Fuzzy Logic and Neural Networks
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Lecture – 14
Applications of Fuzzy Sets (Contd.)

Now, we are going to discuss, how to use the concept of this Fuzzy C-algorithm to solve one numerical example.

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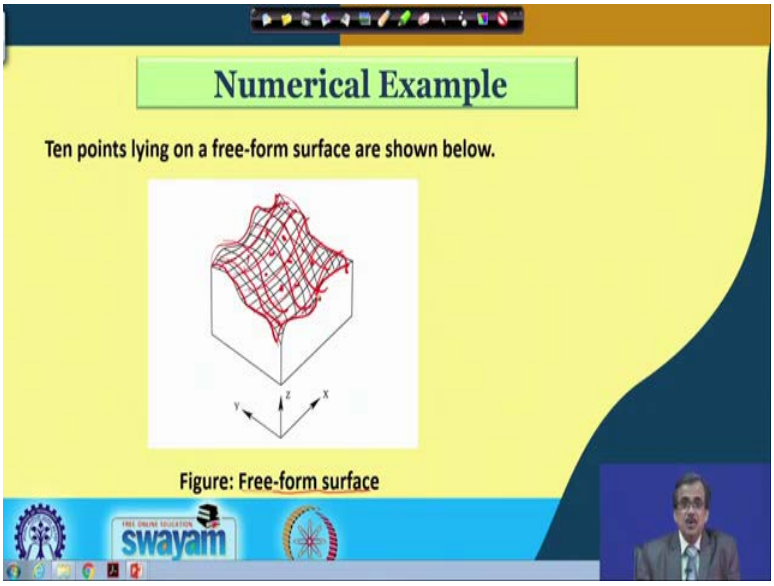

Now, the numerical example, which I have taken here, is a very practical example. The example is as follows: supposing that, we want to carry out some sort of machining operation to generate the free-form surface. Now, the free-form surface is actually a bit difficult to do the machining. Now, a very good example of free-surface could be the surface of your this mouse, this is an example.

Now, how to do this type of machining or how to generate? So, this type of surface, now this is a simple one, but if there are so many such ups and downs for example, the surface which I have considered here, how to carry out the machining. Now, if you see this particular surface, what you do is the nature of the surface is something like this, say I have got that this is the three dimension like x, y and z and the nature of the surface is something like this. So, this could be actually the nature of the surface and here if you see, we have got this type of undulations.

So, there are large number of undulations here and we will have to do the machining just to generate the free-form surface. Now, what you do is, we take the help of some milling cutter. Now, as there are so many such up's and downs on this particular surface, this milling cutter will have to utilize in an optimal sense and truly speaking, the machining has to be done cluster-wise.

So, this surface, we try to divide into a number of clusters based on the similarity and after that, for a particular, we do the machining in one way and for a another cluster, we will have to do the machining in another way, and to decide that particular strategy of machining, so that we can get a very accurate free-form surface, we may take the example or we may take the help of this type of clustering or the fuzzy clustering.

Now, let us see, how to solve so this type of problem. Now, here, for simplicity, what I have done is, I have considered the 10 points only lying on the freeform surface. And, I have tried to show you like how to do the clustering, so that we can select the machining strategy accordingly. Now, let us see, how to proceed with this type of the clustering.

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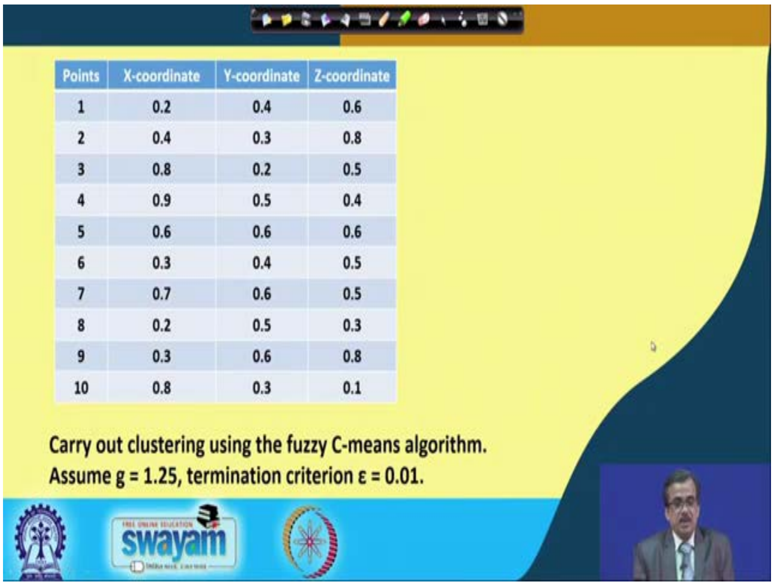

