Fundamental of Fluid Mechanics for Chemical and Biomedical Engineers
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Lecture 42
Flow in Pipes: Types of Problems

Hello, in this module, we are looking at the pipe flow problems. Especially, how we can solve pipe flow problems using one-dimensional equations, using Bernoulli's type equations, but the difference from Bernoulli's type equation is that we can take into account the frictional losses and other losses due to the flow separation etcetera, at different fittings, valve, inlet, exit, etcetera.

So, we combine all those losses in two categories, major and minor losses. And we discussed in the previous two lectures that what could potentially be the major losses and what are the different minor losses that we should be taking into account when solving problems or when designing a pipeline system.

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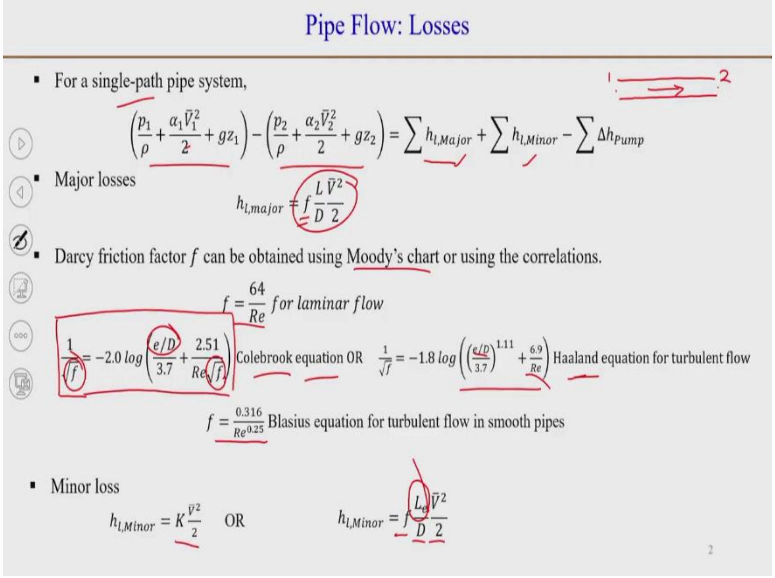

So, to summarize that, we can say that for a single path system. So, by a single path system, I mean that when you have a pipe network, you could have a system of pipes for example, you may have that a pipe is dividing into multiple paths, this may further get divided into different paths. So, here the flow can go here or here and then in this, so you can have multiple paths.

And solving such a problem becomes even difficult, because we need to take into account or we need to solve that how much flow will be going into different paths. So, we will have more variables in terms of flow rates that Q 1, Q 2, Q 3 in different dot dot branches. So, what we are talking about here that if it is a single path pipe system there is no bifurcation etcetera.

Then we can use this equation where the total mechanical energy at point 1, so if I take a piping system and say take two points in the pipeline it might be, what I have drawn here is, is a straight pipe, but it may have a pipe with bends, elbows, and fittings etcetera. So, we can, in the entire length we can assume that it does have everything. It may also have a pump for pushing the liquid through it.

So, we will not be looking at the pumps in today's lecture, but for the sake of completeness we have this equation and this equation at point 1 we have, the total mechanical energy at point 1 here and the flow happens from point 1 to point 2. So, when the flow happens the mechanical energy will be decreasing because of the losses. So, this term gives you the mechanical energy at point 2 and that will be =the loss in energy between point 1 and point 2 will be =major losses and minor losses and if there is a pump in the system, then pump will be adding energy to the system.

So, we have this in terms of - Δ h, so it is a, you can say a negative loss or you can bring it with the mechanical energy at point 1. So, the total energy that is there in the system and then the losses. So, if we look at the major losses, we defined that major losses are the losses in a pipe of constant cross-sectional area, horizontal pipe and to define losses we defined a friction factor because losses depends on L/D, the velocity and the friction factor.

So, we define this Darcy friction factor and we looked at that it can be obtained using correlations that have been developed for laminar as well as turbulent flow. And those correlations have been plotted in terms of chart what we call moody chart. So, for laminar flow one could obtain a very simple and neat correlation which is $f = 64/Re$ and the friction factor depends only on Reynolds number.

Whereas, for turbulent flows it depends on the Reynolds number as well as roughness of the pipe, because the pipe roughness can enhance the turbulence in the flow. Whereas, in the laminar flow the fluctuations that are coming or that are being generated, they can get dissipated because of the dominant viscous forces. Now, so, there are two correlations that we suggested, one is Colebrook equation and other is Haaland equation.

And if the pipe is smooth, then we do not need to take into account the roughness of the pipe, then we can use Blasius correlation which is a function of Reynolds number only. So, if we look at Colebrook equation, here we have the roughness of pipe where e is the roughness and it is non-dimensionalise by the pipe diameter, so e/D. If you look at the structure of this equation, you have f on the left-hand side and you have f on the right-hand side and this is in the log.

