Fundamental of Fluid Mechanics for Chemical and Biomedical Engineers
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Lecture 41
Flow in Pipes: Minor Losses

So, in the previous class we talked about pipe flow and then we derive a relationship for the losses in pipe or the frictional losses in pipe. We divided these losses in major and minor losses. So, in the previous class we talked about major losses, and in this class, we will focus on minor losses. So, let us just recall what we discussed in the previous class.

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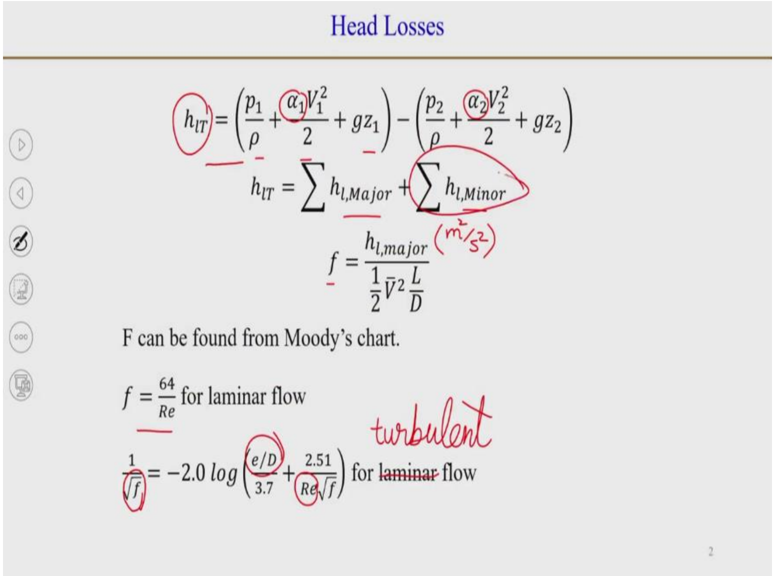

That using Reynolds transport theorem for energy, we could derive an equation of this form. Which looks like a modified form of Bernoulli's equation, here you have all forms of energy, pressure, kinetic energy, and the potential energy at two points, point 1 and point 2. And it says that the frictional losses or all other losses can be combined in a term which we call h_{lT} or head losses.

Now, we could divide these head losses into two parts, one is what we call major losses and another one is minor losses. So, major losses we discussed and we said that, in a pipe of cross sectional, in a pipe of constant cross-sectional area, where the flow is fully developed, the total frictional losses is what we call the major losses and the pipe is horizontal. So, we showed that h_{lT} can be $\Delta P/\rho$, where ΔP is the pressure difference between the two points in a pipe.

Now, we defined a factor using dimensional analysis, we found out that how we can relate pressure drop with other parameters, geometrical parameters, defining the pipe geometry or the flow parameters and fluid properties; density and viscosity. And from that, we finally found a friction factor which we call Darcy friction factor and defined $h_{l, major}$ where the unit is meter^2 per second^2, this is divided by half V bar^2 × L/ D and V bar is the average velocity at a cross section because in laminar as well as turbulent pipe flow, the velocity is not uniform across the cross section.

So, when we take a velocity at a particular cross section, because this analysis is one-dimensional analysis. So, to do that, we take the average velocity. And so, this brings us to another difference from the Bernoulli's equation that we have these values here α 1 and α2 which we call kinetic energy correction factor or kinetic energy coefficient and we derive the relationship for the kinetic energy coefficient.

The kinetic energy coefficient, if you use the parabolic velocity profile, it comes out to be about 2 and if you use the turbulent velocity profile say 1/ 7th power lower than the value is about 1.06. So, you take the value approximately equal to 1 for turbulent flows and for laminar flows the value is 2. Because most of the flows that we deal in chemical engineering applications, the flow is turbulent.

So, often we use the value of kinetic energy correction factor as 1, but say other applications where you are looking at flow in say micro fluidics, there we need to take this kinetic energy correction factor as equal to 2. So, the friction factor that we have, we could find from moody chart that we saw yesterday or we can represent it in terms of equations for laminar flow, it is f =64/ Re and for turbulent flow we can use Colebrook relationship, so this should have been a turbulent flow.

