## **Fundamentals of Fluid Mechanics for Chemical and Biomedical Engineers Professor. Raghvendra Gupta Department of Chemical Engineering Indian Institute of Technology, Guwahati Lecture 6 Introductory Concepts 2**

Today we are going to talk about some further introductory concepts and first we will talk about viscosity.

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There is this name Rheology which is basically the study of relationship between stress and rate of strain. When we defined a fluid we said, a fluid is defined as a material which undergoes deformation continuously when it is under a shear stress. So, when we apply a shear stress on a fluid it deforms continuously.

So, then the question comes let us see the relationship between the shear stress that has being applied on the fluid and the rate of deformation. So, the relationship between rate of deformation and a shear stress this itself has become a very active and very important part of our branch of fluid mechanics which is called rheology. So, when you find a new material or when you make a new material, generally as a material scientist one need to characterized the material.

And if it is fluidic or semi fluid or semi solid kind of material then one need to understand its rheology. For example, if you are looking at a polymeric solution or new a polymer you would like to characterized its rheology or flow behavior. How it behaves or what is it flow behavior under different range of stresses. So, the relationship or the science in which we study the relationship between stress and rate of strain or rate of deformations is known as rheology.

And if we plot on an XY plot the relationship between  $\tau$  which is shear stress and  $\dot{\gamma}$  which is rate of deformation.  $\gamma$  is deformation and  $\dot{\gamma}$ , dot generally represent the rate. So,  $\dot{\gamma}$  is rate of deformation. If we plot the relationship on an XY plane plot a between  $\tau$  and  $\dot{\gamma}$  then we can have a different kind of relationship it may be linear which is generally common for most of the pure liquids.

But if you have particle suspended it or cells suspended in the fluid or some other materials suspended into it then you may observe different behaviors. So, depending on that you can have a different naming of the fluids. Now a relationship so when you have a, a relationship between stress and rate of strain generally you would like to represent it in a mathematical form and that mathematical form is termed as constitutive equation.

So, sorry there is spelling typo here, so this is constitutive equation that gives the relationship between stress and rate of strain. Now, this we call rheology and when you want to measure it, so the instrument that you use to measure the rheological behavior of a fluid is called rheometer. If it is a Newtonian fluid in that case, we are measuring basically the viscosity as we will see in a few minutes. So, in that case those equipment are called viscometers but if we, you are looking at in general the relationship between stress and rate of strain then such equipment are known as rheometers.

So, basically in rheometers what you do, you apply a definite or a known shear stress on a fluid and measure the rate of strain. Or you, the particle undergoes a certain or a known rate of deformation and then you measure the force and from that the stress on the fluid or stress generated in the fluid.

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So, we have already looked at stress in the previous lecture. Where we talked about then, when there are surface forces applied on a fluid, then there are stresses generated and the stress is a tensor which has nine components and so on. We will just briefly look at what is rate of strain. So, this  $\dot{y}$ as I said,  $\gamma$  is deformation and  $\dot{\gamma}$  is rate of deformation or we also call it, because it is shear rate, so it is basically it is caused by shear stress, so the strain that is caused is shear strain.

And the rate of shear strain or the rate of strain is what we call shear rate. So, there are different names that are used for it. And shear strain if we define that there is the relative displacement of particles in the body. So, when you compress a material or a fluid that means you are applying normal stresses on a fluid and it may undergo normal strain.

But when we talk about shear strain that is the relative displacement of particles in the body. So, if you consider, to understand this let us consider flow between 2 parallel plates. So, you have a plate at the bottom and a plate at the top. And you apply a force (*F)* on this plate and because of this force the fluid between them it starts undergoing deformation, it starts moving.

So at, at time t+ $\delta t$ . So, you start let us say you start deforming it or you start applying force on the upper plate. So, this plate is stationary and the upper plate is moving. So, when you apply a force on the upper plate it starts moving and the fluid between them it will start deforming. So, in time δt the deformation in the fluid is equal to δα. So, the particles have moved by an angle of δα. And at the top the distance moved is δl.