So, if you want to solve this equation, you cannot do it explicitly, you might need to assume a value of f first and then get a new value of f and then again do the same exercise and keep doing it until you converge to a value of f that you started at the start of the iteration and that you get as your final answer if they are same then you say that value has converged. So, to simplify the calculation process a bit, Haaland developed another equation which is explicit.

So, there is no f on the right-hand side. So, if you know the roughness of the pipe, if you know the Reynolds number you can calculate it directly and the results are not far off and when you compare with those obtained using Colebrook relationship. So, that is about major losses, minor losses can be represented or can be combined either in the form K $V^2/2$ or in analogy with major losses, it can be also written as f L e/D $V^2/2$, where the effect of the components it may be valve, fitting, inlet, exit, etcetera that come into Le or what we call equivalent length.

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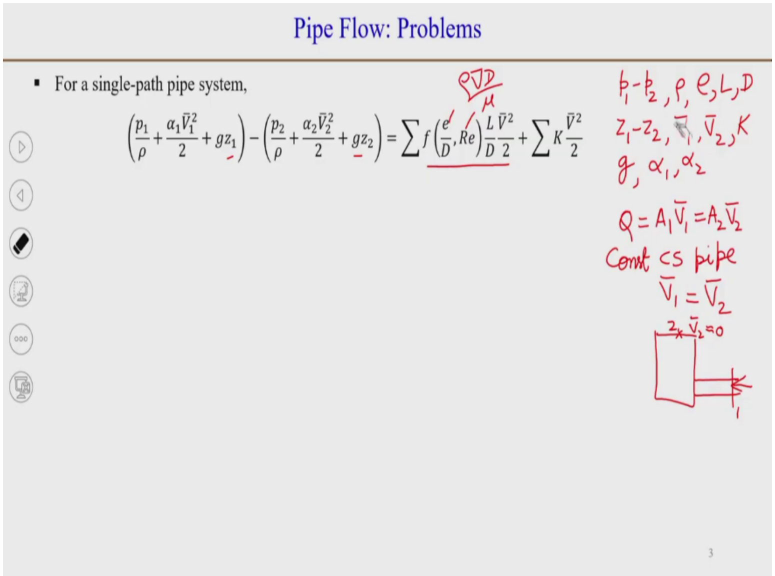

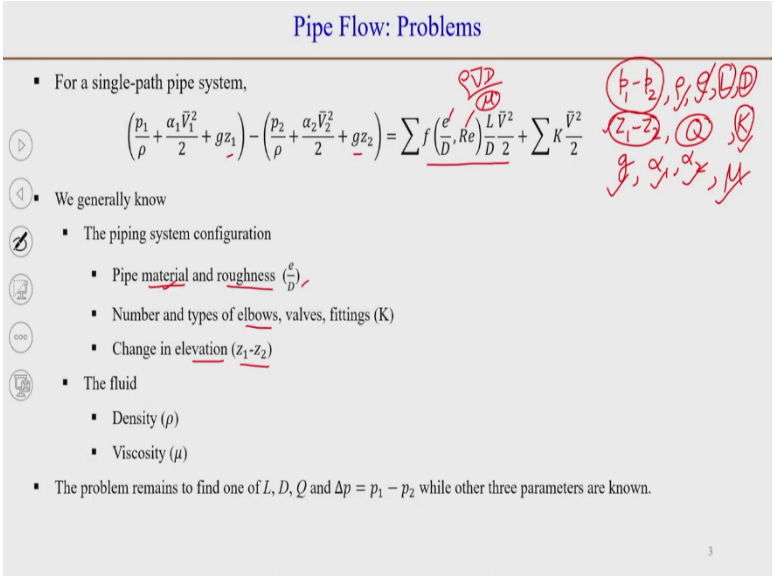

Now, we can write this down here, the equation. So, we can replace the major losses by the equation f L/D $V^2/2$ and to take into account of the fact that f is a function of e/D and Re. And Re is ρ V bar D/μ and this is K $V^2/2$ all the minor losses. Now, if we look at that, what are the variables involved in this. So, we can list down those variables here, let us say p 1 and p 2 or we can combine say this will be p 1 - p 2 /ρ.

So, the pressure drop p 1 - p 2 we will have ρ, as well as we will have z 1 - z 2 and some constants, so for example, g, our acceleration due to gravity is a constant and you have z 1 - z 2. So, you can combine those together then you have α 1 and α2 which are kinetic energy factor and they have values of 2 for laminar flow and close to 1, for turbulent flows.

