# Fundamentals of Fluid Mechanics for Chemical and Biomedical Engineers

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**Lecture 7**

## Flow Visualisation

Hello, the topic that we are going to discuss today is flow visualization. So, when we look at a flow most of the time, we are following something that is in the flow. For example, when we see the smoke coming out of chimney and by looking at the smoke or by looking at the plume of smoke, we get to have some idea about the, the flow that is happening in the atmosphere. Similarly, if there is a river flowing and you put a particle or put a leaf in the liver, in the river then following the path of that leaf you can see that where the that particular river stream is going or what is its path.

When you want to study the flow behavior or visualize the flow, we generally introduce a tracer in the fluid. in the liquids, it is often a dye which can follow the fluid, which does not mix, which diffusivity of which is low.

And, when we do it in gases, we introduce a smoke, sometimes it is might, in the liquids it might also be done with, with bubbles or, or particles which are non buoyant. Non buoyant means, they have the same density as that of the fluid so the particles do not have any motion because of buoyancy. So, when we visualize the flow, we see different lines, especially when we are trying to study the flow, we can plot different lines depending on the path. So, in today's lecture what we are going to discuss is three such different lines.

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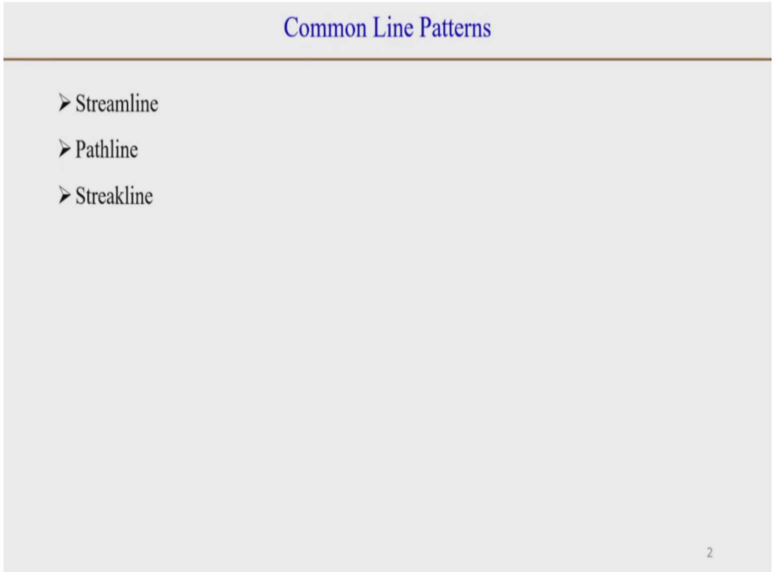


And, these are named as streamlines, pathlines and streaklines. So, what is their definition and how we can give a formal mathematical definition to these lines. So, if you want to compare your flow visualization experimental studies with some theory, then you can compare following this mathematical framework.

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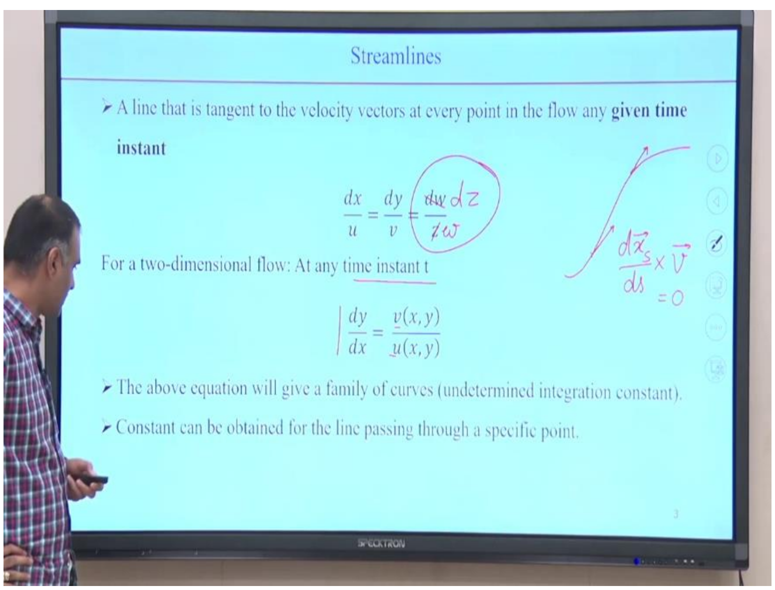


So, let us look at streamlines first. So, streamlines, you might have read in school probably. Streamline is a line that is tangent to the velocity vectors at every point in the flow at any given time instant. So, if you plot, if you plot a streamline then at every point on the streamline, the tangent at every point that represents the velocity at that particular point. So, that means, if you take the position vector of the streamline, let us say dx subscript s vector and have its cross product with the velocity vector.

