the temperature is medium with different values of your membership. Now, this can also be written following this particular rule as follows: like you're a(x) is nothing, but the membership function value of the temperature on first day, supposing that this is 0.15 on the first day.

So, this is moderate temperature on the first day, the temperature is called moderate with this much of membership function value on the second day, it is called moderate with this much of membership function value, say 0.3 divided by 3 is actually slash 3, it is actually slash 3, and so on. So, on the first day, the temperature is moderate with this much of membership function value, second day the temperature is moderate with this much of membership function value, on third day it is moderate with this much of membership function value, and so on. So, this is the way actually, we can represent the discrete fuzzy set.

Now, if you see the literature, the same discrete fuzzy set can also be represented in another form for example, on the first day the temperature is called moderate with this much of membership function value. On the second day, it is called moderate with this much of membership function value, and so on. So, this is another way of representing the discrete fuzzy sets. Now, this is how to represent the discrete fuzzy set, now if I just see how to represent the continuous fuzzy set. Now, here for this continuous fuzzy set, this is the way we will have to represent; that means are A(x) is nothing, but so, in place of summation, we are using the integration.

So, $A(x) = \int_X \mu_A(x)/x$ and once again, this is actually not the true integration and this is actually not the true division. So, here actually what you do is, in continuous fuzzy set, we are going to represent or we are going to fit a curve to represent the fuzzy sets. Now we are going to look into all such issues in details.

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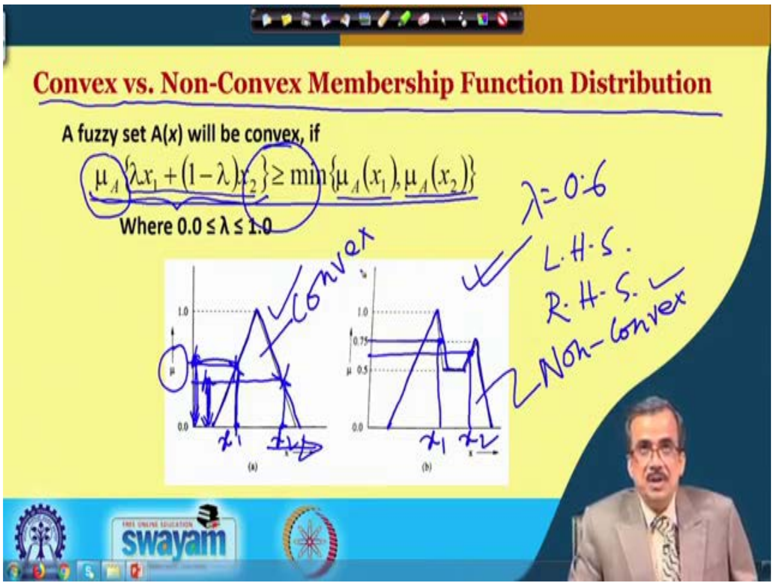

Now, before I go for that, like how to represent the continues fuzzy set more clearly.

So, I am just going to concentrate on one concept that is called the concept of convex versus non-convex membership function distributions. Now, this is very important, it is important in the sense, supposing that this is the temperature denoted by x and this is the membership function value μ. Now, the membership function distribution, it could be something like this and it can also be something like this, like it will increase then decrease, it will remain constant once again it increases then it decreases, and here, the membership function value will go on increasing, then it will reach the maximum then it will go on decreasing.

So, this is one type of membership function value and this is the second type of membership function value, in both the types, the value of μ is going to vary from 0 to 1, if I take say 1.0 here and if I take another point here, supposing that this is corresponding to x_1 and this corresponds to your x_2 and there is a μ value, membership function value. Now, if I want to check, whether it is a convex membership function distribution or a non-convex. So, this is the rule to be followed, a fuzzy set is called convex, if this particular condition gets fulfilled, that is, $\mu_A\{\lambda x_1 + (1-\lambda)x_2\} \geq \min\{\mu_A(x_1), \mu_A(x_2)\}$, now what is x_1 and what is x_2?

So, this is the x_1 a particular value of x or the temperature, this is x_2 another value of temperature, that is x_2 and its corresponding μ is this much and its corresponding μ is nothing, but this much. So, this is your $\mu_A(x_1)$ and this is your $\mu_A(x_2)$. So, I can compare to find out what is the minimum value, take a particular value of λ lying between 0 and 1, say you take λ equals to say, 0.6. So, can I now find out what should be the numerical value corresponding to your left hand side? The answer is yes.

So, I can find out the numerical value of the left hand side, because I know the λ, I know the value of x_1, x_2. So, I can calculate. So, I will find out a value of x. So, corresponding to that, I can find out the μ from this distribution and similarly, on the right hand side, what you can do is I know what is $\mu_A(x_1)$. So, this is my $\mu_A(x_1)$ and this is my $\mu_A(x_2)$, I can compare to find out the minimum.