So, for turbulent flows we will take the values of α1 α2 =1, then you have V 1 bar and V 2 bar, so they can be related with flow rate. So, Q will be A 1 V 1 bar = A 2 V 2 bar. Now, in most of the cases what will we have that if the pipe of constant cross-sectional area pipe, then we will have V 1 bar = V 2 bar. Other case might be that if you have a pipe connected with a reservoir and so, let us say if the flow is coming into this reservoir is going out of this reservoir.

So, in this case, let us say the point 1 and point 2 and because the reservoir cross-sectional area is significantly larger. So, in such a case you will have V 2 will be close to 0, it is very, very less than 1. So, you will have only V 1 there. So, then other unknowns will be the roughness of the pipe and the length of the pipe, the diameter of the pipe, and different K values.

So, say in place of V 1 and V 2 we will just replace it with by Q here. We will know, what is the acceleration due to gravity, α1 α2 are known if we know that the flow is laminar or turbulent and if we know the piping system configuration that for example, what is the pipe material. So, generally the roughness of pipe is a function of pipe material, you will have a different roughness for cast iron and different roughness for galvanized iron and then it will also depend on the lifetime of the pipeline because due to corrosion, there will be the piping surface or the pipe surface will be corroding and over time the roughness factor may change.

So, for a piping system configuration roughness will be a function of pipe material + other factors. So, if you know the piping system then you will know the roughness. You will also know in the piping system that how many types of different fittings, valves, elbows etcetera. You have how many bends do you have, then for a given piping system you will know the change in elevation.

So, you will know either z 1 or z 2 or in general what is the difference between the two because you will take one of them as a reference. So, say z 1 is 0 and z 2 is some value. So, let us see what are the factors that we have already known, we know or knowing the piping configuration we know z 1 and z 2, knowing the how many elbows etcetera, what type of inlet is there, where the fluid is going. So, we will know the value of K for different components and then we will know what is the roughness.

The fluid density and viscosity, viscosity come into picture in the Reynolds number. So, we will know the viscosity of the fluid, if we know the fluid. Then we will know its properties, density, and viscosity. So, if we look at what we basically do not know is p 1 - p 2, L, D, and Q. So, there are four things that we need to know L, D, Q, Δ p. So, some of these we will know and one of these we might not know and the problem will boil down to that we need to know one of these factors when we know other three.

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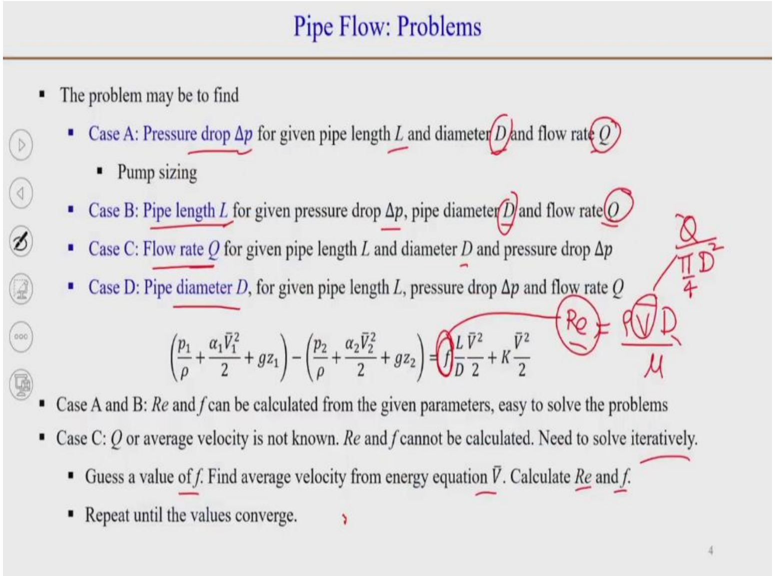

So, we can categorize those problem in four type of problems. Let us say first that we do not know the pressure drop through the pipeline and why we do we need to know pressure drop through the pipeline, it is important, say for example, if you are designing a piping system and if you need to select what size of pump, what the specifications of pump are required, and that will be determined because what is the loss, what is the head loss, what is the pressure loss that is happening through our pipeline based on that you will select a pump.

So, you will need to calculate Δ p So, it is important for pump sizing. Now, you will know that other three factors L, D, and Q. The other type of problem might be that you do not know pipe length the but you know that what kind of pump you are going to use, you know what is Δ p, what is the pipe diameter and what is the flow rate. So, you might need to calculate in some cases, the length of a pipe that is another problem. Then the next problem might be finding out the flow rate. So, for a given pipe length, diameter, and pressure drop, the problem is that we need to find the flow rate.

And the fourth one that the pipe diameter is unknown, you know what kind of pressure drop is going to happen in between two stations and what is the length of the pipe, what is the flow rate, but you need to determine the best diameter and generally when you buy pipes, it comes into standard sizes. So, you might be using a say 1/4-inch pipe 1/8-inch pipe or 2-inch pipe or 3-inch pipe.