Points	X-coordinate	Y-coordinate	Z-coordinate
1	0.2	0.4	0.6
2	0.4	0.3	0.8
3	0.8	0.2	0.5
4	0.9	0.5	0.4
5	0.6	0.6	0.6
6	0.3	0.4	0.5
7	0.7	0.6	0.5
8	0.2	0.5	0.3
9	0.3	0.6	0.8
10	0.8	0.3	0.1

Now, as I told that we have considered, for simplicity, only 10 points, so that I can show you the hand calculations but truly speaking, on the surface you will have to generate a large number of points like 1000 points, 10000 points something like this. But, for this numerical example, I have just considered 10 points selected at random and these points are lying on the free-form surface and let us see like how to do the clustering?

For example, for the 1st point, the x dimension is 0.2, y coordinate is 0.4 and z coordinate is 0.6 and so on. So, for each of the 10 points, we have got x, y and z coordinates and as I told, these points are, in fact, those lying on the free-form surface. Now, what is our task? We will have to carry out the clustering using Fuzzy C-means algorithm and we are going to assume that the level of cluster fuzziness, that is, g is 1.25 and termination criterion, that is, ε, we have considered 0.01. Now, let us see how to proceed with the clustering.

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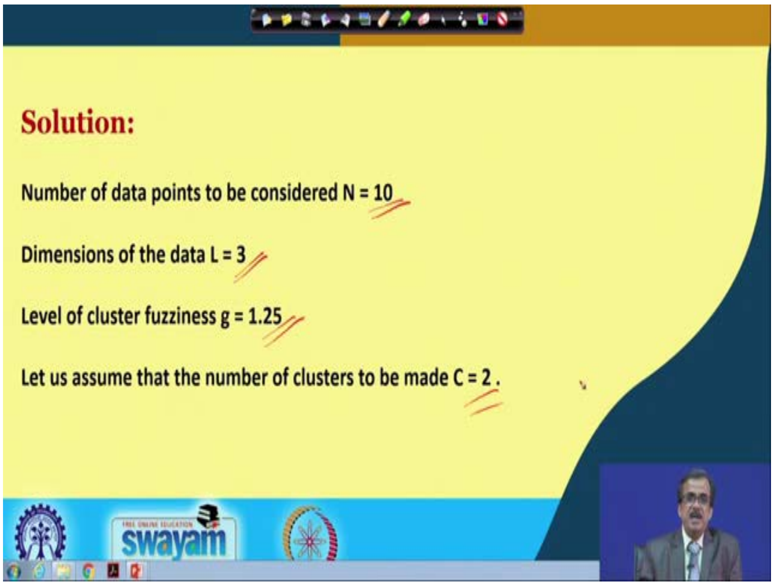

Now, here, the number of data points we have considered that is equal to 10, for simplicity, only 10 points I have considered and each data point is having the three dimensions like your x, y and z.

The level of cluster fuzziness, we have assumed that g equals to 1.25 and let us assume that there could be only two clusters because I have considered only 10 points. So, it is better to go for only two clusters and let us see how does it work, how to explain the working principle of this Fuzzy C-means algorithm to solve this numerical example.

