Fuzzy Logic and Neural Networks
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Lecture – 10
Applications to Fuzzy Sets (Contd.)

So, corresponding to the four fired rules, this is actually the fuzzified output, the combined fuzzified output which you have got.

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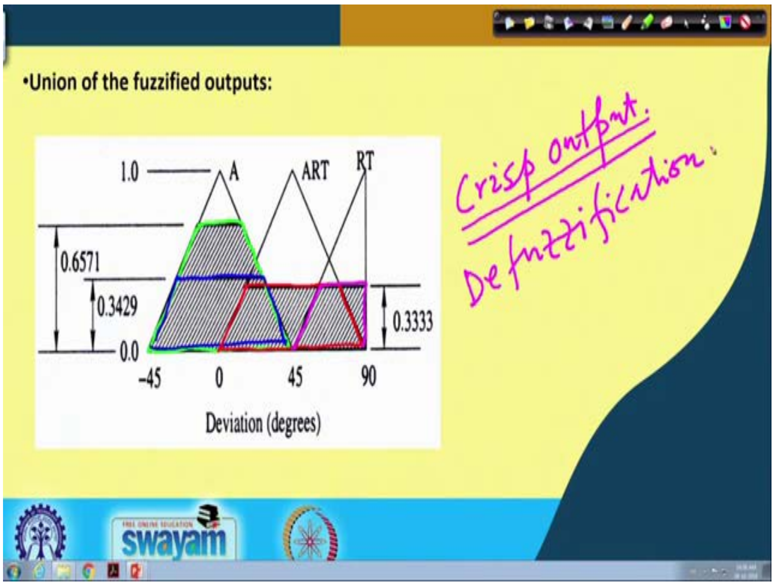

And, as I told, our aim is to determine what should be the crisp output. And, to determine the crisp output actually, we will have to take the help of defuzzification. Now, I am just going to discuss the different methods of defuzzification. Now, let us try to concentrate on the methods of the defuzzification.

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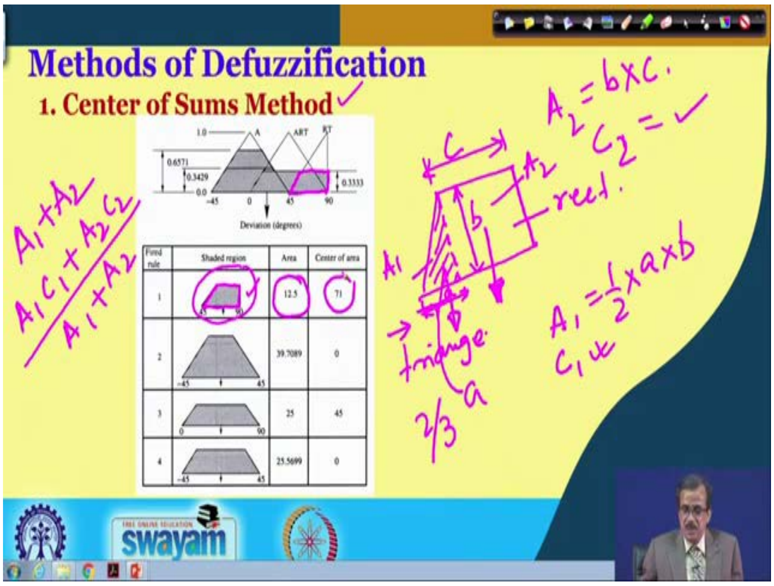

Now, the first method for which I have already discussed the principle, and now I am just going to apply to solve this numerical example. Now, the first method, this particular method is your the center of sums method, now let us see according to this, how to determine the crisp output. Now, if you see corresponding to the different fired rules, we will be getting the different shaded regions as the fuzzified output; for example, say if I concentrate on this particular output first.

So, this particular output corresponds to a particular fired rule and this shaded portion, I have just drawn it here, and what is our aim? Our aim is to find out like what should be the area of this particular shaded portion, and where should be the center of area. Now, this is very simple now to find out the area and center of area, you can do it in different ways. Now, one of the very simplest ways could be something like this, for example, say I have got this type of say truncated area.