So, you might need to select that 2-inch pipe or 3-inch pipe or one of those standard pipes you might need to choose. So, we can categorize in four type of problems. Now, we look at from the problem-solving point of view, we have this our equation and, in this equation, f is a function of Re and roughness. So, roughness we already know if we know the piping system configuration, if we know what kind of material we want to use, then we will know the pipe roughness, but this Re is function of ρ V bar D/μ.

So, V bar is Q/π D^2 or π/4 D^2. So, Reynolds number has in its definition D as well as Q. Now, if for two kinds of problems, the first problem Δ p and L, we know D and Q. So, it is easy, we can simply calculate because we know the flow rate, we know the diameter. So, we will be able to calculate Reynolds number knowing what fluid it is and once we calculate Reynolds number, we can calculate friction factor for the known roughness.

So, it is the calculations are easier and straightforward. But for case C let us say when the flow rate is not known. So, if the flow rate is not known, then we will not be knowing Reynolds number in advance. So, if we do not know Reynolds number then we also do not know f. So, to solve this equation, we will need to solve it iteratively, we need to assume some value of f we can try to make a guess based on the information that is available at our disposal.

Then, we can guess the value of f and then find what is the average velocity or the flow rate and then again from these obtained values, the values that we have obtained for flow rate or mean velocity we can calculate Reynolds number and once we calculate Reynolds number, we will calculate f again and we will do it I iteratively until we converge. So, until we get the same value of f as we started with in solution and until we get the same Reynolds number. So, the process becomes iterative.

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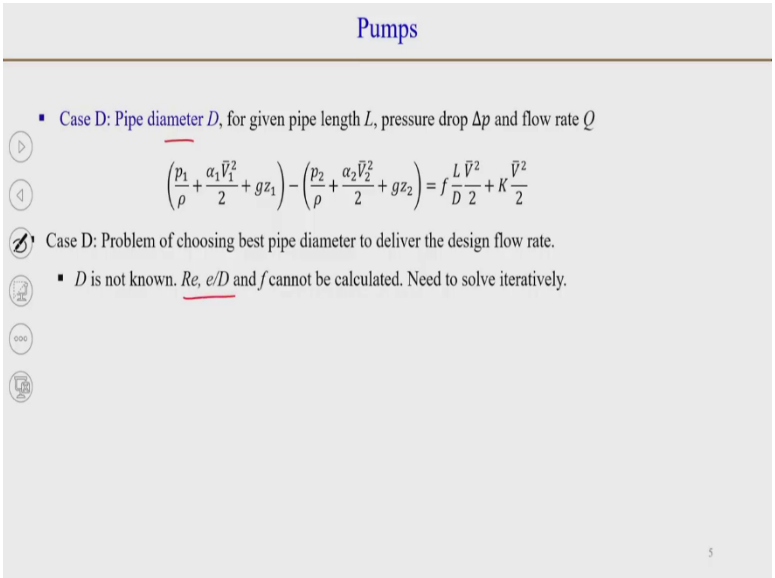

And the same goes for the last case where we do not know the pipe diameter because again pipe diameter meter is in the definition of Reynolds number. So, we need to do it iteratively, we will need to assume a value of f to start with, and then we will need to calculate the diameter, once we get the new diameter, we will calculate Reynolds number find the value of f once we find the new value of f, again calculate diameter until we get a convergence that the same value of f is coming as your solution that we started with and the same value of diameter. So, in this lecture we will be solving some problems where we need to do such iterations.

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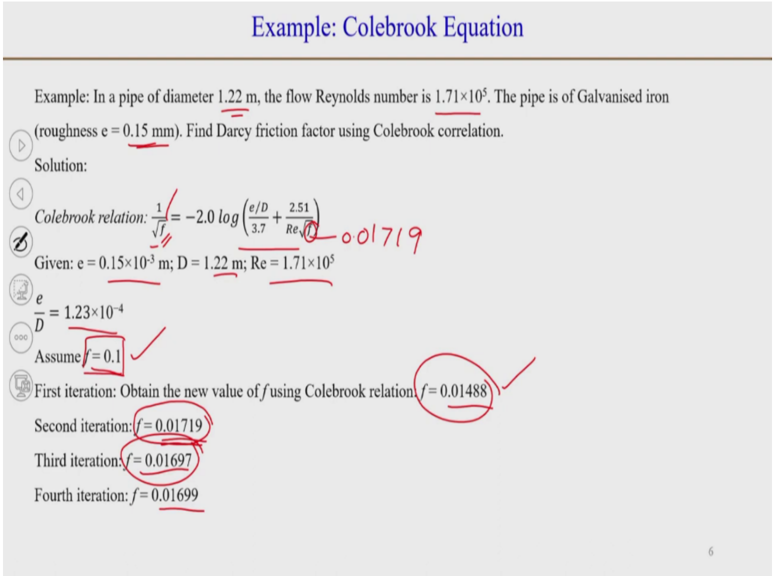

So, the first problem is when we are solving Colebrook equation. So, Colebrook equation we said that it is iterative. So, let us say if you have been given the flow Reynolds number and the pipe diameter and roughness. So, you know e and D you know the Reynolds number. So, using this Colebrook relationship, you need to find out what is the value of f. Now, we can write down that what is my e.