So, if you have $\frac{dx_s}{ds}$ which is the slope of this multiplied by velocity vector then that will give you zero because they are in the along the same lines or they are parallel to each other. So, if you simplify this what you will get $\frac{dx}{u} = \frac{dy}{v} = \frac{dw}{z}$. Now, x y, z are the, are the coordinates at that point in the streamline and u, v, w are the velocity component along x, y and z directions.

So, if you have a two dimensional flow where this should have been the other way around, so this should have been dz and this should have been the w. Sorry for the typo. So, if the flow is two dimensional then this component will not be there, z component of velocity will not be there, w is equal to 0. And then, you will have $\frac{dx}{u} = \frac{dy}{v}$. So, you can easily find the slope of this streamline by rearranging this equation in this form that $\frac{dy}{dx}$ which is slope of the streamline, that is equal to $\frac{v}{u}$.

Another point to see here is the streamline is plotted at a time instant t. So, if your flow is steady then your streamline is not going to change, but if your flow is unsteady, the velocity is dependent on time. So, at a particular point the velocity may change with time because the flow is unsteady. In that case your streamlines will be changing with time. So, when you talk about streamlines, you are talking about streamline at a time instant. Say, t is equal to 3 second and streamline at t is equal to 4 second will be different if the flow is not steady.

So, if we know the velocity vector y and x component of velocity in a two dimensional flow in xy plane, then by integrating this equation you will get the streamlines. But you will have a integration constant. So, when you vary that constant what you will get is a family of curves. Now, if you want to determine a particular streamline, then you will need a condition to, to determine that constant. So, let us say if, if you are asked that find the streamline passing through a particular point $(x_{0,} y_0)$, then you can find that constant by substituting the values and find a particular streamline.

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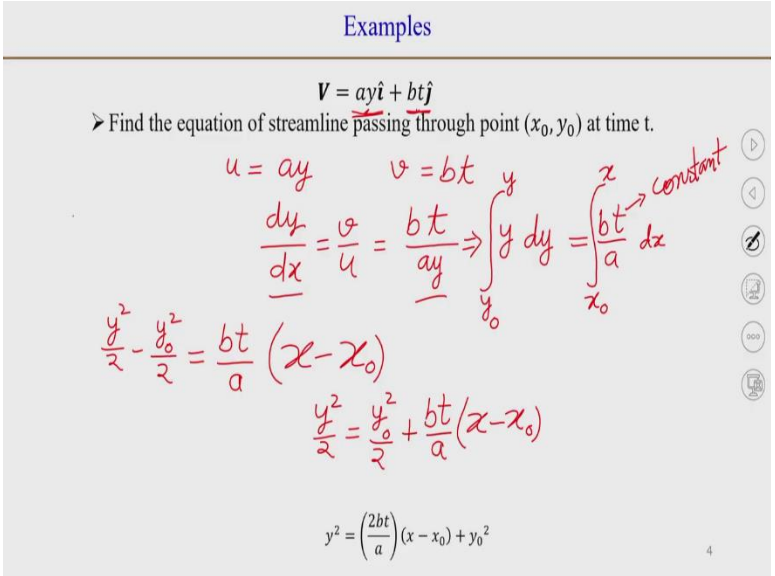


So, let us look at an example here. We have the velocity vector a y i + b t j. So, it is a two dimensional flow. The x component of velocity, it is dependent on y coordinate and y component of velocity, it is dependent on time. Now, if we want to find the equation it is, sorry, it is not the y coordinate or the y, it is y component of velocity. So, if you want to find the equation of a streamline that is passing through point x0, y0 at time t. Note, that because the velocity is dependent on time, so when finding out the streamlines, you will need to know the time instant you which, at which you want to plot the streamline.