So, this type of truncated area I have got, ok. Now, how to find out the area and center of area, it is very simple. So, you just divide into two regular regions. So, this is nothing, but a triangle. So, this is nothing, but a triangle and this is nothing, but one rectangle. Now, for this particular triangle, very easily, I can find out what should be the area. Now, if I know, this particular dimension is say a and if I know this particular dimension is say b. So, the area this is A_1. So, this is $A_1 = \frac{1}{2} \times a \times b$ and where will be the center of area? The center of area is C_1.

So, starting from here, it is two-third, one-third. So, this is nothing, but this particular dimension starting from here. So, it will be you two third a. So, I can find out this is nothing, but your two third a. So, it is two-third, one-third. So, if this is the situation, very easily I can find out what is C_1. Similarly, for the second region the sub region that is the rectangular one. So, if I know so, this particular dimension is say C. So, very easily I can find out what is A_2 so, for this area is A_2.

So, A_2 is nothing, but b multiplied by c, and very easily you can find out what is C_2, that is the center of area from symmetry, I can find out very easily. And, once you have got this particular thing, you just add what should be the total area. So, the total area is nothing, but A_1 plus A_2 and if you calculate this will become 12.5 and the center of area. So, how to find out the center of area? It is very simple that is nothing, but

$$\frac{A_1C_1 + A_2C_2}{A_1 + A_2}.$$

Now, this will give you the center of area. Now, for this actually, the shaded region if you find out the center of area so, you will be getting as 71, ok. This is how to find out actually the area and center of area corresponding to one fired rule and its shaded region as the fuzzified output. Now, if you follow the same principle for the other fired rules, for example, say if I concentrated on say another fired rule. So, this particular fired rule that is your so, this is from minus 45 to 45. So, this is your this particular thing.

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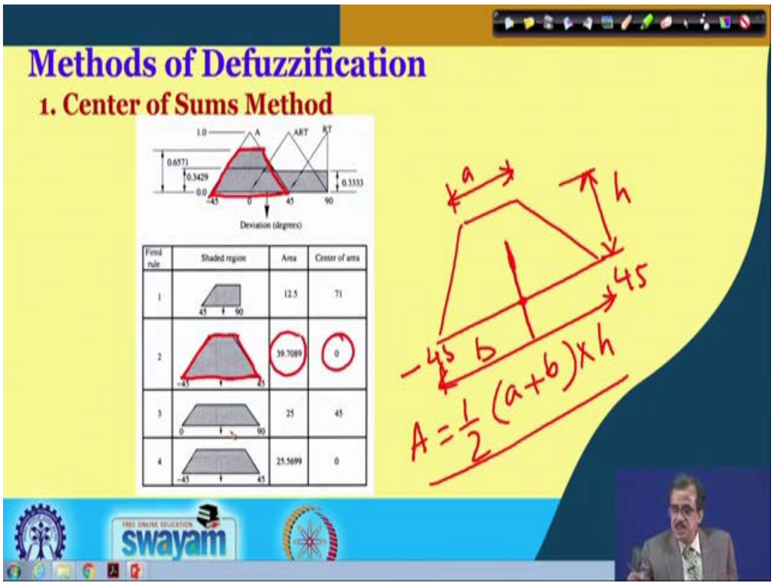

So, this particular shaded region, if we consider, very easily you will be able to find out what should be the area and center of area. Now, how to find out the area and center of area it is very simple, it is just like a trapezium. So, for this trapezium, if I know this particular dimension say a and this particular dimension say b and if I know so, this particular dimension say h. So, very easily I can find out this area is nothing, but $\frac{1}{2}\times(a+b)\times h$. So, this is nothing, but the area.

And, if you calculate, you will be getting this as the area and how to find out the center of area, from actually, it is symmetric, I can find out the center of area; if it is minus 45 and if it is plus 45. So, the center of area should pass through 0. So, this is the way actually you can find out the area and center of area. The same method you can also adopt for this, now if you just do here.