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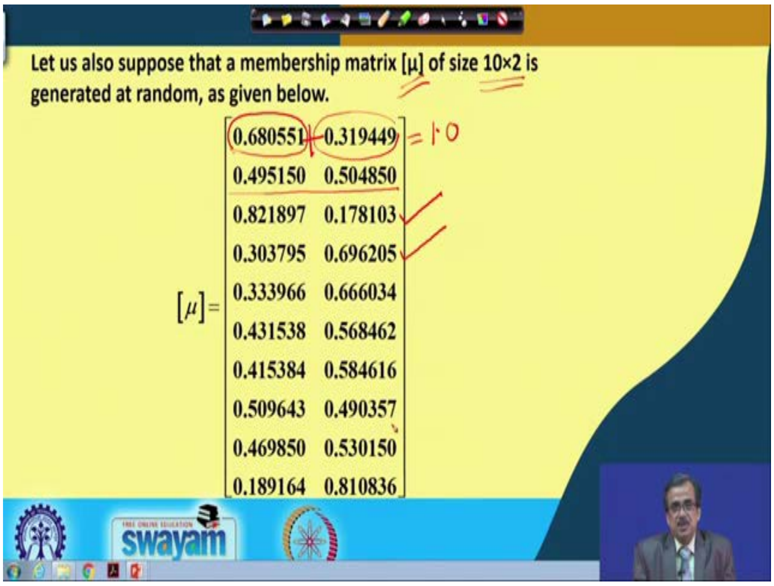

Now, what you do is, so initially, we assume the membership matrix that is denoted by μ and what is the size of this particular μ, it is nothing, but 10×2. Now, why 10×2, because we have got 10 data points and we have considered only two cluster centres.

Now, corresponding to the first data point, so it has got the membership with the two clusters. So, this is the membership with corresponding to the first point with respect to the first cluster centre. So, this is the membership value corresponding to the first data point with respect to the second cluster centre. So, this is the membership function value and if you add them up, this will become equal to 1.0 and the same is true for each of these entries.

So, this is the membership values for the second data point, membership values for the third data point, for the fourth data point and if you add them up, you will be getting 1.0 and these particular matrix the μ matrix of size 10×2. This is generated at random initially. Now, let us see, how to proceed further and how can you do the clustering.

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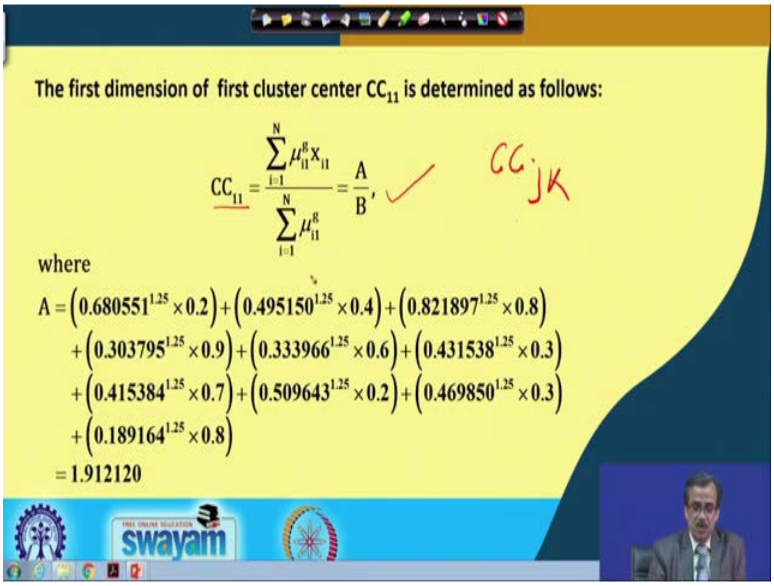

Now, I am just going to determine, what should be the first dimension of the first cluster centre and that is denoted by is your CC_11. If you remember that particular expression which I use CC_jk, now what is that? That is nothing but the k-th dimension of the j-th cluster centre and CC_11 is nothing but the first dimension of the first cluster centre.

Now, the first dimension of the first cluster centre, I am just going to find out. Now, how to do it, the same formula which I have derived, $CC_{11} = \frac{\sum_{i=1}^{N} \mu_{i1}^{g} x_{i1}}{\sum_{i=1}^{N} \mu_{i1}^{g}} = \frac{A}{B}$. Now, let us see how to determine so, this particular A and B? Now, to calculate A, you concentrate here, so μ_{i1}^{g} and i varies from 1 to N, N is the total number of data points. Now, what does it mean the moment I put i equals to one. So, this is nothing but μ_{11}^{g}. What is μ_{11}? μ_{11} is nothing but the membership value of the first data point with respect to the first cluster centre.