So, e is 0.15 mm or 0.15 into 10^{-3} -meter, diameter is 1.22 meter and we know the Reynolds number. So, from this we can calculate what is e /D that comes out to be 1.23 into 10^{-3} and we already know that and also number. Now, the question comes what is the value of f. So, we can guess a value of f and in general, you might remember that when we talked about Colebrook relationship, we said that f = 0.1 is generally a good guess.

So, we can assume f = 0.1 and we can do the first iteration So, we can substitute the value of f = 0.1 here. And calculate what is this new value of f and when you do substitute all the values here, then you will get f = 0.01488. So, it has become f, the new value of f = 0.01488. Now, we need to do the same exercise. So, now, in the next case what we will do, we will use f = 0.01488 and find what is the new value of f.

So, in the second iteration the new value of f that we obtained here is 0.01719. So, the next will be that we can change in this equation, we can substitute the new value here 0.01719 and we do the third iteration. So, when we do the third iteration, the next value of f is 0.01697. So, if you look at the two values are already very close to 0.017, if you restrict it to three digits only at the percentage difference will be very small.

But to see how accurate can it become, so in the next iteration, if we go, the fourth iteration using the value 0.01697 in here, we will get the new value in the fourth iteration 0.01699 and when you do again 0.01699, next value will be again f value is 0.01699. So, if we look at ideally, we needed, first time we use the value 0.1 and second time we use the value f = 0.01488 and we got a value. So, we got a value which are well converged in the second iteration itself and the third iteration would confirm that it has converged. So, we might need to do three iterations only here. So, that how you solve iteratively for Colebrook equation to calculate friction factor.

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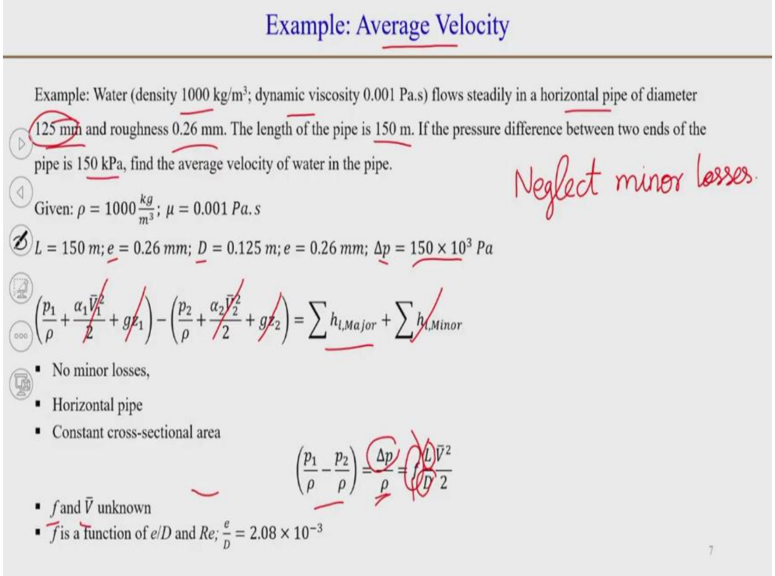

Now, we will do a problem where we need to calculate flow rate or let us say average velocity because flow rate is nothing but average velocity multiplied by cross sectional area of the pipe. So, the problem states that the water is flowing, you have been given the density and viscosity of water, it flows steadily in a horizontal pipe. So, horizontal pipe means, you have z 1 = z 2 of diameters.

So, you have been given the diameter of the pipe, the roughness of the pipe, and length of the pipe. It says that the pressure difference between two ends of the pipe. So, you have also been given Δ p and we need to find what is the average velocity. So, you have been given the factors, ρ, and μ, L has been given, roughness factor has been given, D is given, this has come twice and Δ p is given and we need to find velocity.

Now, in this pipe, there is no mention of any losses and the language does not suggest that there are any minor losses. There are of course, when you have a pipe, there will be frictional losses. So, there are major losses present in the system. But the language does not suggest or nor anything has been given explicitly that there are any minor losses. So, we will neglect and minor losses.

So, that is the assumption that we make. So, we can write down the equation, the energy balance equation there are no minor losses, so this term will go to 0. The pipe is horizontal, so z 1 = z 2, this term will cancel out. Now, pipe is of constant cross-sectional area because its diameter is constant. So, V 1 will be =V 2. So, this term will also cancel out.