So, this is Colebrook relationship where the friction factor is a function of Reynolds number and roughness factor, where e is the roughness factor. You might notice that this equation is an implicit, meaning that you have an f on both sides of the equation. So, it will need to be solved iteratively, you will need to guess a value of f first and then find the new value f until it converges. So, now, we will try to focus on minor losses.

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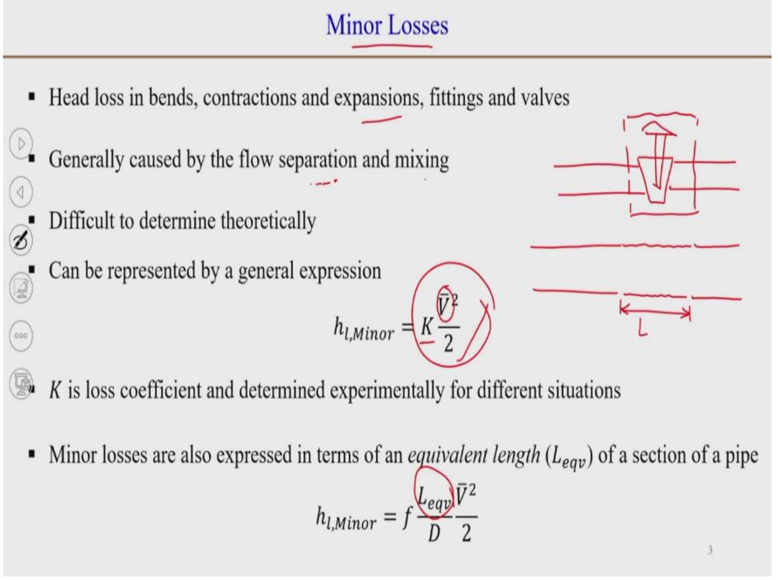

So, as we said yesterday, that actually apart from the frictional losses in the fully developed flow in the pipe, anything else that we have can be brought under minor losses. So, in a piping network you will have inlets and exits. So, the losses because of the geometry of the inlet or because of the exit because it is going, the flow is going to a reservoir or because of bends, because of elbows, Tees, because of valves, and reducers and expanders all those will be combined under minor losses.

So, the physics is not so simple, it is very difficult to find a theoretical value for the loss in case of minor losses, because when we talked about major losses, it was a fully developed flow in a pipe of uniform cross-sectional area and we could derive the friction factor or the equation for the loss for laminar flow analytically and for fully developed turbulent flow we could relate it with the experimental data. Now, when we use different fittings, different valves etcetera, then there will be lot of variability in the fittings etcetera that we use in this.

So, the minor losses can be very different even for a say 90-degree bend or the valves from two different manufacturers, same type of valves from two different manufacturers, manufacturers can give different values of minor losses. So, the losses that are called or that are caused here, if you look at all these there is say flow is expanding or the flow is contracting, you have bends and you have valves where the geometry inside the valve might be complicated.

In all that might cause flow separation, all that cause a bit of mixing, may have secondary flows. So, all those combinedly can cause irreversible loss in pressure. So, when we talk about head losses, all the losses are irreversible meaning that those losses cannot be recovered that is dissipated as we saw yesterday, that all those losses were u2 - u1 - $\dot{Q}/\dot{m}$, so they all those were dissipated in terms of heat energy and that it might be transferred. So, those losses come by from flow separation or mixing or both.

Now, we can have a general expression for minor losses and the commonly used expression is say $KV^2/2$, where V^2 is, V V bar is the mean velocity and K is called minor loss coefficient. So, you can have the K value or you can get it from handbooks, you can get it from manufacturers data sheet and it is always, if you are looking at a practical application then it is always useful or it is always advisable to use manufacturers data sheet to find the K values for different components in your piping system.

And this K can also be determined experimentally So, what we can say that the loss that is present in a piping system because of the presence of a particular component it may be a reducer, diffuser, or elbow that all loss is combined in this term $KV^2/2$. The other way to represent is it in terms of equivalent length. So, if you remember the formula for $h_{l, major}$, it was $f L/D \times V^2/2$. So, in analogy with that we have here because, we could define an equivalent length.