So, we can write down the x component of velocity u is equal to a y and y component of velocity v is equal to b into t. You can write $\frac{dy}{dx} = \frac{v}{u}$ , where v is bt and u is equal to ay. So, if you rearrange it, you will get y dy = bt by a dx. And on integration you will get y square by 2 is equal to, when you integrate you will integrate with the constants and because we know that at time t, with it is passing through point $x_0$ $y_0$ so you can take the integration limits from $y_0$ to y and $x_0$ to x.

Now, you also note that because we are talking about streamline at time t is equal to $t_1$, $t_2$, 3 second, 4 second, so we are talking about at a particular time. So, this time when you do you will be treating as a constant. So, when you integrate you get

$$\frac{y^2}{2} - \frac{y_0^2}{2} = \frac{bt}{a}(x - x_0)$$

y squared by 2 - $y_0$ squared by 2 is equal to bt by a, which is constant into when you integrate dx you will get x and substituting the limit you will get x - $x_0$.

So, the equation you will get is y squared by 2 is equal to $y_0$ squared by 2 + bt by a into x - $x_0$. That is the equation of streamline. Or if you multiply everywhere by 2, so you will get y squared is equal to 2bt by ax - $x_0 + y_0$ squared.

$$y^2 = \left(\frac{2bt}{a}\right)(x - x_0) + y_0^2$$

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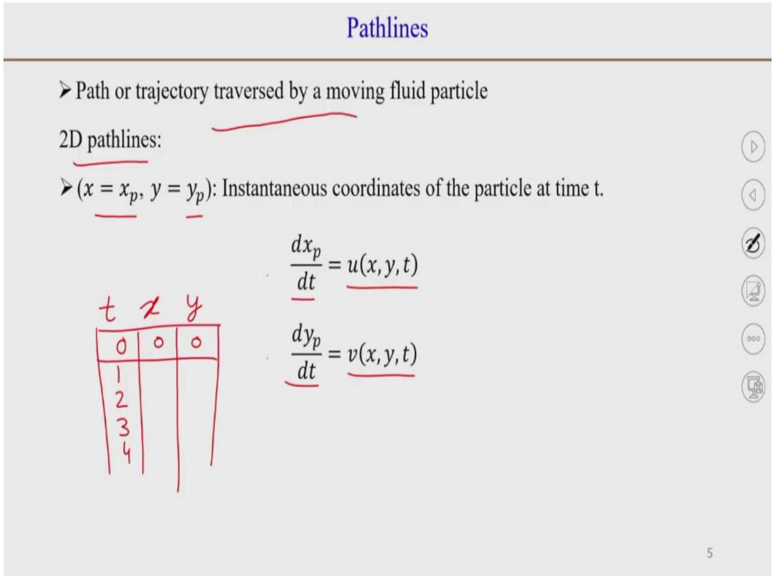


So, let us move to the next line which is called pathline and pathline is simply the path or trajectory that is followed or traversed by a moving fluid particle. So, if you take a particle or if you take a solid particle, you dip it in any flow or you introduce it in the flow, the particle should be non buoyant, that it is following the motion of the fluid only. And, if you keep following the motion of this particle, the line traced will be what is called pathline.

So, at every point the velocity of the fluid particle will be the fluid velocity. So, for simplicity, let us talk about 2D pathline again in xy plane or in xy coordinates. So, if we say that the coordinate of this particle is $x_p$ and y coordinate is $y_p$, then we can write down instantaneous at time t

$$\frac{dx_p}{dt} = u(x, y, t)$$

$$\frac{dy_p}{dt} = v(x, y, t)$$

$dx_p$ by dt is equal to u xyt and $dy_p$ by dt is equal to v xyt, where x, y is the space coordinate and $x_p$, $y_p$ is the coordinates of the particle at time instant t.

So, we can integrate these equations and you will find out when you integrate this equation, you will get $x_p$ as a function of time. You integrate this equation you will get $y_p$ as a function of time. And to plot the pathline, what you can do is you can eliminate time. So, you will get a equation of pathline or if you just want to simply plot, you can have 3 columns.

Say in, if you are plotting in an Excel sheet, you can or you can make a table where you are plotting time, x and y. Of course, again to find out the particular pathlines, you will need the initial position of the particle. So, at time t is equal to $t_0$ what is the x location of the particle and what is the y location of the particle. So, $x_0$ and $y_0$.