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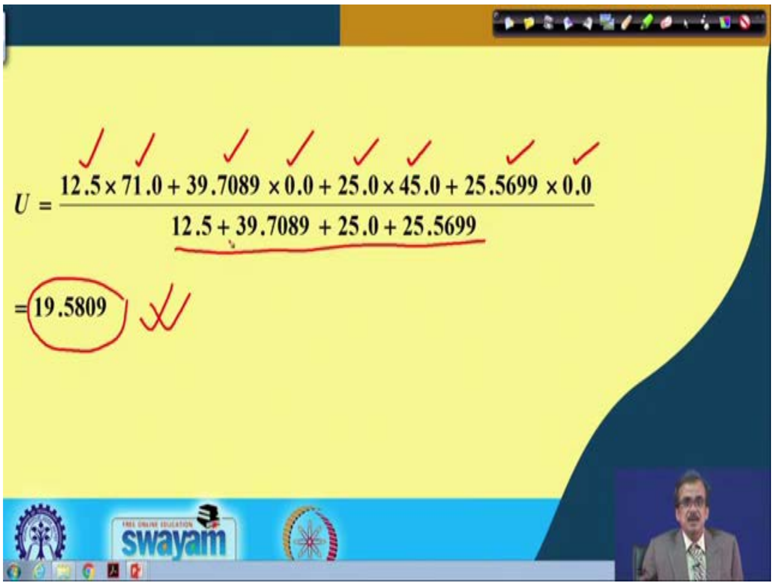

So, what you can do is your for this particular area so, 0 to 90. So, 0 to 90 is nothing, but this. So, I can use the same principle of how to determine actually your area and center of area.

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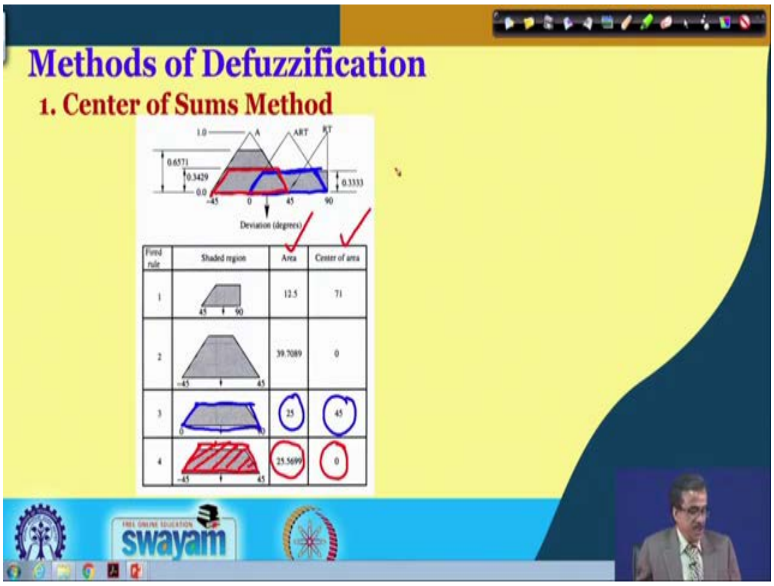

So, this shaded region. So, I can find out area and center of area following the same method and for this, you will be getting the area is nothing, but is your 25 and the center of area. So, this is 0 and 90 center of area will be your 45. And, next we concentrate on is another this thing another shaded area and this is nothing, but is your another shaded area is something like this and that means, I am here. So, this particular shaded area and I can find out following the same principle, what is the area and what is the center of area. The area will become equal to this 25.5699 and the center of area will be 0. Now, for each of the fired rules, I am able to find out what is area and what is center of area.

Now, according to this your the center of sums method, the crisp output can be determined as like area that is your area multiplied by the center of area for one fired rule, then area multiplied by center of area for another fired rule, area multiplied by the center of area for another fired rule, then area and center of area and here we put sum of all area values. So, here, it will be getting actually the crisp output. So, using your center of sums method so, I can find out so, this that is your 19.5809 as the crisp output. Now, I will see for the same problem actually how to use the other method of defuzzification.