So, this is nothing, but is your right hand side. Now, if this particular condition holds good, then we say that this particular membership function distribution is a convex membership function distribution and if it is of this type for example, say it is increasing, decreasing, remaining constant, once again increasing and decreasing and if you take one value here, say this is my x_1, if you take another value here.

So, this is my x_2, I can find out the corresponding μ and if you just check this particular condition, there is a possibility that this particular condition will not hold good and that is why, this type of membership is known as non-convex membership function distribution and this is nothing, but a convex membership function distribution, I hope the idea behind this particular convex versus non-convex membership function distribution is clear to all of you.

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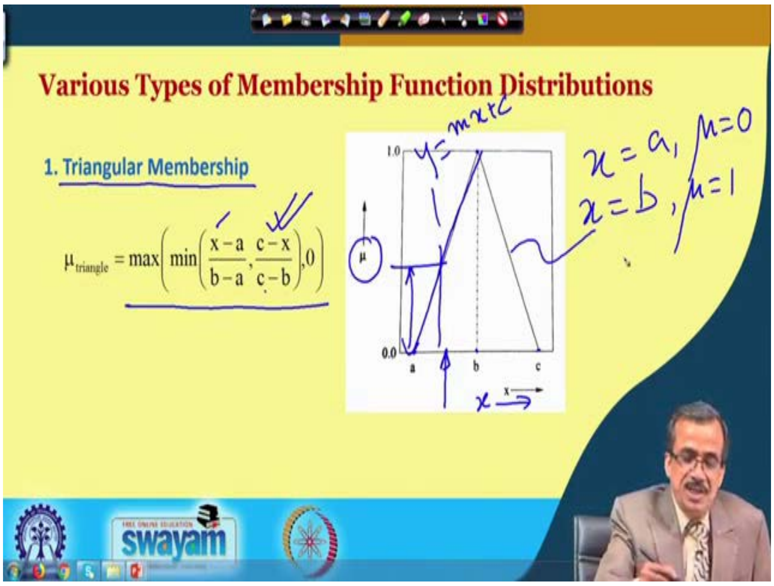

Now, then comes here, how to represent the membership function distribution.

Now, you see the membership function distribution has been represented using both linear function as well as the non-linear function. Now here, I am just going to concentrate on this particular triangular membership function distribution; that means, this is nothing, but μ varies from 0 to 1 and this is the variable say temperature or humidity or whatever may be now here, exactly at a, the membership function value is 0; at x equal to b, the membership function value is 1.0 and once again, at x equal to c, the membership function value is actually equal to 0.

Now, mathematically how to represent, it is a very simple because, this is the equation of a straight line. So, I can use $y = mx + c$, there is no problem I can find out one expression. Similarly, I can find out another expression for this particular the straight light and I can find out corresponding to a particular value of x, what should be the value for this particular μ? It is very simple, now if you see the literature.

So, this triangular membership function distribution has been represented using this particular expression and also, with the help of some max and min operator. For example, say $\mu_{triangle}$. So, this is the triangle membership function distribution. So, $\mu_{triangle} = \max(\min(\frac{x-a}{b-a}, \frac{c-x}{c-b}), 0)$. Now, if I put x equals to a here, say x equals to a, if I put what will happen? So, if I put x equals to a. So, this will become 0 and this will become non 0 and c minus a divided by c minus b. So, this will become greater than 1. So, 0 comma a value greater than 1 and the minimum will be 0 and the maximum between 0 and 0 will be 0. So, at x equals to a. So, mu will become equal to 0. Similarly, you can check what happens at x equals to b. So, at x equals to b. So, this will become b minus a divided by b minus a. So, this will give rise to 1 and here. So, c minus b divided by c minus b will give rise to 1.0.

Now, the maximum between 1 and 0 is 1. So, at x equals to b, the μ will become equal to 1 and similarly, you can find out what will happen at x equals to c and once again, you will be getting μ becomes equal to 0. So, this is the way actually, we can represent the triangular membership function distribution.

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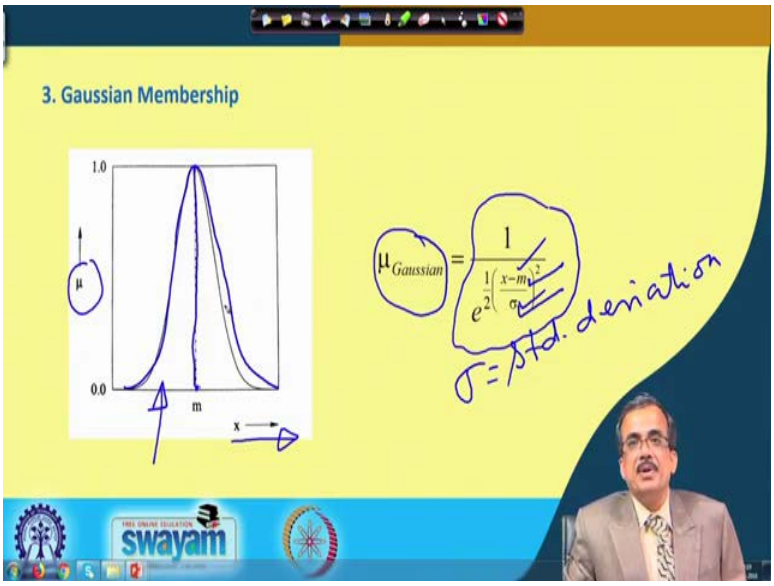

