Fuzzy Logic and Neural Networks
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Lecture - 16
Applications to Fuzzy Sets (Contd.)

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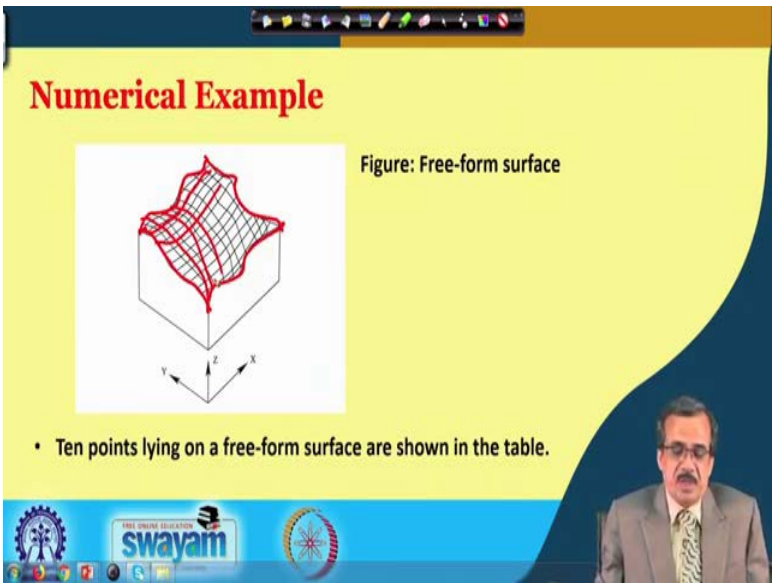

Now, we are going to discuss one numerical example to explain the working principle of this entropy-based fuzzy clustering. Now, I am just going to take the same numerical example, which I took for the previous algorithm, that is fuzzy C-means algorithm.

Now, once again, let me take the same example. So, this example is nothing, but the example of the free-form surface and let me repeat, what I will have to do is, this is the free-form surface and this particular free-form surface, I will have to generate, I will have to do the machining to get this type of the free-form surface. So, how to get it? Now, to carry out this particular machining with the help of milling cutters, so before that, what we do is? We do clustering based on similarity.

Now, here we are going to discuss. So, how to use? So, this entropy-based clustering to solve this clustering problem or how to achieve the suitable clusters?

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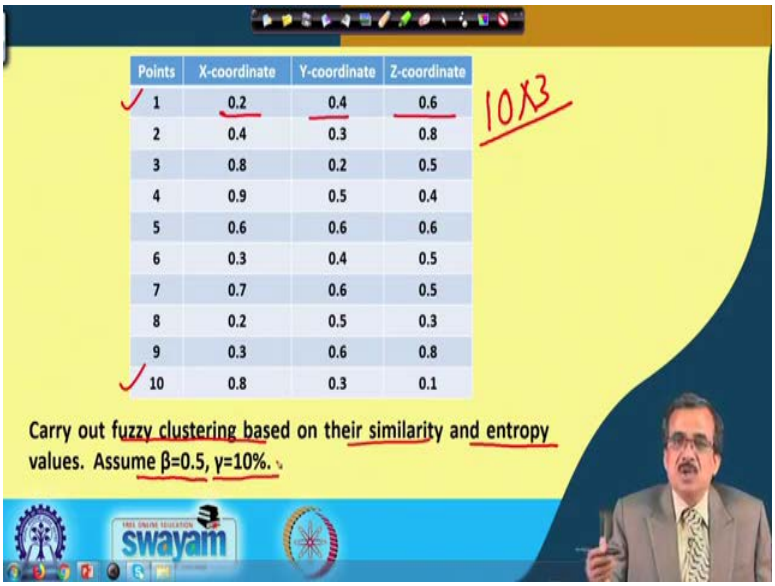

Now, let us take the same example, and here, what I am going to do is, the same set of 10 points, we are going to consider and which are lying on that free-form surface and let me do this particular clustering using entropy-based clustering. Now, these data points are nothing, but the 3D data points; that means, corresponding to the first point, I have got X dimension, Y dimension and Z dimension.

Similarly, we have got 10 number of data points. Now, in fact, if I want to do the optimization for the practical problem related to the machining of free-form surface, we will have to take a very large number of data points, might be 10000, 20000 data points, but here, for simplicity, I am just going to consider only 10 data points. And, for each of the data points, I have got three dimensions. So, in the matrix form, this particular data can be represented by your 10×3 there are 10 rows and 3 columns.

Now, here, I am just going to carry out the fuzzy clustering based on similarity and entropy and we assume that the threshold value for this particular similarity that is β is nothing, but 0.05. And, to determine whether there is any such outlier, we consider the concept of γ and we assume that γ is equal to 10 percent. Now, here, let us see like how to use this particular concept to solve the clustering problem.