So, you will start with say, if the particle is at time t is equal to 0 at position x is equal to 0 and y is equal to 0 and then you can just keep finding out what the location of the particle is. And then you can plot x on, an xy plane these two. So, you will get the pathline in xy plane.

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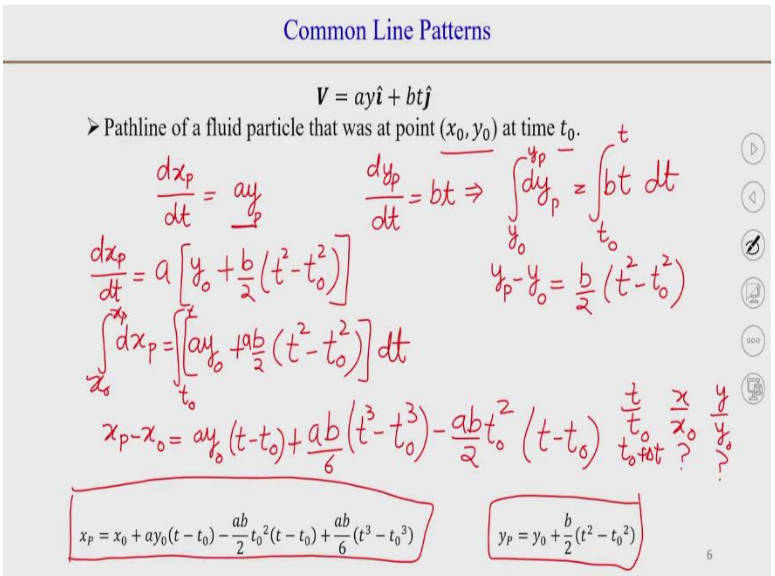


Now, let us look again the same example. You have velocity $V = ay\hat{\imath} + bt\hat{\jmath}$

and you want to find the pathline of a fluid particle that was at a point x0, y0 at time $t_0$. So, if we write down d$x_p$ by dt that is equal to ay and d$y_p$ by dt is equal to bt. Since this equation has a y in it, so let us find y first. So, this will give us d$y_p$ is equal to bt dt. And when you integrate, we can integrate it from time $t_0$ to t and $y_0$ to $y_p$.

So, after integration we will get from this term $y_p$ - $y_0$, this is equal to bt squared by 2. So, b by 2 t squared - $t_0$ squared. We can substitute this here. So, we will get d$x_p$ by dt is equal to, because why we need to do this, because y as we can see from here of course, this will be the $y_p$. So, as we can see from here, $y_p$ is a function of time. So, when we integrate with respect to time, we need to know the functional dependence of $y_p$ on time. So, we will substitute this a into $y_0$ + b divided by 2 into t squared - $t_0$ squared. So, we can write d$x_p$ is equal to a$y_0$.

And we can integrate this d$x_p$ from $x_0$ to $x_p$ a $y_0$ not + b by 2$t_0$ t squared - $t_0$ squared this within bracket multiplied by dt and we integrate it from $t_0$ to t. So, when we write we will get $x_p$ - $x_0$ is equal to, we are integrating with respect to t a $y_0$ is a constant. So, a$y_0$ into t - $t_0$ + we have left one a here, so there should, this should have been ab by 2. So, ab by 2 and if we take these two terms separately and integrate, so ab by 2 into t cube by 3. So, we can write ab by 6, t cube - $t_0$ cube.

The next term will be - ab by 2 $t_0$ squared which is a constant. So, when you integrate you will get another t - $t_0$. So, when you rearrange you will get $x_p$ is equal to $x_0$ + ay t - $t_0$ + ab by 6 t cube - $t_0$ cube - ab by 2 t squared t - $t_0$.

So, these are the coordinates of the particle as a function of time. And to plot pathline, you can either substitute, you can find out the value of time, rearrange this equation algebraically and find t as a function of $y_p$ and you can substitute here. So, you will get a equation in terms of $x_p$ and $y_p$ by eliminating time t. Or if you want to plot it, you can simply make a table in terms of t, x and y.

And when you solve this, when you write it down in terms of t, x and y. So, you will start with at time $t_0$ particle is at position $x_0$ $y_0$. And then you can write down the values, say $t_0$ + some delta t and find out the values, what is x value, what is y value. Keep doing this and then you can plot on an xy plane x, y that will be your pathline.