So, membership value of the first data point with respect to the first cluster centre. Now, if you see the previous thing the membership value of the first data point with respect to the first cluster centre, so, this is actually the numerical value ok. Now, similarly the membership value of the second point second data point with respect to the first cluster centre this is nothing but the μ value. Similarly, the membership value of the third data point with respect to the first cluster centre is nothing but this particular the numerical value. And, corresponding to your the first the data point, if you see, the X dimension is nothing but 0.2, second data point if you see the X dimension is your 0.4, third data point the X dimension is 0.8, and so on. Now, I am just going to use all such information here.

Now, A is nothing but this particular expression and I am just going to put first your i is equals to 1. So, it is $\mu_{11}^{g} \times X_{11}$. Now, X_11 means what? The first data point first dimension. So, that is 0.2 and this is the membership value raised to the power 1.25. The next is i equals to 2; that means, your μ_{21}^{g} that is your the membership value of the second data point with respect to the first cluster centre and this is nothing but this multiplied by X_21. What does it mean? It means that the second data point first dimension and that is your 0.4.

Similarly, when i equals to 3, this is the scenario, then i equals to 4, i equals to 5, i equals to 6, i equals to 7, 8, 9 and i equals to 10. So, by following this and if you just simplify, I will be getting one numerical value for A and that is nothing but 1.912120. So, this is nothing but the numerical value for A. So, I hope this is clear to all of you and now, I will have to find out what is B.

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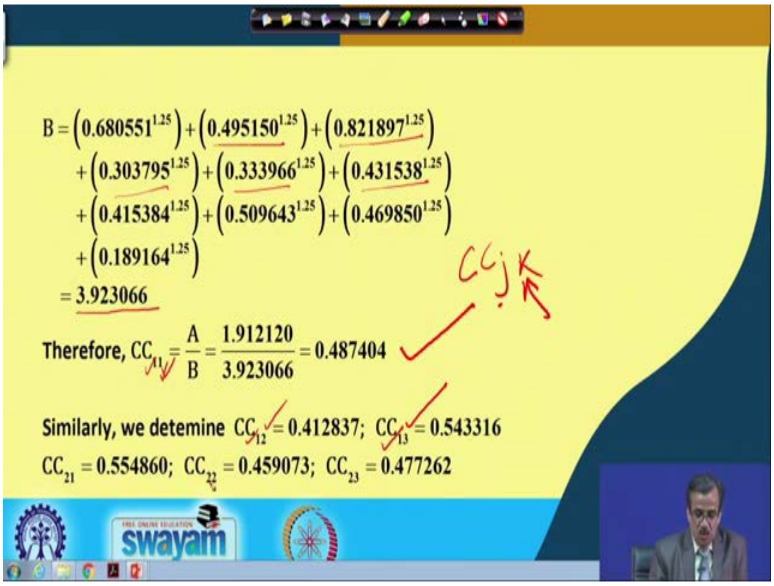

Now, to find out this particular B, B is nothing but, so μ_{i1}^{g} and i varies from 1 to N.

So, I put i equals to 1, next time i equals to 2, i equals to 3, then at the end, i equals to N. Now, if I do that then, very easily, I will be able to find out B, that is nothing but 0.680551 raised to the power 1.25 plus 0.495150 raised to the power 1.25 and these corresponds, in fact, i equals to your 2, similarly i equals to 3, 4, i equals to 5, equals to 6.

So, this is the way actually, we can find out and we can consider all the data points and ultimately, you will be getting the B. And, once you have got this particular B, so CC_11 that is what that is nothing but the first cluster centre, the first dimension. So, the first dimension of the the first cluster centre, let me once again write CC_jk is the k-th dimension of the j-th cluster centre.

And, CC_11 is the first dimension of the first cluster centre and I will be getting 0.487404 and by following the same procedure, I can find out what is CC_12 and that is nothing but the second dimension of the first cluster centre. Then comes your CC_13 and that is nothing but the third dimension of the first cluster centre and that is your 0.543316 and similarly, we can find out CC_21 that is the first dimension of the second cluster centre, CC_22 second dimension of the second cluster centre, that is 0.459073.

And, by following the same procedure, I can also find out CC_23 that is your the third dimension of the second cluster centre. So, I will be getting this particular the numerical value.