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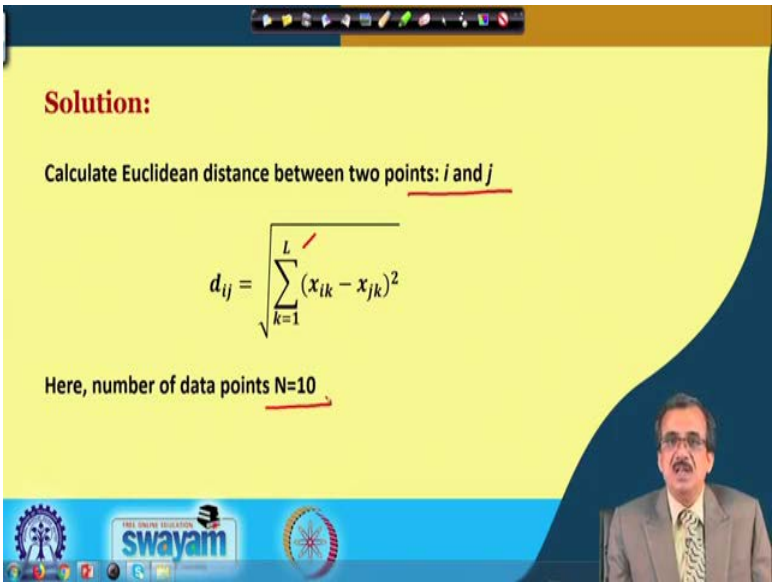

Now, the first thing, we do is, we try to find out the equilibrium distance between the two data points i and j. So, this formula, I have already discussed. So, $d_{ij} = \sqrt{\sum_{k=1}^{L} (x_{ik} - x_{jk})^2}$. And, here, we are going to consider actually the 10 data points, that is N is equal to 10.

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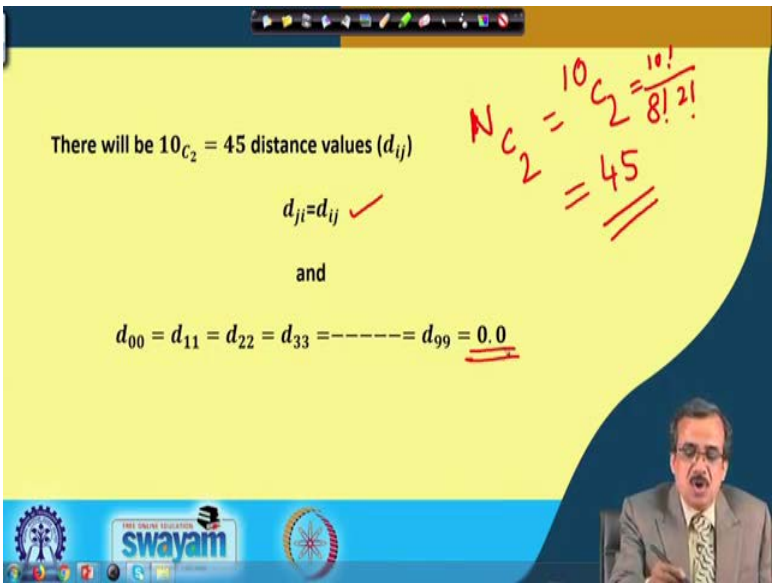

Now, using this actually, we can find out what should $\bar{d}$ and all such things. Because, here so, $^{10}C_2$ that is nothing, but is our $^{N}C_2$. So, $^{N}C_2$ is actually the total number of

distance values, which we will have to consider and calculate. And, here, it is nothing, but is our ^10 C_2 and that is nothing, but is your 10 factorial divided by 8 factorial and 2 factorial. So, it is 9 multiplied by 10, 90 divided by 2 so, I have got actually the 45 distance values. And, here, d_ji is nothing, but d_ij and the diagonal elements are all put equals to 0 because d_00, d_11 is nothing, but equal to 0.

So, using this particular information, very easily you can find out, what is your $\bar{d}$?

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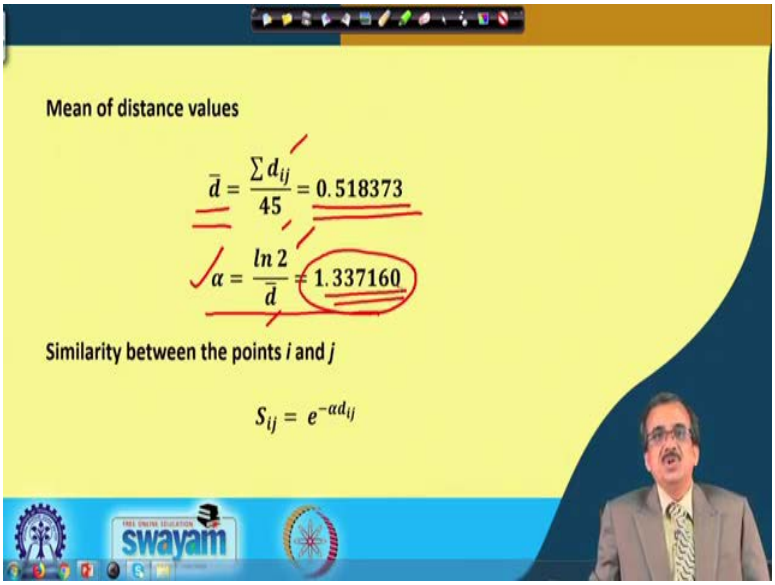

That is your the mean distance, that is $\bar{d}$. And, once you have got this particular $\bar{d}$, that is summation d_ij divided by 45 is something like this 0.518373 and once you have got this particular $\bar{d}$ so, very easily, I can find out this α . α is nothing, but ln 2 divided by is your $\bar{d}$ and if I calculate for this particular data point. So, this will become equal to 1.337160. And, once you have got this particular value for the α now we are in a position to calculate what should be the similarity, that is, S_ij and that is nothing, but $S_{ij} = e^{-\alpha d_{ij}}$ and α is equal to 1.337160.