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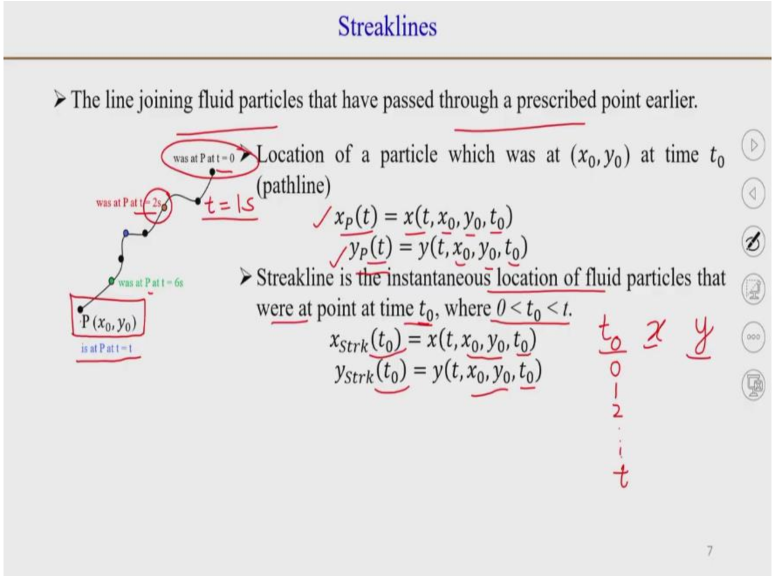


Now, streakline. So, streakline it is slightly cumbersome or slightly difficult. I generally find it difficult to comprehend, but let me try to explain it. So, the streakline is a line that join fluid particles that have passed through a prescribed point earlier. So, if we visualize it, it is the line at a time instant t and it is talking about that a fluid particle is at point P $x_0$, $y_0$. So, this is a streak source and it is the instantaneous position of a fluid particle or instantaneous position of all those fluid particles which have passed through point P at earlier times.

So, if you look at this picture, what it tells us that this particle which is now here at this location, it was at time t is equal to 0 it was at point P. This particle at t is equal to t second, it was at point P. This particle was at point p at t is equal to 1 second, it was at P. This particle was at point P at t is equal to 6 second and so on. So, basically if you look at, say, you have a school and in a particular class, let us say, class twelfth the students are given roll number from 1 to 50.

There are 50 seats in the class and the students are given roll number 1 to 50. And over the years, each year one student will have a roll number, say 27. Now, if you connect the dot over say, last 10 years and find out the instantaneous position of these students or instantaneous locations of these students, connect these dots, then what you get is streakline. So, streakline is plotted at time t and it is the history of all those points, which were at point P at time t0.

So, it is location of a particle. If you want to define it mathematically, we already know from the definition of pathline or when we write down the pathline, by solving the equations, we can find

$x_p$ and $y_p$. The two equations will give you the location of a particle at time t, which was, the particle was at time $t_0$ at point $x_0$, $y_0$. Similarly, $y_p$ t is equal to this equation will give you the position of the particle, y coordinate of the particle which was at time t0 at point $x_0$, $y_0$.

Now, if we see it in a different line or see this equation in a different line then you will get the equations of a streakline. So, streakline is the instantaneous location of the fluid particles that were at time $t_0$ that were at point p. So, what we need to find is the, or what we need vary now is $t_0$. Here when we are trying to plot the pathline, we know that the particle, we are following the position of a particular particle, so at time t0 this particle was at point $x_0$, $y_0$ and $t_0$ is fixed.

And we write down the table of our t, $x_p$ and $y_p$ because time is varying. Whereas in this case, we are finding or we are plotting the streakline at a time instant t and we are looking at all the particles which were at time t0, which is varying from 0 to t those particles were at time $x_0$, $y_0$. So, while these will be a function of, $x_p$ and $y_p$ are function of time, these will be function of streaklines, these will be function of $t_0$.

When you want to find the streakline, what you can do is you can make a table, $t_0$, x, y. And at time t, so you can use these equations and find that what is the position of the particle between, so all the particles which were, so let us say if you substitute $t_0$ is equal to 0 in this equation and you are finding the location of the particle at time t. So, time t is fixed, let us say it is at t equal to 5 seconds. So, you will have to say that at t is equal to 5 seconds and $t_0$ is equal to 0 x0, y0 remains same. What is x streak, same y streak.