And this plate is moving with a velocity δu. At time t+δt this angle becomes  $2\delta\alpha$  the deformation becomes 2 δl. So, the rate of deformation will be the deformation of the fluid which is the angular deformation is δα. And because we are interested in the rate of deformation, so deformation per unit time  $\delta \alpha$  /  $\delta t$  and when the limit of time  $\delta t$  goes to zero, we will have a the derivative that  $\frac{d\alpha}{dt}$ will be the rate of deformation where,  $\alpha$  is the angular deformation or the shear strain and  $\frac{d\alpha}{dt}$  will becomes rate of deformation.

Now, from this we can write that δl is the travelled by the fluid in time delta t. Because the upper plate is moving with a velocity δu. So, in time δt the fluid just which is adjacent to it, it will travel the same distance as the plate has moved. So, the distance travelled will be δu δt.

So that means,  $\delta l = \delta u \delta t$ .

Because we are taking infinitesimal deformation, so for this small angle  $\delta \alpha$  we can write the  $\delta l$ which is basically an arc and this distance, the distance between plate if it is  $\delta y$  then we can say that because this distance is  $\delta y$ 

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\delta l = \delta \alpha \delta y
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So, we have two relationship that

Δl is equal to δu by δu into δt.

And it is also equal to  $\delta \alpha \delta y$ . So, by rearranging what we can get is that  $\frac{d\alpha}{dt}$  is equal to  $\frac{du}{dy}$ . So, we have this relationship we can substitute in place of delta alpha or delta t we can substitute it with delta u by delta y. And then we can take the limit. So, what we get is the rate of deformation which is δα is over δt is  $\frac{du}{dy}$ . So,  $\frac{du}{dy}$  is what u is the velocity so  $\frac{du}{dy}$  is velocity gradient.

So, because this is one dimensional flow. So, the rate of deformation for one this one dimensional flow is  $\frac{du}{dy}$ . So that is why you might see a that in general when you define viscosity it is defined that shear stress is proportional to the rate of strain which is given by  $\frac{du}{dy}$ . And we can generalize the rate of strain in three dimensions. So, when we write in three dimensions of course we will have three velocity components along x, y and z direction u, y and w. And then you will have gradients along x, y and z directions. So, you will have 9 components for rate of strain. So, rate of strain is also a second order tensor and it will have 9 components.

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So, let us define viscosity. And viscosity is basically it is, it can be defined or it is defined for the fluids where there is a linear relationship between stress and rate of strain. And that is common for most of the fluids. So, we talk about the two most common fluids that we are concerned with which is water and air. Both of them they show this property that the shear stress is directly proportional to rate of a strain.

So, then we can put it proportionality constant here. And that proportionality constant is what is called viscosity. So, if we plot on an XY plot a graph between shear stress and rate of strain then the slope is, what is called viscosity. Now this in known as Newton's law of viscosity. So, Newton's law viscosity simply states that the rate of deformation or rate of strain is directly proportional to the shear stress applied on it.

And the proportionality constant is what is called viscosity. This viscosity is known as absolute viscosity or dynamic viscosity. And the fluids that follow this Newton's law of viscosity they are called Newtonian fluids. So, we can write down the units in different systems. For example, if you write in fundamental quantities in terms of fundamental units so the dimension of viscosity will come out be  $ML^{-1}T^{-1}$  so the unit will be kg/ms.

Or if you write in terms of Pascal so the because the unit of  $\tau$  is  $N/m^2$ . So,  $N/m^2$  is what is call Pascal and the unit of rate of strain we just now find out that  $\frac{d\alpha}{dt}$ . So, the unit of rate of strain will be second inverse or  $s^{-1}$  or time<sup>-1</sup>. So, unit of viscosity will be Pascal divided by second inverse or Pascal second.

In CGS system, there is another unit of viscosity which is commonly used is called poise. So, these are different units of viscosity. Now,  $\mu$  we call it dynamic viscosity whereas there is another term which is called, pronounced as ν. And this ν is called kinematic viscosity which is ratio of dynamic viscosity and density. So, it is basically the ratio of absolute viscosity or dynamic viscosity and density of the fluid.