So, let us say if you have a piping system in which you have a valve in between and if you replace this valve with a pipe which has the same amount of minor losses as because of the presence of this valve and the length of the pipe that will be required to create the minor losses is what we call equivalent length. So, when you do that, your equation become simpler you will have just simply l + l equivalent. So, you could use either of these expressions. Generally, you will find in most of the cases the values are given in the form of K or minor loss coefficient.

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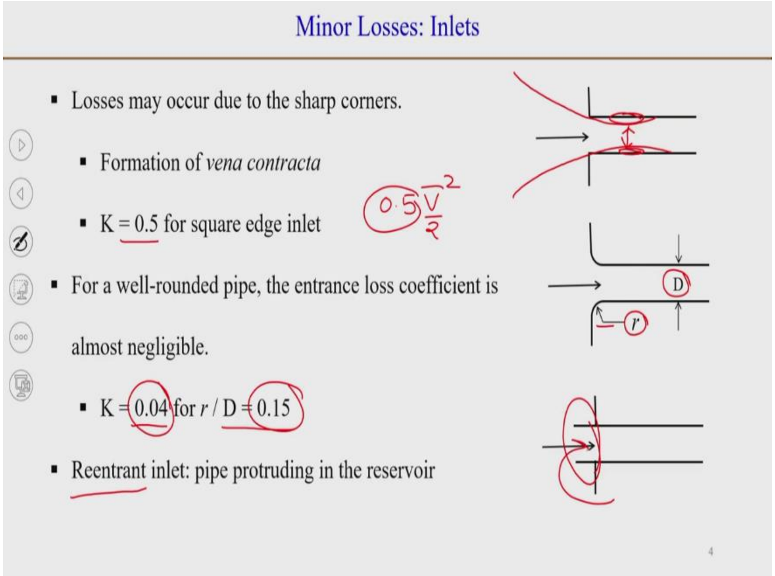

So, now, we will discuss briefly for each component that what can be the causes of different minor losses or what could be the typical value or expressions if available. So, if we talk about inlets, if you have that the flow from a reservoir is coming into a piping system and the corners are sharp here then the losses are large. So, it is like you have a large reservoir and suddenly the flow area decreases here and if you plot the streamlines, you may get some recirculations in this of course, it will be inside the pipe only and that recirculation will cause mixing and you will get a vena contracta formation here.

So, the actual flow area is reduced and eventually when it goes further downstream, it will attain a fully developed flow. So, the loss due to sharp corners can be significantly high and for such a^2 edge inlet the value of K is about 0.5. So, the minor losses will be 0.5 V bar^2/ 2. So, if you use rounded pipes, a rounding can help quite a bit and say if this r/ D, where r is the curvature here and D is the pipe diameter, if it is 0.15, then this value reduces to say 0.04.

So, rounding may help to reduce, the value has come down from 0.5 for sharp edges or for^2 edge to value of 0.04 when the edges are rounded. Another kind of inlet that you might see is that the pipe might be slightly protruding inside the reservoir. So, it is not just at the edge of the reservoir, but it is inside the reservoir and you might see this for example in the household tanks, there is pipe in the tank that is protruding in and this is called Reentrant inlet.

And the value here can be quite large because the flow, for example might need to turn here. So, the value can vary depending on the pipe length, etcetera between 0.5 to 1, the value of K. A typical value you could say is about 0.8. So, that is about inlet and the takeaway from here is that we should be having the rounded inlet as far as possible, as and when possible.