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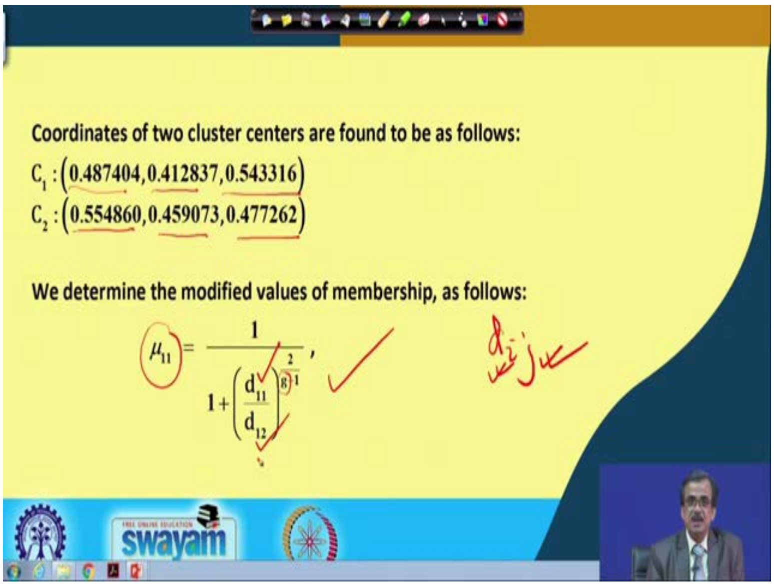

And, once you have got this particular the numerical value now, the coordinates of cluster centres are known and these are determined. So, we have got this C_1, that is the first cluster centre and it is corresponding the first dimension, second dimension and the third dimension because, these are all 3 D data and for the second cluster centre, the first dimension second dimension and the third dimension. Now, once you have got it what will have to do is, you will have to update this μ.

That means, I will have to update the membership value of a particular data point with respect to the clusters. So, how to update now to update that? So, this formula I have already derived. So, this mu_11 is nothing but 1 divided by 1 plus d_11 divided by d_12 raised to the power 2 divided by g minus 1. Now, we will have to substitute the values for this particular g, that is the level of cluster fuzziness which is 1.25 here we will have to find out the Euclidean distance, that is, d_11 and d_12.

So, if you remember the d_ij is the Euclidean distance between the i-th data point and the j-th cluster centre, this d_11 is nothing but the Euclidean distance between the first data point and your the first cluster and d_12 is the Euclidean distance between the first data point and the second cluster.

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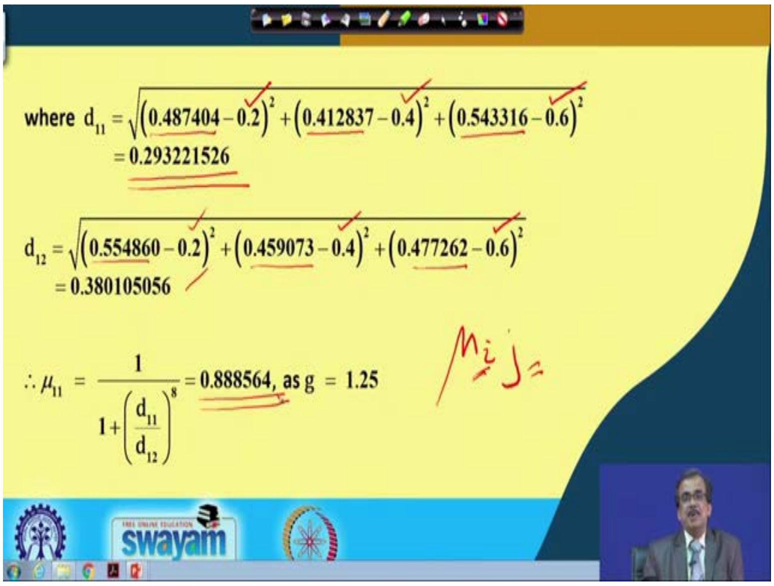

So, how to determine that? To determine this actually, mathematically, we can calculate this d_11, as I told the Euclidean distance between the first data point and your the first cluster centre, and these are nothing but the dimensions of the first cluster centres. And, corresponding to the first data point this is my x, y and z coordinates and using this, so I can find out the Euclidean distance d_11 and this will become equal to this. And, following the same procedure, I can also find out d_12 is nothing but square root the first data point and second cluster centre.