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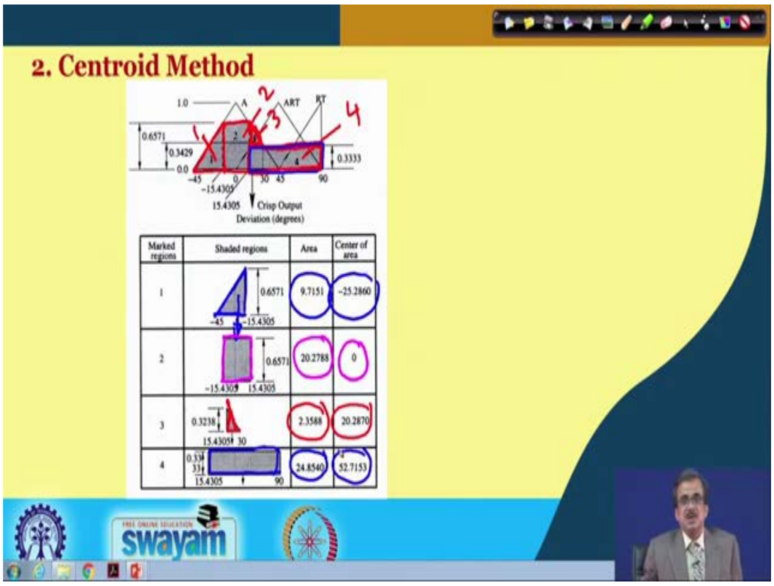

Now, the principle of centroid method actually I have already discussed. Now, here actually, what we do is, we try to divide the whole region, the whole fuzzified output after considering all the fried rules. We try to divide into a number of some regular sub-regions. For example, say if I consider all the four fired rules, the combined output is nothing, but this.

So, if this is nothing, but the combined output, say this is the combined output, after considering all four fired rules. So, what we do is, we try to divide this into a number of regular sub-regions. Now, the sub-region could be something like this, for example, say one sub-region could be this. So, one is nothing, but a triangle, next we can go for the second sub-region that is nothing but the rectangle, then here, we have got a triangle. So, this is nothing, but 3 and we have got actually here, one such your rectangular sort of thing and this is nothing, but is your 4.

So, this particular shaded portion the combined control action, is divided into 4 sub-regions regular sub-regions. Now, if I concentrate on each of these particular regular sub-regions for example, say if I concentrate on sub-region 1. So, this is actually the sub-region 1, that is nothing, but a triangle. So, I know the dimension. So, very easily, I can find out what is this particular area of the triangle and I can also find out what should be the center of area that is your two-third one-third principle, and I will be able to find out the center of this particular area.

Now, I concentrate on your the second one, that is nothing, but the rectangle. So, if I consider so, these particular rectangle here. So, what will be getting is your so, this is the rectangle. So, very easily I can find out what is the area and the center of area. So, this will become equal to 0. Next is your, I can concentrate on the third region and that is nothing, but your the triangle. So, this small triangle, that is this particular small triangle if I consider; I will be getting this type of triangle here, and I know the dimensions. So, very easily, I can find out what is area and center of area, then I can concentrate on this rectangle.

So, your this particular rectangle and I can find out the area and the center of area. So, this is the rectangle, area is nothing, but this and center of area is nothing, but this. Now, for each of the sub-regions, I know the area and center of area, now I can find out what should be the crisp output corresponding to this.