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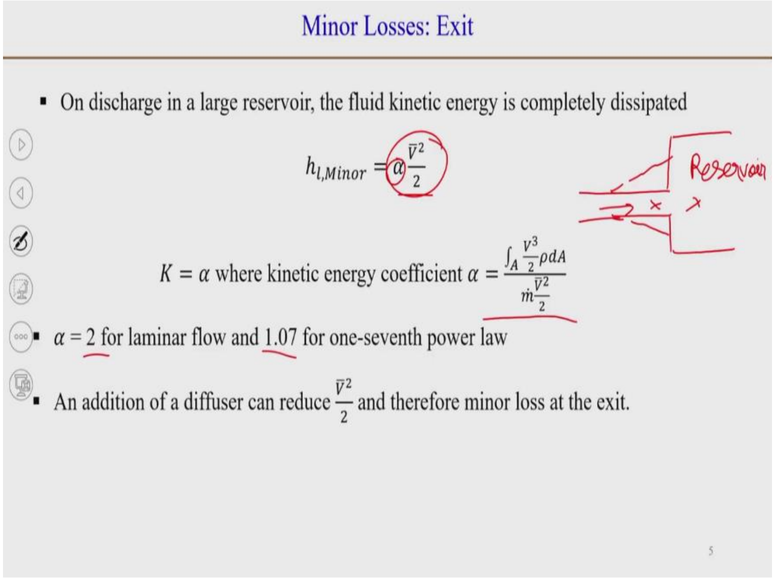

So, the next comes, Exit. So, when you talk about exit if the flow is generally, it is going out in a large reservoir. So, basically if you have a pipe and from this pipe the flow will be going out to a reservoir. So, the velocity because the reservoir area is large, so the velocity of the fluid as it goes from here to the reservoir from the pipe to the reservoir, velocity is almost going to 0 or to be 0 in the reservoir.

And if you assume that there is negligible change in the height which is a fair assumption if you take these two points, point 1 and point 2, then the losses or the change in pressure it will be because the kinetic energy is going to be 0. So, you could say that this will be $\alpha 1$ V 1^2/ 2 - $\alpha 2$ V 2^2/ 2. So, $h_{l, minor} = \alpha V^2/ 2$, where $\alpha$is, as we discussed earlier that $\alpha$is kinetic energy coefficient and we derive this formula for kinetic energy coefficient yesterday, the value 2 for laminar flow 1.07 for one-seventh power law.

So, if you want to reduce it, we could not do much but except that we try to reduce the velocity at the exit and this can be done if in place of this if you include a diffuser and that may reduce the velocity. So, addition of a diffuser, when it goes into the reservoir, when the fluid goes into the reservoir if you add a diffuser then the velocity will be reduced slowly in the diffuser itself and this value αwill remain same but V will get reduced.

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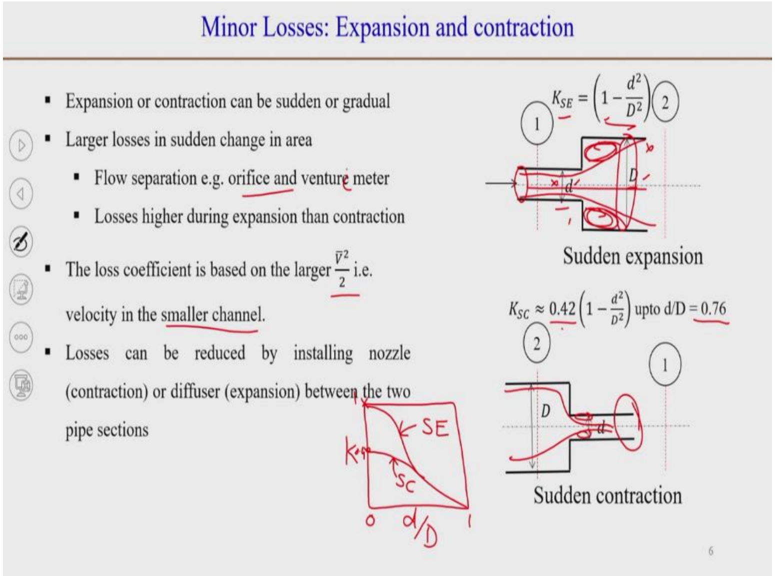

Then, you have reducers and expanders. As we saw that inlet and exits are also a kind of reducer or expander. So, these expansions can be either sudden, you have a sudden change in area for example, sudden expansion as you can see here or sudden contraction. So, sudden expansion and contraction again because for example, when you have the flow expanding and the streamlines will be something like this and you will have, flow separation happening in this region because, as the area of flow increases, when the area of flow increases the diameter has increased and as a result, the velocity will decrease, when the velocity decreases the pressure at this point will become higher than pressure at this point.