And you do it by varying $t_0$. So, you will get the instantaneous location of the particles which were at different times, say t is equal to 0 second, 1 second, 2 second at point $x_0$, $y_0$. So, the equations will remain same, the only thing is changing here is the $t_0$ becomes a variable which is between 0 to t. And when you plot, you will get the equation of the streakline.

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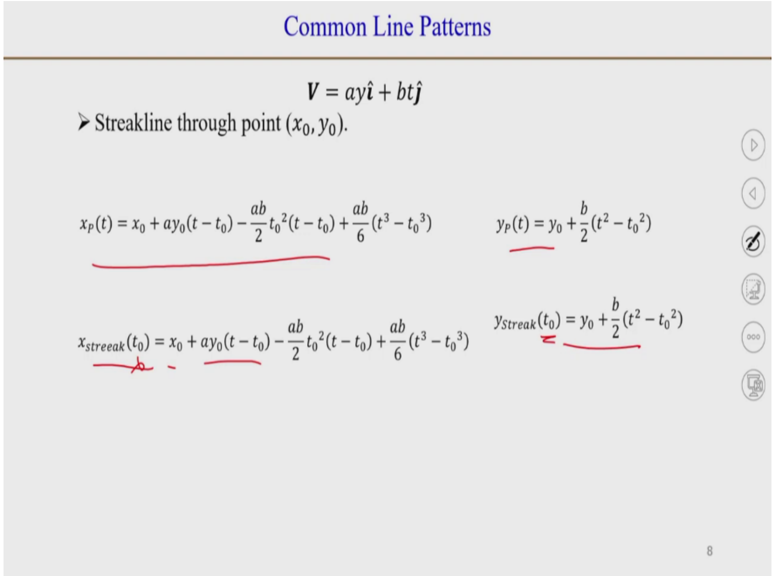


Now, if we look at the same example, and we need to find the streaklines through source point, $x_0$, $y_0$ at time t. So, we write down the first equation of pathline that we obtained earlier and substitute this, the equation of the streakline as a function of $t_0$, $x_0$ + a $y_0$ t - $t_0$. So, the equation remains same, what we have changed is varying $t_0$ and we will need to now vary $t_0$ and find out x and y which will give us the streaklines.

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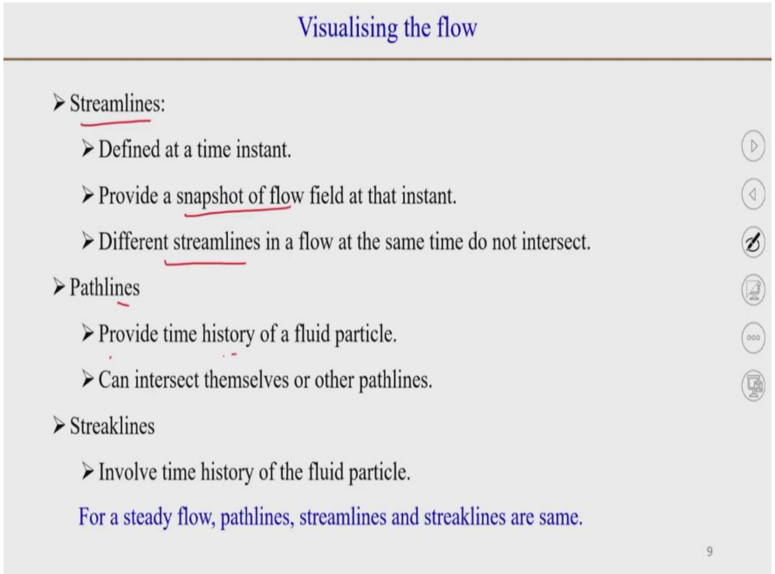

Now, if we look at all of these, all three lines together, so streamlines they are defined at a particular time instant and it gives us a particular snapshot of flow field at that particular instant. Because at a particular point the streamline, the slope of the stream line gives the velocity vector. So, the steam lines cannot intersect because if two streamlines intersect that means, you will have two directions at that particular point.