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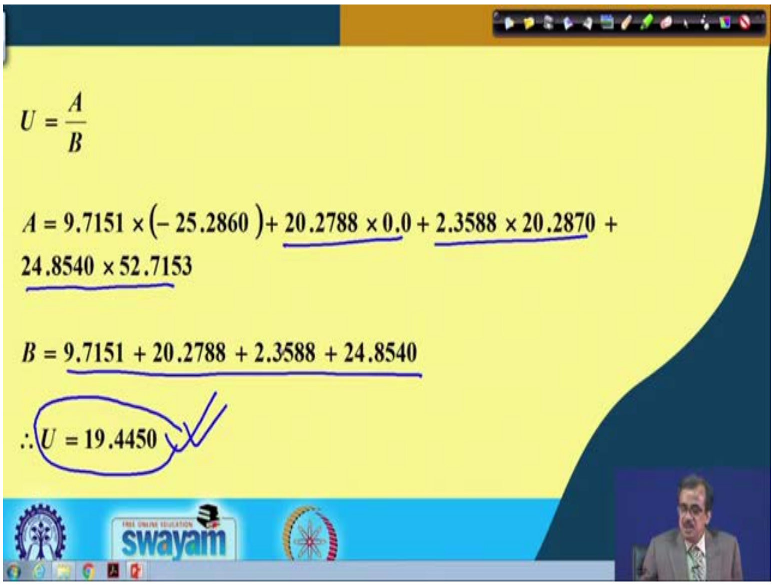

And, the crisp output corresponding to this is nothing, but U; U is a nothing, but A divided by B, where A is your the area and centre of area, area and center of area then area and center of area, area and center of area. There are four regular sub-regions, and here, we add all the area values. And, if we just calculate A divided by B, you will be getting the crisp value, that is nothing, but 19.4450.

So, this is nothing, but the crisp output corresponding to the fired rules. So, this is the way actually, you can find out the crisp output using the centroid method. Now, then comes your another method that is called the mean of maxima.

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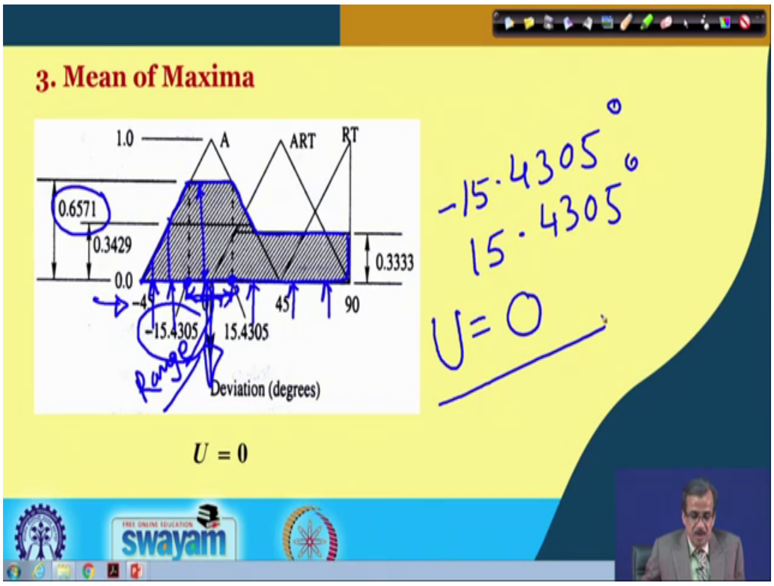

Now, the principle of this particular mean of maxima; I have already discussed now let us see how to use the same principle to find out the crisp output. Now before I discuss once again the mean of maxima, the first thing we will have to do is. I will have to indicate actually the combined control action, that is your fuzzified output considering all the four fired rules.

Now, if I just indicate, I will be getting this is actually the combined control action, the fuzzified output. So, this is the area, ok. Now, what you do is. So, we start from here and we increase this particular variable in this particular direction and we try to find out. So, corresponding to these, we try to find out the μ, corresponding to this we try to find out μ, here we try to find out μ we try to find out μ. So, we can find out that the μ is going to vary or it may remain constant or different values for this particular your deviation angle.

Now, if I just look into this. So, there is a possibility that it will be getting a range for the deviation, the range is something like this. So, this particular range corresponding to which actually you will be getting the maximum value for this μ. So, starting from here up to this, the μ is something like this, that will remain constant and we are able to reach the maximum value for this particular μ. And, if you calculate for this problem, this will become equal to 0.6571.