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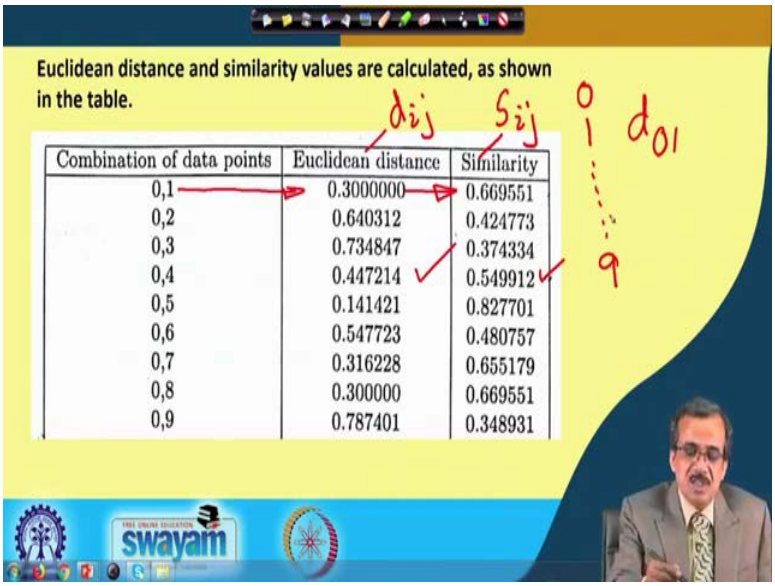

Now, if I just calculate, what should be the equilibrium distance value and what should be the similarity. Now, we can find out so, there are 10 points. So, the way I am marking the first point is marked as 0, the second point is marked as 1, similarly the 10th point is marked as 9. So, what we do is we try to find out, we try to first determine the equilibrium distance between 0 and 1 that is nothing, but d_01. So, this is nothing, but is your d_ij and this is nothing, but the similarity, that is, S_ij.

So, what we do is, the distance between 0 and we try to calculate the way I have already discussed and once I got that particular distance value and knowing the value of α. So, we can also find out what is the similarity? Now, similarly the equilibrium distance and similarity or the different data sets for example, say different data combinations like 0 and 2, 0 and 3, 0 4, 0 5, 0 6, 0 7, 0 8 and 0 9, we can find out. So, for different combinations of these data points starting from 0 so, I can find out the equilibrium distance values and I can find out their similarities, the exactly the same way, the way I have already discussed.

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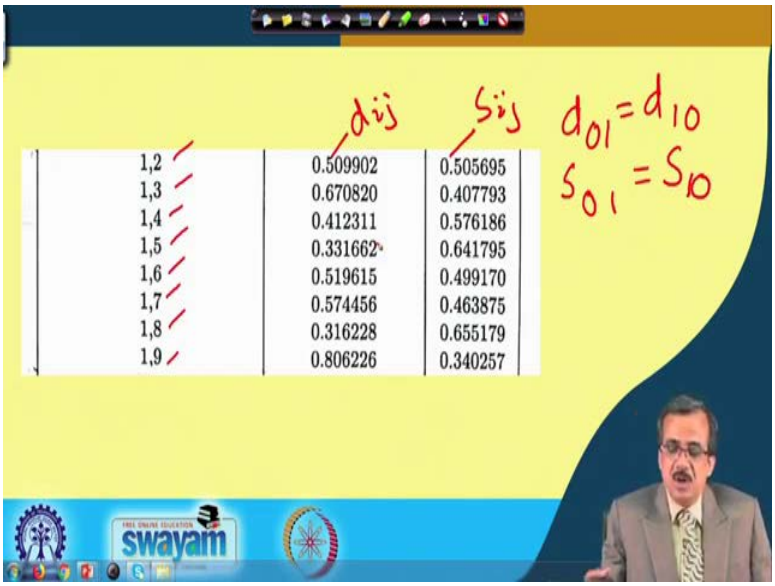

Now, the next is, I will have to find out the distance and your similarity values. So, this is nothing, but d_ij and this is your S_ij and I can find out the distance and similarity between 1 and 2, 1 and 3, 1 and 4, 1 and 5, 1 and 6, 1 and , 1 and 8, 1 and 9. Now, this 1 0 I should not determine because I have already calculated 0 1 and the distance between 0 1 that is your d_01 is nothing, but d_10 and we consider similarity between 0 1 is nothing, but similarity 1 0. So, starting from 1 so, 1 2 3 and so on up to 1 9 I can find out the distance and your the similarity values.