Now, we can also substitute that h l major = f into L/D $V^2/2$. So, what we have now is p 1 - p 2/ρ or Δ p/ρ, Δ p is p 1 - p 2, Δ p/ρ = f L/D $V^2/2$. So, we can substitute the values here, we can substitute the values and we need to find f and V bar and f is a function of Reynolds number.

So, Δ p is known 150 kilo Pascal, P is known, L is known, D is known and what we not know is f and V bar. And F a function of Reynolds number which is a function of V bar. So, we will need to solve this problem again iteratively. So, we need to assume to calculate V, we need to assume some value of f. So, what we could do is we know already what is the roughness factor.

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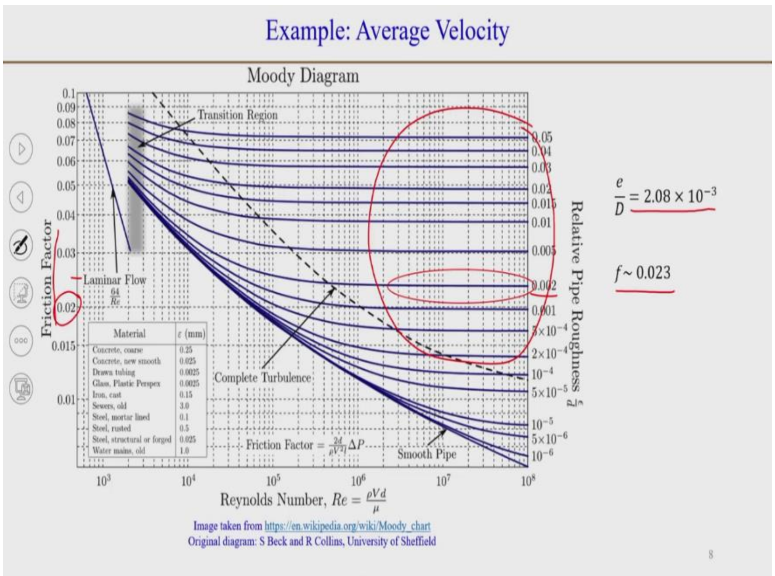

So, let us look at the moody chart, you could have done or you could also solve this problem using say Colebrook equation or Haaland equation. I just wanted to show you a different example. So, we could use moody chart also. So, for example, if you look at the values here at high Reynolds number the value of roughness factor, value of sorry, friction factor it becomes independent of Reynolds number.

So, for example, my roughness factor is 2 into 10^{-3} or 0.002. So, for 0.002 if I see what is my friction factor that is close to 0.023 and it is not so important that you take a very accurate value, because finally, what you need to do is you need to iterate and you will use the formula, so you will reach an accurate value. So, if you start with a good guess, so I have done here f = 0.023 you might even guess it, f = say 0.02 and you will end up with the same final answer. So, what we will do is we will substitute the value of f = 0.023 in the formula.

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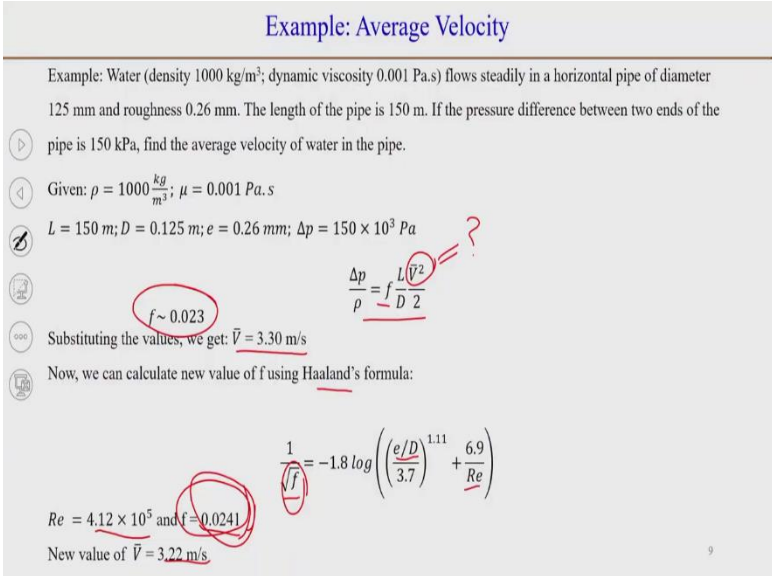

So, we had this formula and, in this formula, f = 0.023 and what we can do, we can calculate what is my V bar. So, when you substitute the values and calculate the value of V bar it comes out to be 3.3 meters per second. So, now, using this value of V bar, we can calculate the friction factor. So, here I have used Haaland's equation. So, if I use Haaland's equation, I do not need to do it, or I do not need to calculate iteratively.