Now, here actually, what you can do is, I can identify the range for the deviation corresponding to which I am getting the maximum value for this particular μ and that particular range starts from here and it will end here. So, this is nothing, but actually the range, where we are getting the maximum value for this particular membership that is μ; and once you have got this particular range. So, what we do is we try to find out the mid value of this particular range as the crisp output. For example, say, here we are getting one value for the range is your minus 15.4305 degrees and the maximum value of the range for which you have got maximum value of μ is 15.4305 degree and its mid value is nothing, but 0. So, your the crisp value is nothing, but 0.

So, using the mean of maxima, I will be getting your the crisp value and that is equal to 0, now you see the problem. Now, by using the three different methods of your defuzzification; in the first method we have got the crisp value for the same set of inputs as 19.5809 and using this particular the centroid method, what you have got is, the crisp value is something like this. So, this is actually the crisp value corresponding to the centroid method and then, using this, your the mean of maxima method actually I am getting this crisp value and that is nothing, but 0.

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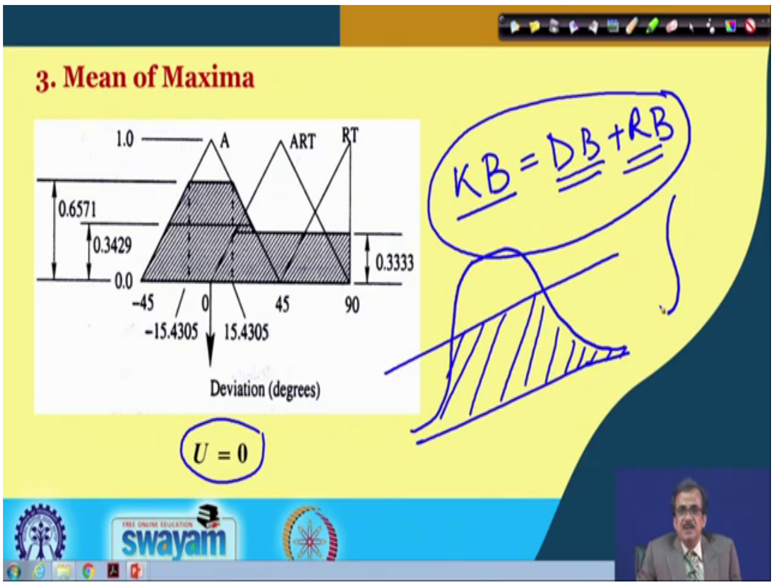

So, even the inputs are the same. So, by using the three different methods of your defuzzification, I am getting three different values, now my question is, which one to be believed and which one is the most accurate. The answer is you can believe any one and you can carry out the optimization because ultimately, if you want to utilize this fuzzy reasoning tool or the fuzzy logic controller, the main thing which you will have to do is you will have to find out, what should be the optimal knowledge base.

Now, how to find out the optimal knowledge base, I have not yet discussed, I will be discussing in details. Now, this knowledge base consist of your data base and the rule base and by using any one of these methods of defuzzification, I will be getting the output and based on that particular output, I am just going to use some optimizer and with the help of training scenario, I am just going to develop what should be my optimal knowledge base, that is the optimal data base and optimal rule base.

Now, once you have got that particular knowledge base, the optimal knowledge base, your fuzzy reasoning tool is going to perform in a very nice way, in a very good way and it does not depend on which method of defuzzification you have used. You can either use the center of sums method or you can use the centroid method or the mean of maxima method and you can find out, what should be the optimal knowledge base that is the optimal data base and the rule base, so that this fuzzy reasoning tool can perform in the optimal sense for any set of input parameters.

Now, here, I just want to compare these three methods of defuzzification once again in terms of computational complexity. Now, this I have already discussed in my last lecture, but once again I just want to mention that in terms of computational complexity, mean of maxima method is the fastest. Because in center of sums method or centroid method, you will have to calculate area and center of area, which is computationally very expansive particularly if you consider the non-linear distribution of this membership.