So, you will have adverse pressure gradient and that will cause a flow separation. So, you will have the flow separating here and the recirculation. Similar to what we saw at inlets, you will have a vena contracta forming in this case and there will be losses. So, this flow separation example, we have already seen that in orifice and venturi meters, so it was venturi meters. So, orifice meter where you have a certain change in area, whereas venturi meters you have a gradual change in area and we saw that the losses are in the venturi meters.

So, the same thing could be done that, if possible, we can try to reduce the sudden change and we can do it gradually, so that the pressure does not get lost and it can be recovered. The losses are higher in the expansions than those are there in contraction. So, one of the formulae for sudden expansion is based on the area changes, you could use Bernoulli's and try to find this, say that =1 - d^2/ D^2, where d is the area of diameter pipe and D is the, d is the diameter of pipe and D is diameter of the larger pipe.

So, this is about 1 - d^2/ D^2, whereas at the sudden contraction there is a value of 0.42 and this is valid say for d/ D less than or up 0.76 and after that this is almost same. So, if you plot k with d by D you will get a graph something like this. So, that will be for sudden expansion and that will be for sudden contraction and the value changes from 0 to 1 here, the k value will be 1 here, this is about say 0.4 of course, it will be slightly away from the graph.

So, these losses can be reduced if you include a gradual change in area. So, when we talk about contraction and it will be a nozzle and when we talk about expansion and it will be a diffuser. The other thing to remember is because we have two diameters here, d and D diameter of a smaller pipe and diameter of larger pipe and then we will have dilemma that when we calculate minor losses, which V to take. V in the diameter pipe or V in the larger diameter pipe.

So, the calculations are based on the larger $V^2/2$ and that will be the case in the smaller channel, so this is done. Either you take, so it will not be that it is based on the entry, it will be based on the smaller diameter channel, it may be an expansion or contraction and convincingly that is how you will find in the k values in the literature.

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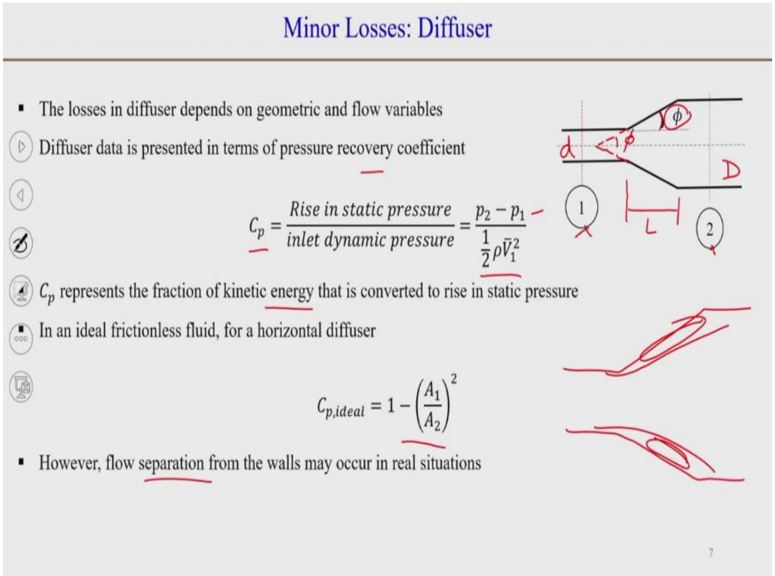

So, if we talk about diffuser. So, diffuser can be characterized by say an angle here, which is defined as phi and the diffuser length. So, for example, the length in which the area changes or the diameter changes from d to D that will depend on this length can be defined in terms of this angle or it is called half cone angles, if you extend it to the point here this angle will also be phi.

So, it will depend on Reynolds number and the geometry of it. If we consider that the flow is frictionless than we can define or we can find out how much is the pressure recovery using simple Bernoulli's equation. So, this pressure recovery is defined in terms of a coefficient Cp here, which is called pressure recovery coefficient. So, that is when you have flow in diffuser and if you consider the flow of an ideal fluid then from point 1 to point 2 because the area is increasing here, so you will have a decrease in velocity and correspondingly a rise in pressure.