And so, if you put the unit, you find out the dimensions it will come  $L^2T^{-1}$ . So, the unit is  $m^2/s$ or it is also called as Stokes. And this is very important because when you look at say, thermal diffusivity or momentum diffusivity the units of both of them come out to be  $m^2/s$ . And kinematic viscosity in analogy with them is called momentum diffusivity.

So, viscosity is basically the property of the fluid which causes the diffusion of momentum because of the molecular motion. So, if you have molecules in all the fluids have molecules and these molecules are travelling continuously. So, because of the collision of molecules there is transfer of momentum in the fluid. And that momentum transfer that is characterize or that is defined by viscosity.

So kinematic viscosity is also called momentum diffusivity. Now, the viscosity of gases it increases with an increase in temperature. Whereas, if you just think of that what happens to a liquid which is thicker and when you heat it what happens to it, it becomes thinner that means it can flow easily. One of the solutions for example, when crude oil it is highly viscos fluid and when it needs to be transported through pipelines and there are generally people put heating stations as different distances so that its viscosity is reduced and one can easily flow the oil.

So, the viscosity of liquids it decreases with an increase in temperature. Whereas viscosity of gases it increases with an increase in temperature because what happens that when you increase the temperature of a gas the number of collisions between its molecules is increases. So, the viscosity of the fluid also or viscosity of the gas also increases.

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Whereas in if you talk about liquids when you increase the temperature the bond or the attraction force between the two molecules becomes weaker so it can flow relatively easily. Now, we talked about Newtonian fluids and the fluids that do not follow Newton's law of viscosity we call them or we group them together as Non-Newtonian fluids. So, for the fluids for which there is no proportionality, direct proportionality between  $\tau$  and  $\dot{\gamma}$  between shear stress and rate of strain then those fluids are called Non-Newtonian fluids.

So, we can have as we see on this graph there are, there can be different instances in which the relationship is not linear or it does not pass through origin. So, the first one is the that there is linear relationship between  $\tau$  and  $\dot{\gamma}$  but up to a certain value of  $\tau$  which we call  $\tau$  y or yield stress there is no deformation. So, when you go in the morning and take paste out of the if you press the paste and the paste comes out.

So, there is a particular amount of force that you need to require for the paste to come out. So, tooth paste is a very common example of this kind of fluid where you need to have a, you need to give a stress or the stress should be above a certain threshold value above which the fluid will start flowing. And this threshold value is known as yield stress. Above that stress the relationship between stress and strain is linear and such fluids are known as Bingham plastic fluids.

So, they show yield stress behavior some common examples are, toothpaste, drilling mud. So, when the oil is taken out from the wells it needs to be their needs to be drilling in the wells and then there is mud which comes out so the viscosity of that mud shows or the rheological behavior that mud shows is Bingham plastic kind of fluid that is shows and yield stress. Clay suspensions also show the same behavior.

Then there is pseudoplastic fluids. So, pseudoplastic fluid shows shear thinning behavior, shear thinning behavior that means the fluid starts thinning as you increase the stress or as you increase the rate of strain. So, if you look at this fluid the slope which is viscosity it is high and then the slope decreases as you increase the rate of strain. So, with an increase in rate of strain the slope decreases such fluids are known as pseudoplastic fluid or shear thinning fluid.

So, blood is a common example of shear thinning fluids at low shear stress values blood behave as a shear thinning fluid. When the shear stress becomes high then blood start behaving as a Newtonian fluid. But at low shear stress values blood behave as a shear thinning fluid. So, other examples of shear thinning fluids are polymers solutions and colloidal suspensions. The shear thickening fluid as the name suggest dilatant or shear thinning fluids.

So, when you apply shear stress on it the viscosity of the fluid increases. So, you can see that the slope increases with an increase in the stress or in the rate of deformation or rate of strain. So, such fluids are known as dilatant or shear thickening fluids. A common example is, the solution of corn starch, a suspension of starch. So, if you take corn starch put it in water and try to stir it. You will find that is increase the speed of stirring, you will find it harder and harder to stir it.