Now, next comes the trapezoidal membership function distribution, it is very simple. So, this is x, the variable say temperature or humidity and this is your μ and now at x equals to a, μ becomes equal to 0, at x equals to b, μ becomes equal to 1 and after that the value of μ will remain constant up to this, then at x equals to c. Once again, the value will be 1 and after that it will start decreasing and at x equals to d, the value of the membership function will become equal to 0, and mathematically actually, this can be represented using this particular formula.

So, this $\mu_{trapeziodal} = \max(\min(\frac{x-a}{b-a}, 1, \frac{d-x}{d-c}), 0)$. Let me take one very simple example, supposing that x is equal to b. So, if I put x equals to b here. So, this will become 1 comma 1 and d minus your b. So, d minus b divided by d minus c. So, this will become more than 1. So, the minimum among 1 comma 1 comma a value more than 1. So, the minimum is 1 and maximum between 1 comma 0 is nothing, but your 1. So, μ becomes equal to 1.

So, at x equals to b μ becomes equal to 1. So, this is actually the trapezoidal membership function distribution, then comes the Gaussian distribution and you know so, this particular Gaussian curve is nothing, but a non-linear curve now here. So, this is actually the membership function distribution μ varies from 0 to 1 and this is the variable say, temperature or humidity, whatever may be and this is your the Gaussian distribution and for this particular Gaussian distribution, the mean is here that is denoted by m and σ is nothing, but is your standard deviation. So, m denotes the mean and σ denotes the standard deviation.

So, $\mu_{Gaussian} = \frac{1}{e^{0.5(\frac{x-m}{\sigma})^2}}$. So, we can find out the expression for this particular μ, the moment you select a particular value of x. So, we can find out. So, knowing the value for this mean and standard deviation, we can find out what should be the value for this particular μ and as I told that this is some sort of non-linear distribution for this membership, ok?

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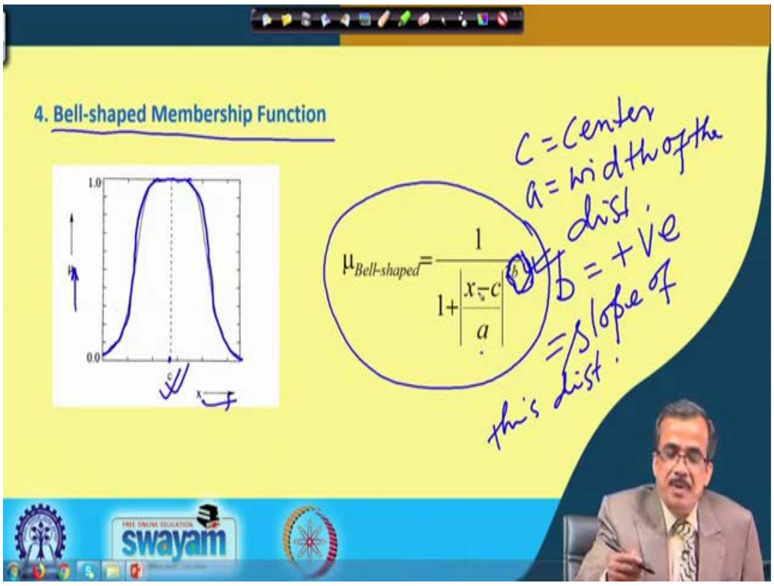

Now, I am just going to concentrate on another very popular non-linear membership function distribution and that is known as the Bell shaped membership function distribution.

Now, the distribution is such so, this is x, this is your μ the membership function value. So, it will start from here, there will be non-linear distribution and it will go on increasing and after that it will reach 1 and it will be kept same then comes your it will go on decreasing something like this, ok. So, this is nothing, but the bell shaped membership function distribution and this is the mathematical expression for your membership function distribution. So, $\mu_{Bell-shaped} = \frac{1}{1+|\frac{x-c}{a}|^{2b}}$. Now, here c indicates that this is nothing, but the centre of this particular distribution, which is visible from here. Now, what does this particular a indicate? Now, a denotes actually the width of this particular distribution. So, a denotes actually the width of the distribution and this particular b, that is considered a positive value, and it indicates actually the slope of this particular distribution.