So, the fluid particle at a time instant present at one particular location cannot have two velocities. So, that is why the streamlines, they do not intersect. Now, the pathlines, what they do is they provide time history of a fluid particle. So, a fluid particle can keep passing through the same point again and again. So, pathlines can again, they can intersect. And pathlines are not at a particular time instant as because it is a history. So, they have, they vary over a particular time. So, unlike streamlines they are plotted over a period of time.

The streaklines, they you plot them at a particular time instant, but they also involve the history of the fluid particle, that a particular, from a, the particles which have passed from a particular point between time 0 and t. The streakline involves the time history of the fluid particle.

But if you look at a particular point on a streakline that will represent the instantaneous position of a fluid particle which was present at time, at some time before at point $x_0$, $y_0$. So, they also cannot intersect. Now, if the flow is steady then the pathlines, streamlines and streaklines they are all same. It is only when the flow is unsteady you will find that these lines are different.

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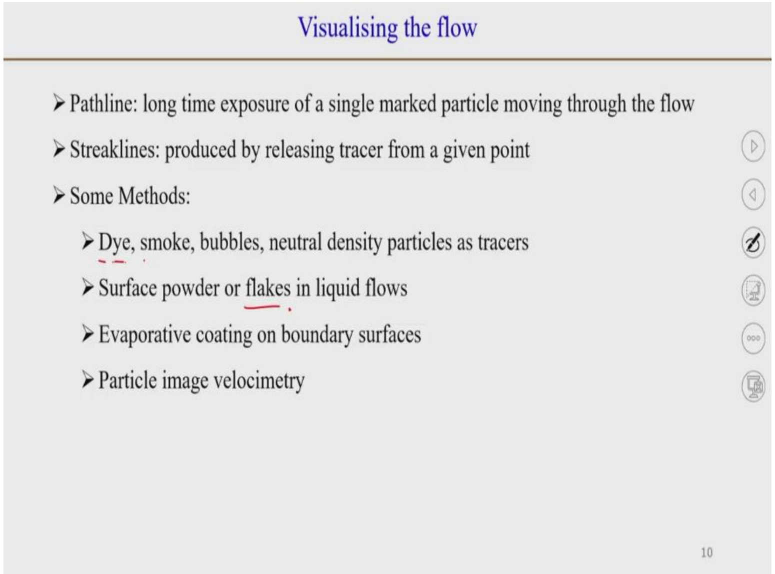


So, when you want to visualize the flow, if you take a long time exposure of a single marked particle where you can mark the position of a fluid particle over time and if you are taking such a picture, a long time exposure of the fluid particle then you will get pathline. If you introduce a tracer at a particular point, if you keep introducing a tracer, let us say a dye, and then following the fluid particle or following the path of this dye, then what you get is streakline.

So, some methods which are commonly used to visualize the flow that one can introduce a dye in a liquid form or in a gaseous form, you can introduce a smoke in gases or bubbles, neutral density particles or which we call non buoyant particles, which can act as tracer. In liquid flows sometimes flakes can be introduced, which can follow the fluid particle or some powder. So, for example, if you want to just do some simple cheap experiments, you can introduce a powder in the fluid which you think or which can follow the fluid faithfully.

In some cases, when you are looking at, because of the heat transfer is involved, then you can introduce on the boundary surfaces where you are looking at where the fluid is going or where the gas evaporated vapor is going, then you can use an evaporative coating on boundary surfaces. Then, there is, there is very popular and novel technique for finding out the three dimensional velocity in the fluid which is called particle image velocimetry.

So, when you do particle image velocimetry, you if the concentration of particle is low then you talk or you call it particle tracking velocimetry. So, basically what you do is you take the pictures of fluid particle at different time instants. So, you take different frames of the particle, you can take the image of a particle or number of particles at low concentrations at time t equal to 0, time t equal to 1, 2 and so, on, where your delta t or the instance between the two photographs which is small so you are certain that you are able to capture the fluid particles. So, then also you will have a fair enough understanding of the, of the flow patterns.

So, in this lecture what we have looked at is pathline, streamlines and streaklines. And, we see or we saw that how to define these lines mathematically. It will help if you can draw these lines in an Excel so that you will have a fair enough understanding of the difference between streamlines, pathlines and streaklines. Especially, when you are talking about pathlines streaklines, if you plot the two equations, then you will be able to appreciate the difference between pathline and streakline. We will stop here. Thank you.