Because the viscosity of the suspension keep increasing with an increase in shear rate. The same is true with sand and you can experience it. When you walk on a beach when you press it you find it further difficult to apply shear stress. So, these are some common examples of Non-Newtonian fluids. All of these Non-Newtonian fluids are time independents fluids. That means their behavior does not depend on time.

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There are some fluids for whom which so the time dependent behavior but before that we can characterize this the flow of Non-Newtonian fluids in terms of mathematical models. So, a common general model we can write down where, the stress is equal to yield stress plus k strain rate raise to the power n. So, it shows the yield stress behavior and nonlinear relationship or power law relationship between stress and rate of strain.

If the fluid does not show the yield stress behavior then we can remove this term or if it is linear then n simply becomes 1. We can also represent Newtonian fluid from this relationship for which τ y or the yield stress will be zero, K will be equal to the viscosity and n will be equal to 1. So, in this equation  $\tau$  y we call yield stress, k is called consistency index and n is called flow behavior index.

Now, if the flow fluid does not show yield stress behavior then we can simply write the relationship as k  $\dot{\gamma}$ <sup>n</sup> and it is known as or it is called power law. For Bingham plastic fluid the yield stress is non zero as we can see here the intersect, the intercept is  $\tau$  y and n is equal to 1. For pseudoplastic fluids the yield stress is zero as we see here and n is less than 1.

You may find some fluids even for example for blood that they may show some yield stress behavior so in that case the yield stress may not be zero. This equation or the yield stress zero is written here just to simply represent the graphs in this figure. For dilatants fluids the yield stress is again zero and n is greater than 1. Now, you may see that one uses the term apparent viscosity for all of these fluids.

So, apparent viscosity is basically the ratio of the stress and rate of strain which we call as apparent viscosity. So, the apparent viscosity of a pseudoplastic fluids or shear thinning fluid will decrease with a increase in a rate of strain and the apparent viscosity of a shear thickening fluid will increase with increase in rate of strain.

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Now, time dependent fluids. So, all the Non-Newtonian fluids we talked about they show, time independent behavior that their viscosity does not change with time if the shear rate or the rate of strain is constant or if the shear stress being applied on them is constant than the viscosity does not change with time. But there may be or there are some class of fluids which shows time dependent behavior that their viscosity changes even when the shear stress or rate of strain is constant.

So, we can classify these fluids as thixotropic fluids whose viscosity or the apparent viscosity decreases as the time progresses their viscosity decreases. Then there is a mirror fluid which is called the rheopectic fluids. So, they show the opposite behavior that their viscosity, apparent viscosity increases as the time progresses. So, increase in apparent viscosity with time. Then, there is another class of fluid which we call viscoelastic fluid which show viscous behavior as well as elastic behavior.

So, elastic behavior when you take out or when you remove the shear stress then the fluid may regain its shape partially. So, that means it shows some elastic behavior such fluids are called viscoelastic fluid. So, these are some classifications of time dependent fluids. In this course, we will generally be dealing with Newtonian fluids where which show or which have a constant viscosity with respect to shear stress or rate of shear. So, only few examples we might include for some particular class of fluids for time independent fluids.

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So, we can classify fluid flows in different manner depending on their compressible behavior or compressibility, depending on their viscos behavior or based on viscosity or their randomness in the fluid or internal or external fluids. So, we will look at this different classifications now. So, we can classify the fluid flows as viscos fluids, so where we have viscosity important such fluids are called viscous fluids.

Now, we can define this non-dimensional number which is called Reynolds number. The ratio of inertial and viscous forces. So, inertia is generally what we define as ρ u square or v square. So, ρ into velocity square divided by viscous force which is  $\mu$  u by L where L is a length scale. So, we can obtain Reynolds number is ρ μ, ρ u L by μ. So, Reynolds number is basically the ratio of or the relative importance it can show between inertial and viscous forces.