So, in this case $1/\sqrt{f}$ is a function of e/D which we know and Reynolds number we can calculate. The Reynolds number come out to be about 4.12 into 10^5 and when we substitute all this, we will get a new value of f which comes out to be 0.0241. And so, once you get a new value of f, 0.0241, you can again calculate from here what is the new value of V bar, what is the average velocity and that comes out to be 3.22 meters per second.

And when you again calculate the new value of f that will be again 0.024. So, within one iteration we have converged, if you would have guessed a value of f, which is slightly far off, because when we use the graph, we were not very far off, our value of α0.023, f =the final value of 0.0241.

So, the value of f that we guessed was not very different from what the actual value was. So, we could converge in one iteration. So, it helps if you start with a good guess. So, we have looked at that how to calculate average velocity or flow rate. Once we know the average velocity, you can calculate the flow rate also by multiplying the cross-sectional area, because we know the diameter of the pipe already.

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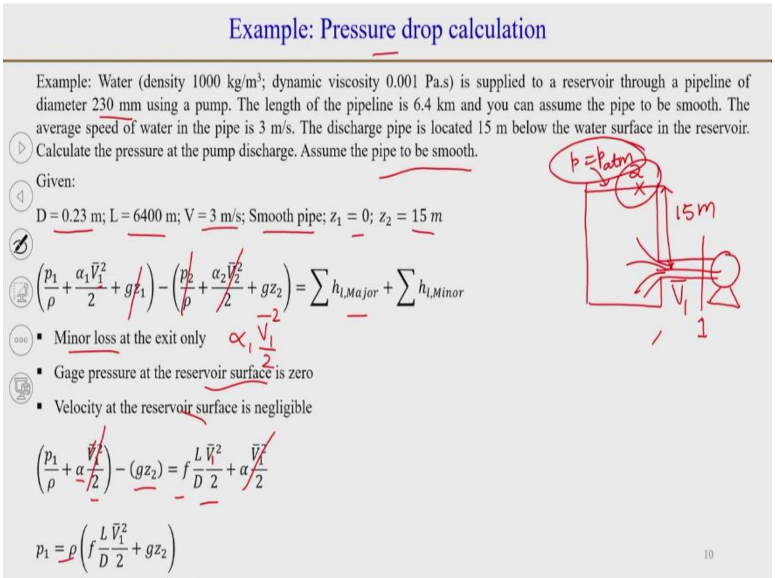

So, we will solve another problem now, where we need to calculate the pressure drop. So, the question reads that water is supplied to a reservoir through a pipeline of diameter 230 mm using a pump. So, if we draw the schematic of the problem, let us say this is a reservoir and to this reservoir, the water is being supplied through a pump and we have been given the diameter and the discharge pipe is located 15 meters below the surface.

So, you can say that the distance, this distance is 15 meters. Now, what we need to calculate the pressure at the pump discharge, so the pump discharge water say here at point 1 and the second point we will consider point 2 here and this is open to atmosphere. So, here the pressure is p atmosphere and the other thing is given that we can consider that the pipe is smooth.

So, now, this time what we will do we will use Blasius equation to calculate friction factor. We know the diameter of the pipe, we know the length of the pipe, we also know what is the velocity of the water in this pipe, we know the roughness that the pipe is smooth, and if we consider these two points point 1 and 2, we know the elevations z 1 and z 2, we can consider that the reference point is point 1 here. So, this is z 1 location and point 2 is z 2. So, again we can write down the equation.

Now, in this equation, we have that V 2 will be 0 because the reservoir area is very large. So, any velocity or the velocity there will be very small when you compare with V 1, that is the velocity in pipe. So, we will neglect V 2. Now, coming to minor losses, so we have not been given explicitly that there is any minor losses, but what we learnt about the losses at the exit, we see here that there is a sudden expansion of the flow, the area is changing suddenly. So, there will be exit losses.

And if you remember what we discussed that exit losses is that the entire kinetic energy of the flow is being lost. So, that exit losses will be $=\alpha V\ bar^2$ and V is here V 1. So, in the pipe the velocity is V 1 bar. So, the minor losses will be $=\alpha 1\ V\ 1\ bar^2/2$. Then the top surface is open to atmosphere, if we consider the gauge pressures, then we can say that the gauge pressure at the reservoir surface that is at point 2 is 0. So, we will consider gauge pressures and p 2 will be 0.

So, remember what we will get an answer for p 1 will be a gauge pressure. We have already talked about that the velocity as the reservoir surface V 2 is 0. So, considering all this, what we will have z 1 is 0. So, this term will also go. So, we will have p $1/\rho$ + $\alpha 1$ because it is just 1, so we will just write it $\alpha V\ 1^2/2$ - g z 2 = major losses f L/2 V $1^2/2$ + $\alpha V\ 1^2/2$. And you might see here that $\alpha V\ 1^2/2$, and $\alpha V\ 1^2/2$ they will cancel out. So, on rearranging what you will get that p $1/\rho$ = f L/D V $1^2/2$ + g z 1 and entire thing multiplied by ρ. So, that will be our p 1.