Now, if we consider the non-linear distribution for example, say if I consider say Gaussian or if I consider say the bell-shaped membership function distribution, and if I have to find out what should be the area and center of area. For example, corresponding to the μ supposing that I am getting so, this is the area. So, for determining this particular area, we will have to take the help of your integration and you know that integration is computationally very expensive and that is why, it is better to use the mean of maxima because it does not calculate the area or the center of area, but still it can find out actually one crisp value. And using this particular crisp value, you can carry out some sort of training or some sort of optimization for the fuzzy reasoning tool so, that we can find out the database and the rule base.

And, once you have got this particular the optimal knowledge base for the fuzzy reasoning tool, now we are in a position to use this fuzzy reasoning tool online because once the training is obtained, once it is trained, now the computational complexity is not a problem. And, within a fraction of second using the fuzzy reasoning tool, you will be getting the output for a set of inputs. So, this is the way actually the Mamdani approach works, but as I told that this particular approach is having some problem, it has got some merits and demerits. This I have already discussed, but once again, let me repeat a little bit now, for this Mamdani approach actually.

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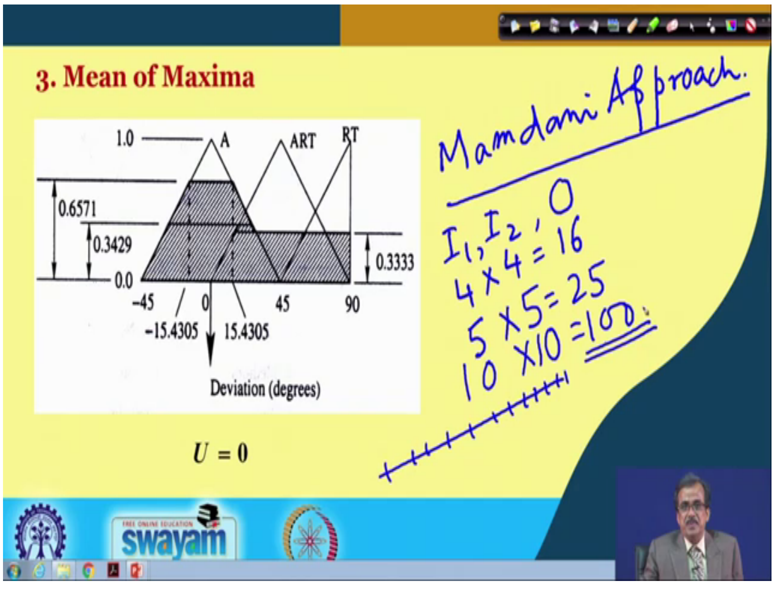

So, it has got very good interpretability, this I have already mentioned now; that means, if you read this particular rule, very easily you can find out what should be the control action; that means, it has got very good interpretability, but the problem is actually the accuracy. Now if we want to get more accuracy in Mamdani approach. So, what you will have to do is. So, you will have to use large number of linguistic terms, let me take a very simple example. Now, supposing that I am just going to develop Mamdani approach having two inputs and one output, I_1 and I_2 are the inputs and output is nothing, but O.

Now, if I use say four linguistic terms for I_1 and four linguistic terms for I_2. I will be getting 4 multiplied by 4 that is 16 rules. Now, similarly if I consider say 5 linguistic terms for I_1, 5 for your I_2. So, I will be getting say 25 rules. Now, remember one thing, if the range is divided into a large number of small-small sub-regions or segments; that means, if I use more number of linguistic terms to represent a particular variable there is a possibility I will be getting the better accuracy.

Now, if I use more number of linguistic terms, for example, for I_1 if I you say 10 linguistic terms and for I_2 if I use a 10 linguistic term, I will have say 100 rules now. So, if I need more accuracy, the number of rules is going to increase and if the number of rules increases what will happen to the computational complexity, that is also going to increase. That means, in Mamdani approach, I may get slightly better accuracy or the precision, but at the cost of more computation and which is also not desirable.

So, as I told that Mamdani approach has got both merits as well as demerits. Now, in the next lecture, I will be discussing another method in which you will see that interpretability may not be so much good, but accuracy could be better that I will be discussing in the next lecture.

Thank you.