So, one can say that when the Reynolds number is low the viscous effects are going to be dominating and the flow will be viscous and we need to take care the, take in to account the viscous effects. When the Reynolds number become high then the inertial forces become dominant. And in such cases, for example when Reynolds number approaches to a very large value one might be inclined to treat the flow as inviscid flow.

Inviscid flow means, you can assume that the viscosity is negligible. So, one do not consider the effect of viscosity or one can neglect the viscosity of the fluid then such fluids are called inviscid fluid. So, in reality there is no such fluid which has zero viscosity all the fluids have some viscous

behavior. For example, the viscosity of gasses is very low of at the room temperature the viscosity of air is of the order of  $10^{-5}$  in kg per meter per second.

Whereas, the viscosity of the water is about  $10^{-3}$  at room temperature. So, there is about two order of magnitude difference in the viscosity of air and water. So, one would incline to treat the flow of gasses specially at the high flow rates or high velocities as inviscid fluid. The beauty of inviscid flow is that when it is also irrotational then it is called potential flow and it can be represented by linear governing equations.

So, when the governing equations are linear mathematician will tell you or when you study a mathematics course you find that it is easier to solve linear differential equations or linear partial differential equations. So, the inviscid flow has been studied for long, for more than two centuries and there have been lot of development in the inviscid flow, which is called generally potential flow.

Now, so people could find lot of analytical solutions and different results. But then it has a limitations. So, one of the very commonly quoted limitation is what is called d' Alembert's paradox. So, d' Alembert he is showed experimentally, he showed theoretically that if you treat a fluid to be inviscid, if you neglect the viscosity then for a moving body which is moving with a constant velocity relative to a fluid the drag is zero.

So, what he suggested or what he found from his calculations that if you take a sphere, the streamlines for an inviscid flow they will be symmetric about it. So, the because there is no viscosity present. So, the shear stress will be zero now when the shear stress is zero the only force that this sphere will experience is the pressure force. And if the streamlines are symmetric then the pressure force on the two halves and the front and the back of the shear will also be symmetric.

So, the drag on this sphere or any such body will be zero. But when you do experiments and measure the drag on sphere in a flowing fluid then this the value of drag is non-zero. You find that there is a certain drag on the body. So, this is why it is called paradox. And it took more than a century to explain this phenomenon. And to explain this phenomenon in early twentieth century in 1904 or so Prandtl, he introduce the concepts of boundary layer. Which could explain, using boundary layer theory he could explain the existence of non-zero drag on a sphere, in a fluid or in a body in a fluid.

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So, we will be using this term boundary layer quite often this course. So, what is boundary layer? The most common example or the easiest way to understand boundary layer is considering flow over a flat plate. So, we have a flat plate here which is a solid wall. And the flow approaches in a over this flat plate. The flow, let us say you have a the flow of air on land. So, this air approaches with a velocity  $V_{\infty}$  which is often termed as free stream velocity.

And the flow here is uniform flow that means there are no gradients in this flow. The velocity of the fluid is  $V_{\infty}$  everywhere. Now, when this flow approaches on a wall he said, the Prandtl suggest, the Prandtl suggested that near this plate in a certain region viscous effects are important, so the velocity there is change in velocity in a certain layer near the wall. Above this layer the velocity is uniform. So, still the velocity of the flow is still  $V_{\infty}$  in this free stream.

But inside the boundary layer, so this layer what we have drawn here is what is called boundary layer. So, in this boundary layer he suggested that the viscous effects are important. No matter how large the Reynolds number is, in a thin layer near the wall the viscous effects will be important and those viscous effects will cause a gradient in velocity near the wall. So, when the fluid approaches because of the no slip boundary condition the flow velocity next to the wall will be zero or the velocity of the fluid next to the wall or at the wall is zero.

And then, because of the viscous effects the fluid layer next to the wall is being drag behind, so the velocity will become zero. And the fluid that will of course move to the upper part of it. So, the layer next to the wall is what we call boundary layer. And in this layer viscous effects are important. Now, as the flow approaches on the plate the boundary layer is laminar.