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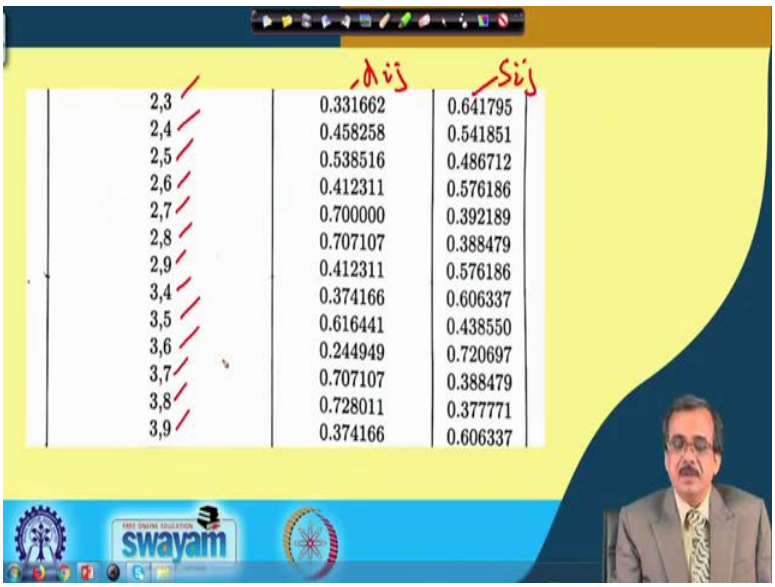

Now, by following the same procedure. So, I can also find out the equivalent distance, that is, d_ij and the similarity that is S_ij between 2 and 3, 2 and 4, 2 and 5, 2 and 6, 2 and 7, 2 and 8, 2 and 9. Now, here I should not determine 2 and 0, because I have already determined 0 2 then 2 and 1 because, I have already considered 1 2. So, I can find out the equilibrium distance and the similarity similarly starting from 3 so, I can find out between 3 and 4, 4 and 5, 3 and 6, 3 and 7, 3 and 8 and 3 and 9 so, using this particular method, I can find out.

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Then, between 4 and 5, I can find out the distance and similarity. Then, 4 and 6, 4 and 7, 4 and 8 and 4 and 9 other things I have already considered. Then, between 5 and 6, 5 and 7, 5 and 8, 5 and 9 then comes within 6 and 7, 6 and 8, 6 and 9, 7 and 8, 7 and 9 and 8 and 9. So, I can find out the equilibrium distance and the similarity. Now, for the different combinations of the data points, we have already calculated their equilibrium distance values and the similarity values. Now, we are going to use this particular information to find out, what should be the total entropy for each of the data points.

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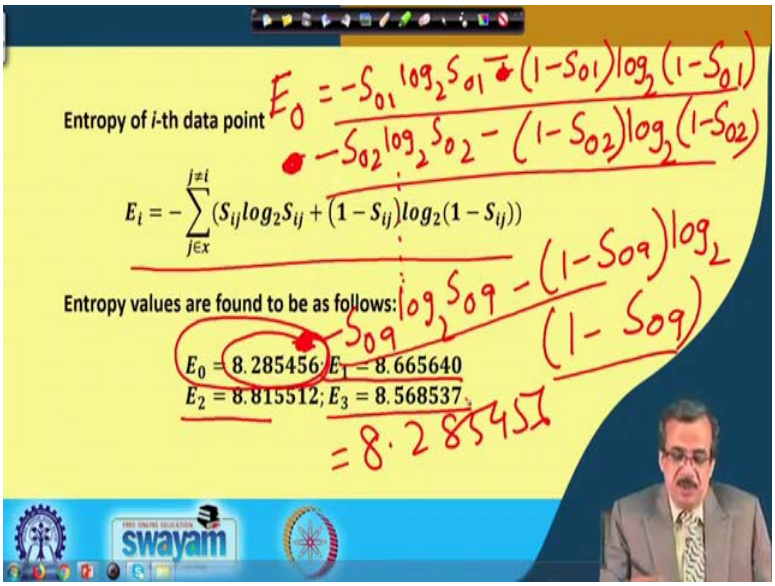

So, here, we have considered, there are ten points; that means, starting from E_0, I will have to find out E 9. Now, let us see, how to find out, the first that is E_0. So, E_0 is nothing, but this, I am just going to use this particular expression. Here, i equals to naught and j varies from what j belongs to x. So, if it is if it is i equals to your 0 and j is not equals to i. So, I will have to start from 1 and I will have to go up to 9.

So, what I will have to do is? So, I will have to put here minus. So, S_i equals to 0 and let me put j equals to 1 then log base 2 then S_01 plus 1 minus, S_ij so, this is nothing, but S_01 then comes your log base 2, 1 minus S, then i is equal to 0 and this is nothing, but 1.

And, here actually, this would be your minus because minus is outside. So, if I write separately here that would be minus then I will have to write down and j equals to 2. So, this will become minus S_02 log base 2 S_02 minus 1 minus S_02 then come log base 2 then comes 1 minus S_02. And, this I will have to write and the last term will be as follows S_09 then comes your log base 2 then S_09 minus 1 minus S_09 log base 2 then comes 1 minus S_09. And, if I just calculate, then I will be getting, E_naught is equal to your 8.285456, the way I have written it here.