So, this is actually the dimension of the second cluster centre and this minus 0.2 square, this minus 0.4 square, this minus 0.6 square and these 0.2, 0.4, 0.6 these are nothing but actually the dimensions of the first data point. So, out of those ten data points, these are the dimensions or the coordinates of the first data point. So, very easily, I can calculate the d_12 and that is nothing but 0.380105056. Now, I can find out what is μ_{11}?

Now, this μ_{11}, let me once again repeat that is actually here, if you see the mu ij that is the membership value of the ith data point with the jth cluster centre. Similarly, the membership value of the first data point with your the first cluster $\mu_{11} = \frac{1}{1+(\frac{d_{11}}{d_{12}})^8}$, ok, this formula I have already derived. And, if we just put all the numerical values and if you calculate, so you will be getting this μ_{11} is nothing but 0.888564. So, I will be getting that particular the membership value.

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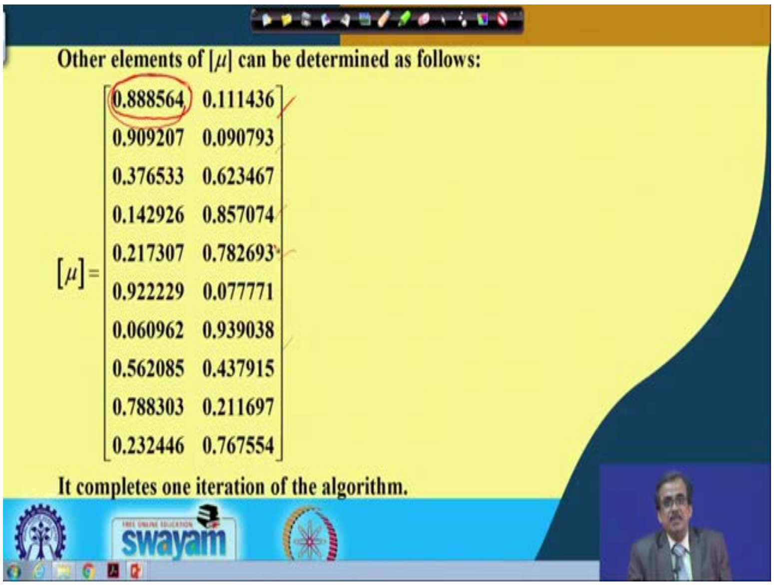

Now, following the same procedure, I can also find out the other values and if you calculate, then you will be getting this thing, I have already calculated, that is 0.888564, just now I calculated. Similarly, the other membership values also, you can calculate. So, these are all updated values. If you remember, to start with the algorithm, initially we actually assumed some numerical values and after that we are updating those values, so we have already discussed how to get this particular numerical value follow the same principle.

So, to get all the numerical values, exactly the same principle we will have to follow. It completes actually one iteration of this particular the algorithm now if you remember. So, at the beginning of the first iteration we started with some initial assumption of the μ matrix and now, you have got slightly modified or updated matrix and the in the second iteration, we are going to start with this particular μ matrix and once again, we will repeat the process and these particular iterations will go on and go on.

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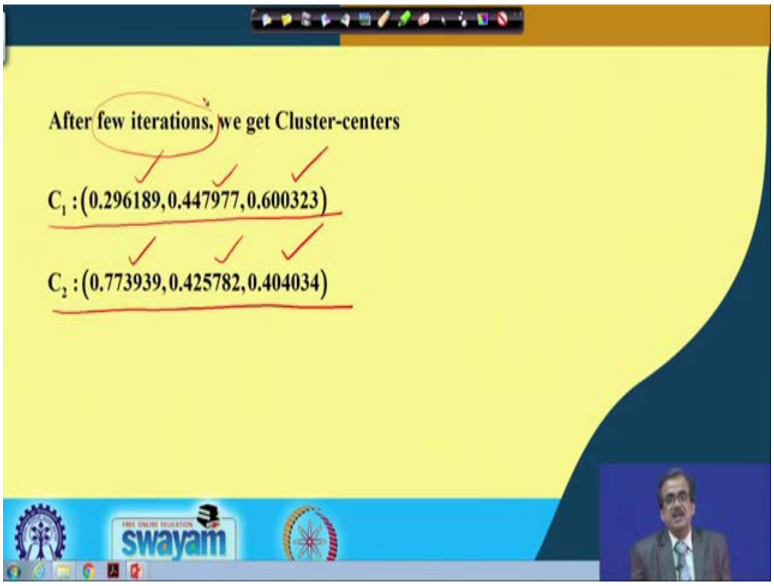

And, after a large number of iterations, there is a possibility that we will be getting the modified, cluster centre and the modified values for these memberships. Now, after a few iteration, there is a possibility that we will be getting the cluster centre something like this. So, this will be the first cluster centre having x, y and z coordinates. The second cluster centre like x, y and z coordinate and this will be getting after running this particular algorithm for a few iterations. So, might be after 20 iterations, 30 iterations. We will be getting so, this type of modified cluster centres.