And then as you can see that the boundary layer grows, the height of the boundary layer growing as you move along the plate. And the flow become turbulent if you keep moving a certain layer. So, the boundary layer when it grows it becomes, it transition somewhere here there is a transition from laminar to turbulent flow. And the boundary layer grows, the rate of boundary layer growth is high from laminar to turbulent flows.

Now, so what you need to remember is that the definition of boundary layer when we use the term boundary layer this a thin layer near the wall in which viscous effects are important which causes a gradient in velocity near the wall. And this the thickness of this boundary layer is inversely proportional to a Reynolds number raise to the some power. So, when the Reynolds number increases the thickness of boundary layer decreases.

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Now, so we have been talking about laminar and turbulent flows quite often and we will be talking about. So, the next classification we will look at is a laminar and turbulent flows. So as the name suggest laminar, so laminar means lamina, lamina means layers. So, laminar flow is when the flow or the fluid is flowing in laminas, so the flow is smoothly in parallel laminas and there is no transfer of momentum or there are no velocities or no eddies which causes mixing.

So, the laminar flow is very smooth. There is a common misconception which when people think that there is recirculation in the fluid when the flow separates or when there are recirculations in the fluid which often we use the term eddy interchangeably then people say that the flow has become turbulent which is not correct. Or people also tend to characterize sometimes that if the flow is unsteady then it is turbulent.

So, we need to remember that the laminar flow it can be unsteady or it can have recirculations. So, what characterize turbulent flows, turbulent flow is basically characterize by chaos or randomness. So, if you look at a laminar flow the fluid particle will be moving in a straight line or if you plot the velocity of fluid. Let us say, x component of velocity in a flow then the laminar flow you may have a steady flow. So, the velocity does not change with time or you may have that there is some periodic variation in the flow.

Whereas, in turbulent flow if you plot velocity with time you will see or you will observe randomness in the flow. So, there is no certain pattern in the flow and that will be not only for u component or x component of velocity you will also observe the same kind of behavior for v component which is y component of velocity or z component of velocity and even for pressure.

So, when you analyze turbulent flows generally you tend to find a mean velocity and the fluctuating velocity. So, you might see that in the turbulent flows people write that u is equal to if the velocity has a mean component and the fluctuating component of velocity. So, the turbulent flows are characterized by random 3-dimensional fluctuations. The pressure drop in because as a chemical engineer or as an engineer in general we are often concerned with the pressure drop of a flow in pipelines.

Because the pressure drop means the energy that we need to provide for the fluid to flow and which is the pumping power. So, the pressure drop is high in turbulent flows. So, one would not prefer turbulent flows but it also provides good mixing. So, as a chemical engineer in many instances you would like your reactants to mix and the turbulent flow provides that capability, that because of the very nature of turbulent flow you would have fluctuation which is in terms of eddies, so it will provide good mixing.

In one of the recently emerging area, which is called microfluidics or chemical micro processing when you talk about chemical engineering there are lot of advantages but one of the problem there is that the flow because the it is what is called micro processing, so the length scales are small so of course the corresponding velocity scales will also be small and consequently the Reynolds number is low the flow is laminar.

When the flow is laminar there is no inherent mixing which be generally get in large reactors. Therefore, lot of effort in microfluidics or in chemical micro processing is directed towards developing mixing devices, passive mixing devices or active mixing devices. So, the turbulent flow offers very good mixing. A good news from chemical engineering perspective. Now, another important area or what we need to look at when does the laminar flow becomes turbulent.

So, if you talk about pipe flow somewhere around a Reynolds number of 2100 or 2300, we say that the flow transitions from laminar to turbulent. So, flow become a transitions to or transition flow between say, 2100 upto 10000 Reynolds number in pipe flow, the flow is called to be in

transition and then for a Reynolds number 10000 or more, the flow is in general it is called fully developed turbulent flow which is true for pipe flow.

If you talk about external flows for example, flow over a sphere or flow over a cylinder, flow over a flat plate in these cases the flow becomes turbulent at a Reynolds number of about  $5 \times 10^5$  or so.