So, this is the way, actually we can find out what should be the total entropy of a particular the point.

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So, these are to be added, in fact, this is not equal. So, all such things, I will have to actually go on adding or go on subtracting then finally, you will be getting this particular expression. So, this should be this equal sign should be replaced and this would be negative sign because this is a summation.

Now, so, following this method so, I will be getting this particular E_0 and similarly actually, we can also find out what is E_1 what is E_2 and then comes your E_3. So, using this actually I can find out what should be the entropy values for the different data points.

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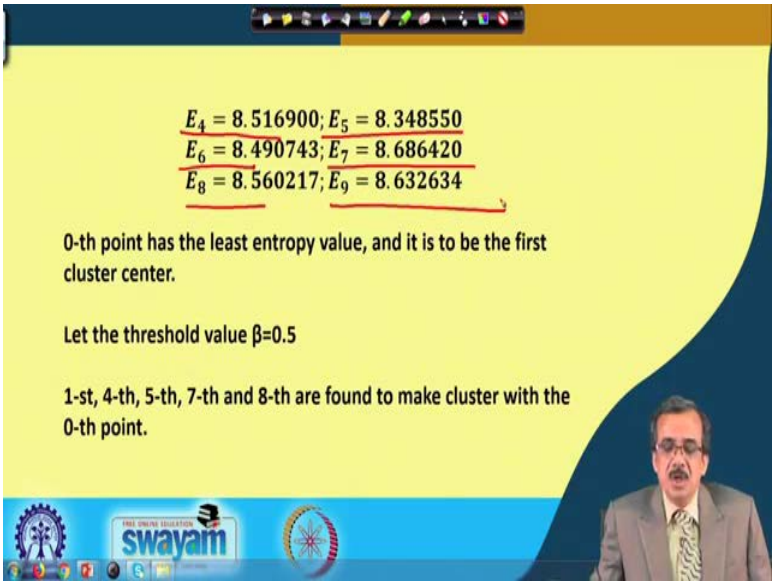

Now, as I told, by following the same procedure, I can find out what should be your E_4 then E_5, E_6, E_7, E_8 and E_9. Now, if I compare, all the entropy values, now if you see the entropy values like E_naught, E_1, E_2, E_3 and your E_4, E_5, E_6, E_7, E_8 and E_9, the minimum in terms of the numerical value, the minimum will be your E_naught. That means your the first point will be selected as the first cluster center that means, your the first cluster center will be the first point.

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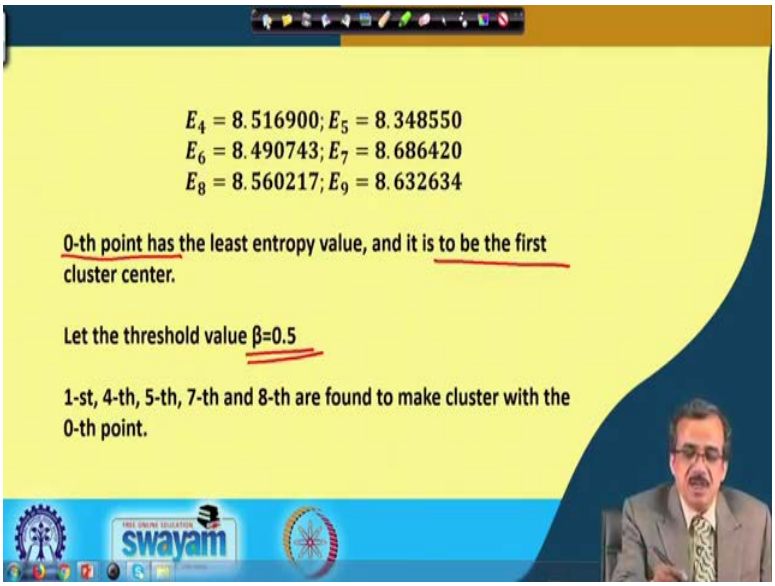

Because, the 0th point that is the 1st point is nothing, but the first cluster center. And, let me assume that the threshold value for the similarity, that is, β is equal to 0.5. Now, if we just go back on the picture of the similarity. So, I can find out the first cluster for example, say the 0 has been taken or has been considered as the first cluster center.

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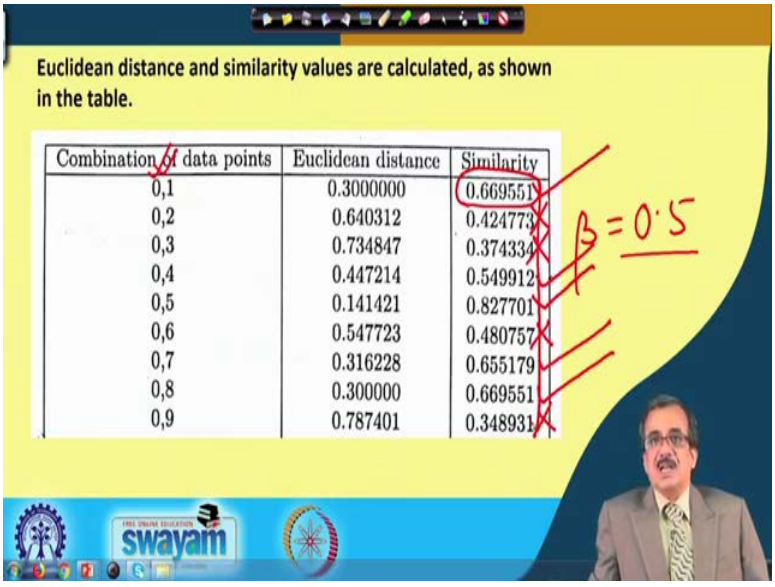