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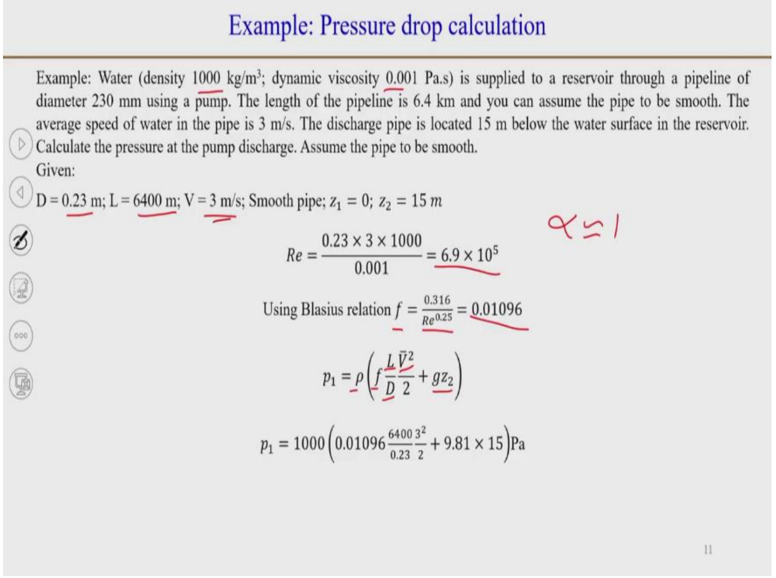

Now, we can calculate Reynolds number, because we know the diameter, we know the velocity, and we know the density and viscosity of the fluid. So, the Reynolds number comes out to be 6.9 into 10^5 that means, the flow is turbulent. So, this also suggests that the value of $\alpha$is going to be close to 1.

And once we have calculated the Reynolds number knowing that the pipe is smooth, we can calculate the friction factor using Blasius relationship $0.316/Re^{0.25}$ is the friction factor and when we substitute the value of Reynolds number, we get the value of friction factor about 0.011.

So, we know the density, we know now f, L is given 6.4 kilo meter 6400 meters, the diameter is given, and the velocity is given 3 meters per second, so 3 meters per second and the elevation of point 2 is 15 meters per second. So, when you substitute all this, you will get the value of pressure about 1520 kilo Pascal. So, this is that how we can calculate, pressure drop it was quite straightforward in the sense we did not need to do any iterations.

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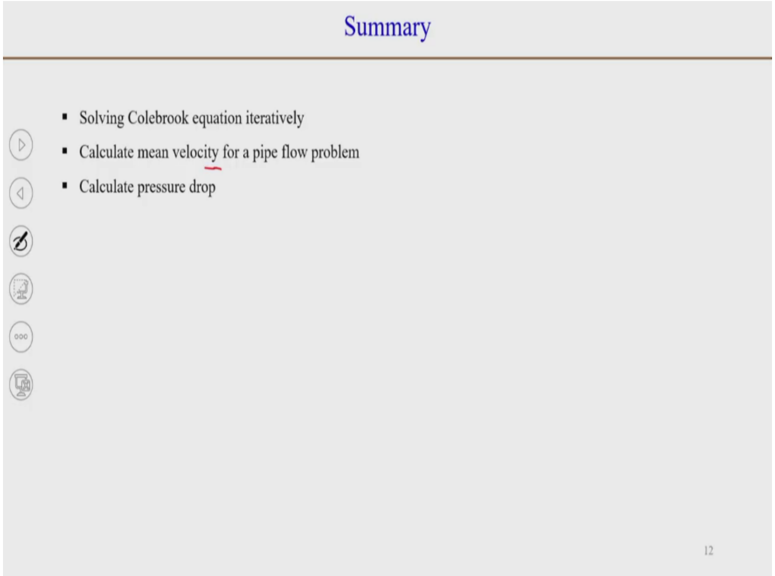

So, what we have done today to summarize is that we looked at that how we can solve Colebrook equation and that how do we need to do iteration and we can make the initial guess, say f = 0.1, we can use as initial guess as an initial guess in all the cases. And then we looked at a problem where we calculated the mean velocity and this needed to be done in an iterative manner because the mean velocity is involved or required to calculate Reynolds number which is required in turn to calculate friction factor.

So, we needed to guess a value of friction factor for that we used Moody's plot we could also use Colebrook equation, we could have assumed a higher value of Reynolds number say of the order of 10^6 because that is where that the value of friction factor becomes independent of Reynolds number. And then we solved a problem where we calculated pressure drop in the system. So, we will stop here. Thank you.