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The next classification we will just look at what is called internal flow. So, internal flow is basically flow in closed conduits, close channels, close pipes and that is what we will look at in this course quite often. There is another class which we can call open channel flow. So, when we talk about rivers, canals which are flowing in a channel but their channel is open at the top. So, these are called open channel flows.

And then external flow. So, flow over a sphere, flow around an airplane they are all external flows. Now, one important point which we might want to go back and look at that when we defined a Reynolds number. So, for a pipe flow we define Reynolds number, the definition of Reynolds number is  $\rho U L$  over  $\mu$ . And what we need to know, what is this is L, and what is this U. So, when we are talking about pipe flow L is the diameter of the pipe.

When we talk about flow over a flat plate, L in place of L what we have is *x* the distance from the, the distance from the leading edge of the plate is how we defines Reynolds number. So, in this case if the distance from the leading edge is x, then in case of flow over a flat plate the Reynolds number is defined as  $\rho$  u x over  $\mu$ . So, as we go along this plate the Reynolds number will keep increasing.

Now if you talk about an external flow. So, if you take flow around a sphere and this sphere has a diameter d. And in this case, the Reynolds number will be  $\rho$  u d over  $\mu$ . So, their d is the diameter of the sphere.



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Another classification is incompressible and compressible fluid. So, the incompressible as the name suggest that the fluids that cannot be compressed or rather better to say the flow in which there is no compression of fluids, they are called incompressible flows. So, incompressible flows

they depend on or they are characterize by a non-dimensional number which is called Mach number, which is defined as the fluid velocity divided by speed of sound in that medium at those conditions.

So, when Mach number is less you can have say, if the Mach number is less than 0.3 than the fluid can be treated as an incompressible fluid. If the Mach number is higher than 0.3 you can treat or you should treat the flow as compressible flows because then the changes in density of the fluid will be significant and those need to be taken into account.

So, from analysis point of view when we talk about incompressible flows we write down the governing equations and our unknown are the 3 components of velocity u, v, w and pressure. If the flow is isothermal. Now, if we are talking about a compressible flow then we do not know the density a priori because the density is also a function of speed, a function of a say pressure and temperature.

So, in such cases of a compressible flow your unknowns will be u, v, w, pressure as well as the density. So, one need to take into account incompressible flows the density variation also. Now, if we look at liquids the common sense will tell us that it is difficult compress liquids. So, liquids will generally be treated as compressible fluids. There are 2 exceptions here, one is called water hammer. So, if you close a valve suddenly there are shock waves and then the liquid also need to be treated as a compressible fluid.

Another example is cavitation, which is when there is phase change. So, when because of the change in pressure if the pressure goes below the vapor pressure and there can be vapor formation or vapor cavity formation in the liquid and that is when the cavitation takes place. So then one need to take into account the change in density of the liquids.

The gases, generally one would think that the gasses should be treated as compressible fluids. But even for gasses, for low Mach numbers the density change, if it is an isothermal flow the density changes are not very large and the gasses can be treated or the flow of the gasses can be treated as incompressible flow. But at higher Mach numbers the gas is need to be treated as compressible flows.

The compressible flow because most of the flow that we observe and the intuition that we have built up over the years is by observing incompressible flows. So, sometime at least at the first instance you may find the theory of compressible flow or some of the results that we obtain in compressible flow to be counterintuitive. One of the common things or what is observed in compressible flow is shock waves which are basically discontinuous change in pressure, density and temperature.

So, the discontinuities in pressure, temperature, and density they are termed as shock waves. So basically, in this lecture what we looked at the rheology, the definition of rheology which is a science, studying the relationship between the stress applied and the rate of deformation. So, the relationship between rate of deformation and stress is called rheology.

And then we looked at different classes of Non-Newtonian fluids. And finally, we classified or we saw a different classifications of flows compressible and incompressible flows, viscous and inviscid flows, laminar and turbulent flows. We also looked at briefly what is boundary layer and what is Reynolds number and how we use it to understand the flow behavior. So, we will stop here. Thank you.