And, now, I will have to find out the similarity between 0 and 2 up to 0 and 9, and if I concentrate on these particular similarity values and β the threshold value of similarity is 0.5.

So, I will have to identify. So, those points whose similarity with the cluster center that is 0 is greater than or equals to 0.5. Now, here if you see the similarity is your 0.669551. So, this is very similar to your the first cluster center. So, this should be considered in the first cluster. The second is 0.424773 that is a less than 0.5. So, this should not be considered, the 3rd point should not be considered, the 4th should be considered in the first cluster.

The 5th should be considered, but 6th should not be considered, 7th should be considered, 8th should be considered, but 9th should not be considered in the first cluster. And, the same thing actually, I have just put it here so, you can see that in the first cluster.

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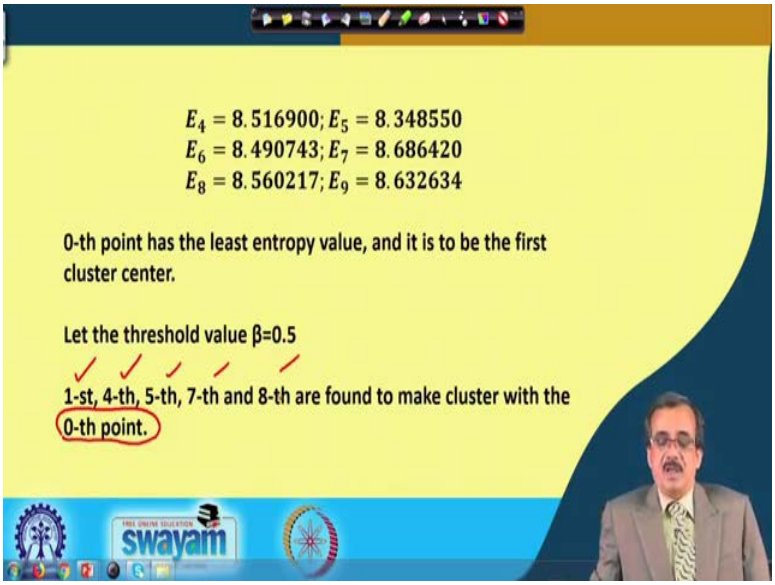

So, we have considered the 1st point, that is, the 0th point is nothing, but the cluster center and the other points will be the 1st point, 4th point, 5^{th}, 6th and the 8th point. Now, if 0th is the 1st point. So, (Refer Time: 16:17) this is the 2^{nd}, 5^{th}, 6^{th}, 8^{th}, and 9th point and this is the way actually we will have to form this particular cluster.

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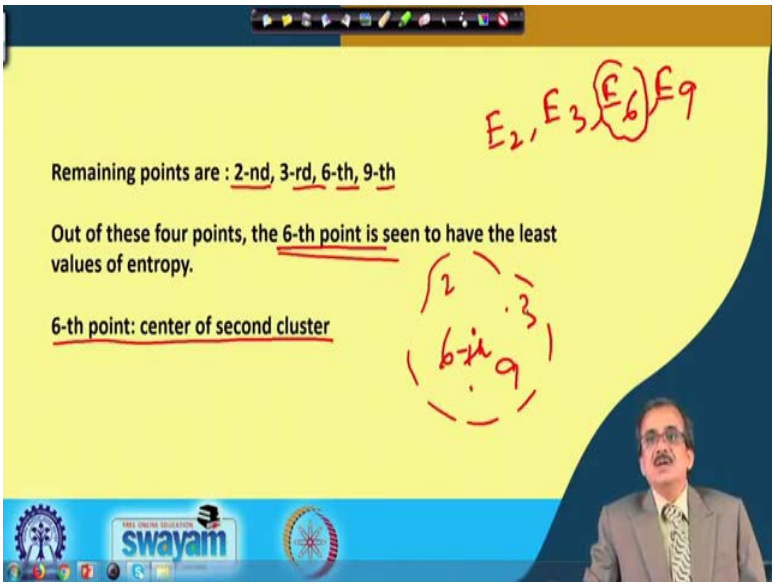

The cluster now, once you have got this particular cluster. So, out of the ten points, a few points have been considered, but we have got a few remaining points, what are those remaining points? The remaining points are nothing, but 2^{nd}, 3^{rd}, 5^{th}, and 9^{th},. And, out of these the E values, if we consider, that is your E_2, E_3, E_6 and E_9, if you compare. So, this E_6 is found to be the minimum. So, the 6th point will be considered as the second cluster center and once you have considered the 6th point as the second cluster center once again you see the similarity of the other points like 2^{nd}, 3rd and 9th with the 6th and those similarity values, we have already considered.