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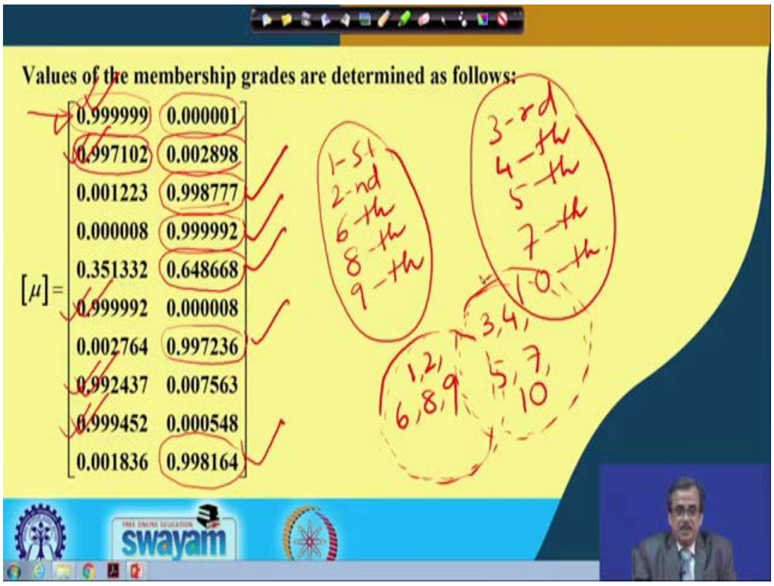

And, corresponding to that, we will also be getting, what should be the modified the μ values, that is the membership function values. Now, let us try to relook, what does it mean, now let me repeat the same thing. Now, if you see this μ matrix and if we concentrate here, what is the practical meaning? The meaning is something like this, so taht the first data point; the first data point belongs to the first cluster with this much of membership value. It belongs to the second cluster with this much of membership value; sum of their values is equal to 1.0.

Now, this shows the membership value of the second data point with the first cluster centre, membership value of the second data point with the second cluster, and so on. Now, you try to locate so, out of these your the ten sets of values and with respect to the first cluster centre, whose values are very near to 1. For example, you can see, corresponding to the first data point and the first cluster centre this particular μ value is very near to 0, this is also very near to 0, this is also very near to 0, this is also very near to 0, this is also very near to 0; that means, your the 1st, then comes here, the 2nd, then 3rd, 4th, 5th, 6th, 6th, 7th, 8th, 8th and your 9th, they are forming a particular group, ok.

And, if you see here, corresponding to this, this is the membership value with respect to the second cluster and it is very near to 1, here also, it is very near to 1 with respect to the second cluster, here it is in between, but you just give the opportunity to join the second cluster. So, this is very near to 1 with respect to the second cluster, this is also very near to 1 with respect to the second cluster, ok.

So, this is one point, this is another point, another point, another point, another point. So, this is the 3rd one, 3rd point, then comes your the 4th point, then comes your the 5th point, then 6, 7 points and this is your the 10th point so, this will form another group. Now, in terms of clusters like say it will form one fuzzy cluster something like this. So, this is one fuzzy clusters. There could be another fuzzy clusters and there could be overlapping also and might be here, the first point, second point 6th, 8th and 9 points are lying and here there is a possibility that 3rd, 4th, 5th, 7th and 10th points are lying. So, I will be getting one such fuzzy cluster here, and I will be getting another such fuzzy cluster here.

So, this is the way using the concept of the fuzzy C-means clustering, we will be able to do the clustering and we will be able to get, say two fuzzy clusters particularly, for this very simple problem having only 10 numerical data like 10 data points and we have seen that this algorithm is able to do this clustering very efficiently.

Thank you.