The slope of the distribution is denoted by b. Now, let us see what happens, if I take a very high value for this b? Now if I take a high value for this particular b, the distribution will be stiffer and if I take a low value for this particular b, the distribution will become flatter one, ok.

So, this is the way actually we control the distribution of this particular function with the help of a, b and c and this bell shaped membership function distribution is very popular.

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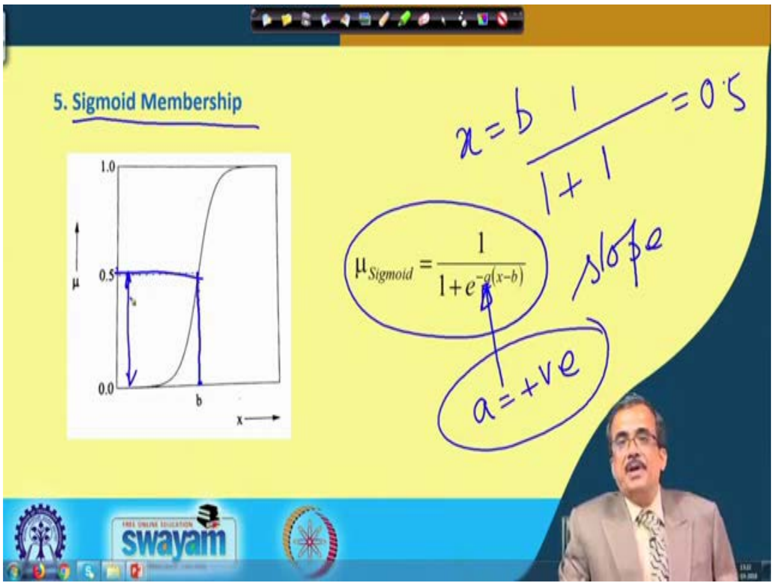

Then comes another very popular membership function distribution that is very frequently used and that is known as the sigmoid membership function and this is actually the mathematical expression: $\mu_{sigmoid} = \frac{1}{1+e^{-a(x-b)}}$.

So, this particular a is nothing, but a positive value and here, I have got a negative sign before a, but a itself is a positive value and a indicates the slope of this particular distribution, the higher the value of this particular a, the stiffer will be the curve and vice-versa.

So, if I put x equals to b in this particular expression. So, this will become 1 divided by 1 plus e raised to the power 0 and that is nothing, but 1. So, I will be getting your 1 divided by 2, that is 0.5. So, corresponding to this x equals to b. So, I will be getting your μ, that is equal to 0.5. So, this is actually the membership function distribution, which we are very frequently using.

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Now, whatever I have discussed in this particular lecture, you can consult the book written by me, that is, Soft Computing Fundamentals and Applications.

So, you will be getting all the details in this particular text book, which is the text textbook for this course.

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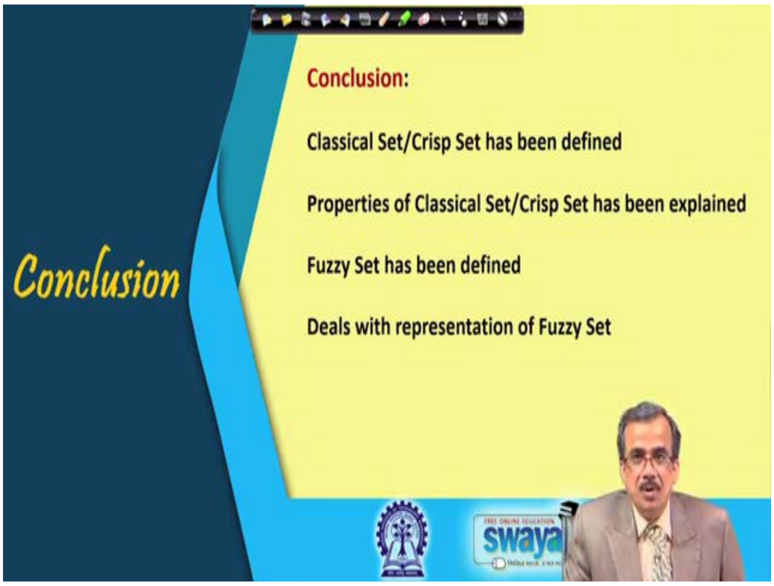

Now, let me conclude, whatever I discussed in this particular lecture. So, to start with I gave a brief introduction to the concept of the classical set or the crisp set. I discussed the 10 properties of the classical set or the crisp set, I tried to find out the reason, why should we go for the concept of fuzzy set, the concept of fuzzy set has been defined with the help of suitable examples and we have seen, how to represent the different fuzzy sets; the fuzzy sets could be either discrete or it could be continuous, but we should be able to represent those fuzzy sets.

Thank you.