So, what we do is, we consider at the center that is nothing, but the 6th point and surrounding that, we have got a points like the 2nd point, then comes the 3rd point then comes the ninth point in this particular the cluster. Now, till now actually whatever we have discussed. So, we have got two clusters.

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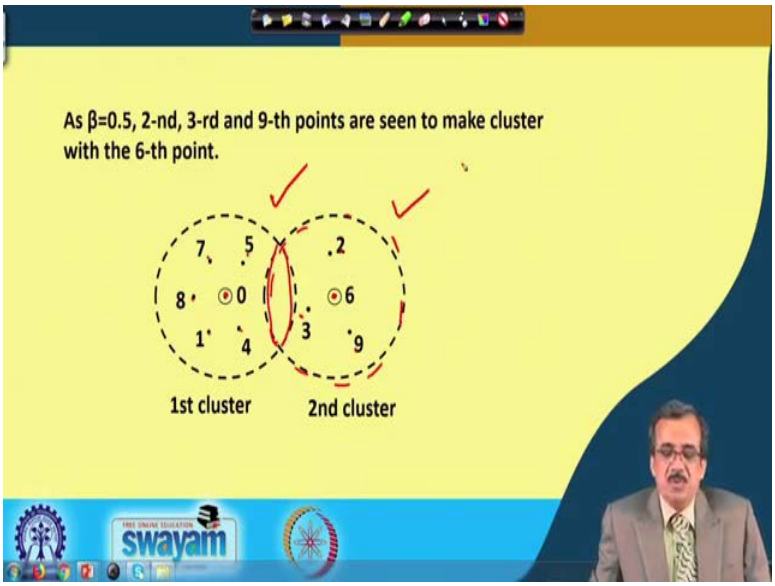

And, if I just draw it. So, might be this is my the first cluster and this is my actually, the second cluster and the clusters are fuzzy in nature. So, there could be some overlapping region also.

Now, here so, 0th it is the first cluster center, and that is followed by the 1^{st}, then 4^{th}, 5^{th}, 7th and 8th points. And here in the second cluster the 6th is actually the cluster center and here the second 3rd and 9th will be your the data points, which are going to follow your the 6th that is the second cluster center. So, this is the way actually, we will be getting two such fuzzy clusters using the entropy-based clustering.

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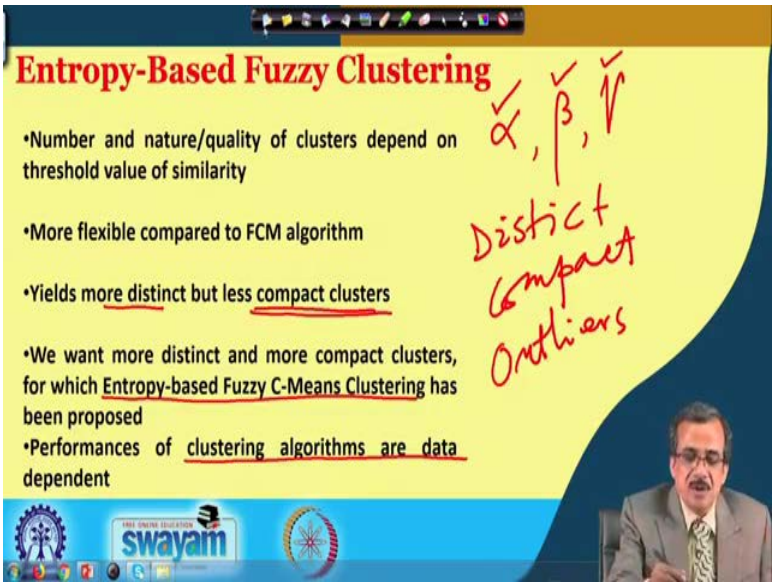

Now, if you see here, the number and nature or the quality of the clusters depends on actually a number of parameters. In entropy-based fuzzy clustering, we have considered a few parameters for example, say we have got α that relates the relationship between your the equilibrium distance and similarity. Then comes we have got β that is the threshold value of similarity, then comes we have got γ which decides your the outliers. So, the performance depends on this particular α, β and γ and we have seen that. So, this particular clustering algorithm is a very flexible and we can yield the distinct clusters here, but the compactness will be less.

Now, this algorithm is also very fast and you will be getting the distinct clusters, but as I told we may not get very compact. And, that is why, actually we we try to combine the merits of fuzzy C-means algorithm and the merits of this entropy-based clustering algorithm. So, just to develop the entropy based fuzzy C-means clustering actually, this entropy-based fuzzy C-means clustering has been proposed by us, and where we tried to consider the merits of these two algorithms and we try to eliminate their inherent demerits.

Now, let me repeat what we need, we need the clusters should be very distinct and we need actually the clusters should be very compact and at the same time the number of outliers should be as minimum as possible. So, we formulated this as an optimization problem and we solved using nature-inspired optimization tool. So, genetic algorithm,

which I am not going to discuss in details in this course, we could find out very distinct, very compact and the number of outlets will become minimum. So, that type of ideal clusters, we could get.

And, moreover another experience which I am going to share with you people, the performance of clustering algorithms are found to be data dependent. So, for different data sets, the performance for the clustering algorithms could be different and actually, it depends on the nature of the data sets.

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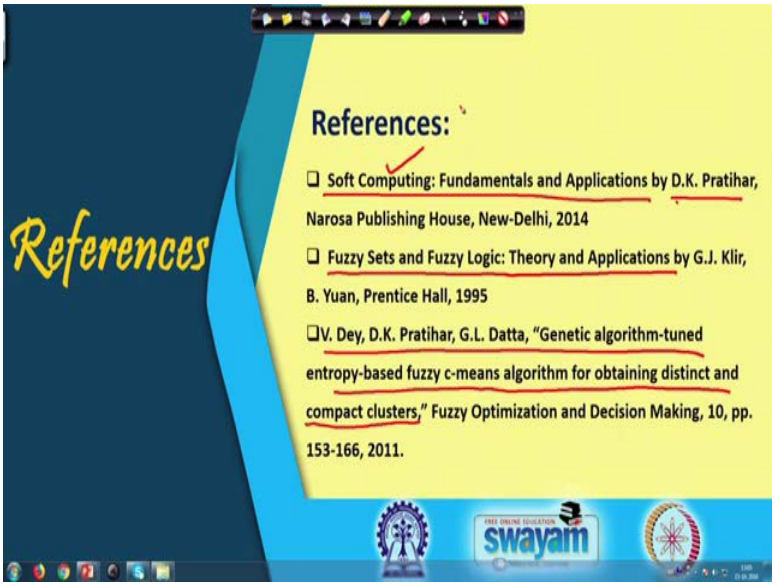

Now, the references, the textbook for this particular course that is the textbook, Soft Computing: Fundamentals and Applications written by me, you will be getting the material, which I discussed, you can also consult the book: Fuzzy Sets and Fuzzy Logic: theory and application by George Klir and for the combined entropy-based fuzzy clustering, if you want to have a look, you will have to look into this paper written by us like Genetic algorithm-tuned entropy-based fuzzy C-means clustering for obtaining distinct and compact cluster.

Now, here, I just want to tell you that in this course, I am not going to discuss the principle of nature-inspired optimization tool or the genetic algorithm, in details. Actually that the working principle of genetic algorithm and other nature-inspired optimization tools has been discussed in much more details in another MOOC program, another MOOC course, that is called Traditional and Nontraditional Optimization Tools

developed by me and this is also , in details, in your textbook, that is, Soft Computing: Fundamentals and Applications by D. K. Pratihar. So, you can have a look of this book.

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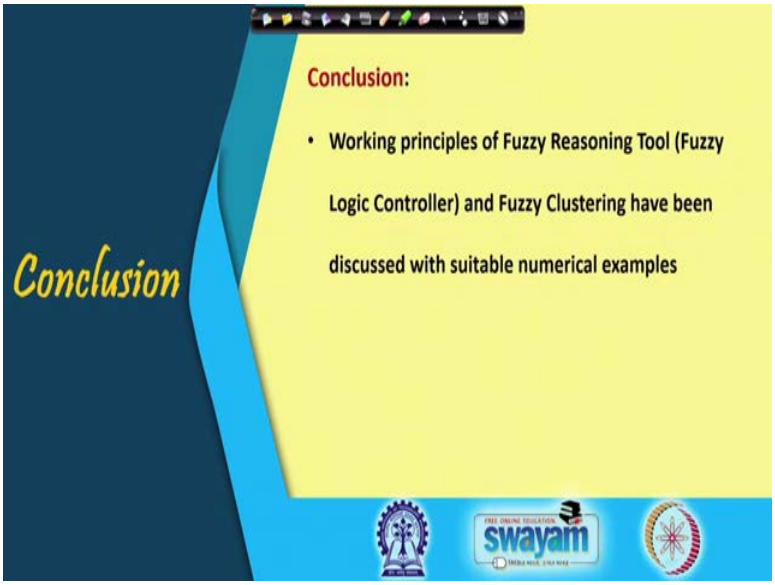

Now, let me conclude, whatever I discussed. So, here we tried to concentrate on the various applications of fuzzy sets. Now, as I told that if you see the literature, the fuzzy set has been used to solve a variety of problems now out of all such problems, two problems I have discussed, in details, like how to develop design and develop the fuzzy reasoning tool in the form of fuzzy logic controller so that we can establish the input-output relationships. So, to establish the input-output relationships, we can take the help of fuzzy reasoning tool or your fuzzy logic controller. And, the principle of fuzzy reasoning tool like your Mamdani approach, then comes Takagi and Savinos approach, we have discussed, in details, with the help of suitable numerical examples.

Now, after that, we started with fuzzy clustering two very popular tools for fuzzy clustering is the fuzzy C-means clustering, another is your entropy-based fuzzy clustering's have been discussed in details, with the help of some numerical examples.

Thank you.