So, that rise in pressure, let us say p 2 - p 1 is the rise in static pressure and this is divided by dynamic pressure half ρ V 1^2 because V 1 is where the velocity is going to be high. So, we will have this half ρ V 1 bar^2. So, that is the pressure recovery and so, theoretically it represents that the fraction of kinetic energy here, which has converted to dynamic pressure and if the fluid is ideal then you will be ideally able to recover this pressure later on.

So, this can be defined further if you write down a p 2 - p 1 using Bernoulli's theorem, you will be able to get this relationship using A 1 V 1 =A 2 V 2 for an ideal fluid. But the world is not ideal you have a lot of flow separation. So, if this is not done smoothly, then you will have, because of flow separation as well as frictional losses, you will have a, losses because of mixing and flow separation happening there. So, there will be a, the Cp will be different from Cp ideal.

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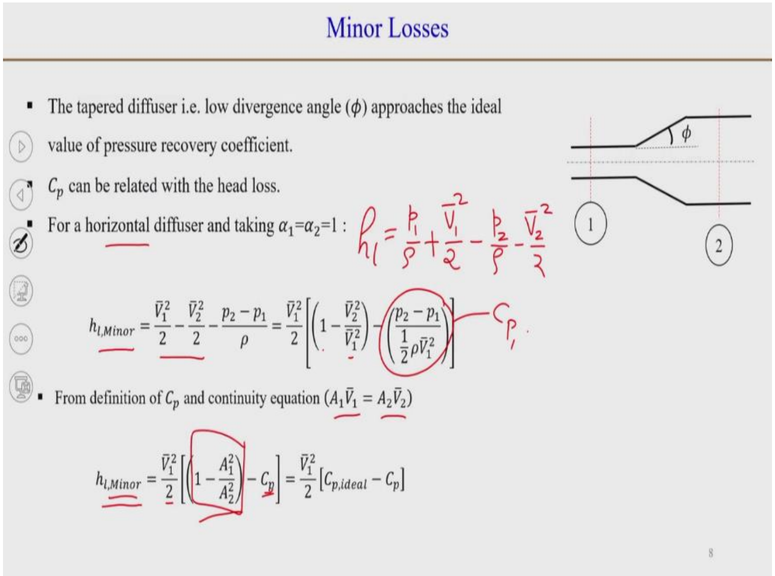

So, you can relate this Cp or the head loss with the Cp value for a diffuser. So, if you write that the minor losses for a horizontal diffuser, so Z 1 will be equal to Z 2 and that will be equal to V 1^2 - V $2^2/2$ and if you write this in terms of p 2 - p 1 because if you write that h l =p 1/ρ + V 1 $bar^2/2$ + g Z 1 and g Z 1 we can cancel out because it is horizontal. So, g Z 1 will be cancelled out with g Z 2. So, we will not write and that will be - p 2/ρ - V 2 bar^2/2, we can write this in this form. So, when you rearrange the losses will be like this.

So, you can write - p 2 - p 1 and then we can rearrange it, we can take V 1^2/2 out of brackets. So, this term will become 1 - V 2^2/V 1^2 - p 2 - p 1/half ρ V 1 bar^2 and we know that this is what, Cp, ideal that we saw in the previous slide. So, and we can write down from A 1 V 1 bar =A 2 V 2 bar and substitute this here.

So, you will have, $h_{l, minor}$ =V 1 bar^2 × 1 - A 1^2/A 2^2 - Cp. So, that is, sorry it is Cp in general, when you have Cp in general and when you substitute this you will get Cp, ideal 1 - A 2 1^2/A 2^2, so that will V 1^2/2, Cp, ideal - Cp. So, this is what you have as Cp, ideal. So, that is the definition of $h_{l, minor}$. Now, knowing the A 1 and A 2 which are the geometric parameters for a diffuser. So, you will know beforehand and if you know the Cp value or the pressure recovery coefficient for a diffuser, then from that value you could calculate, what is the minor losses.

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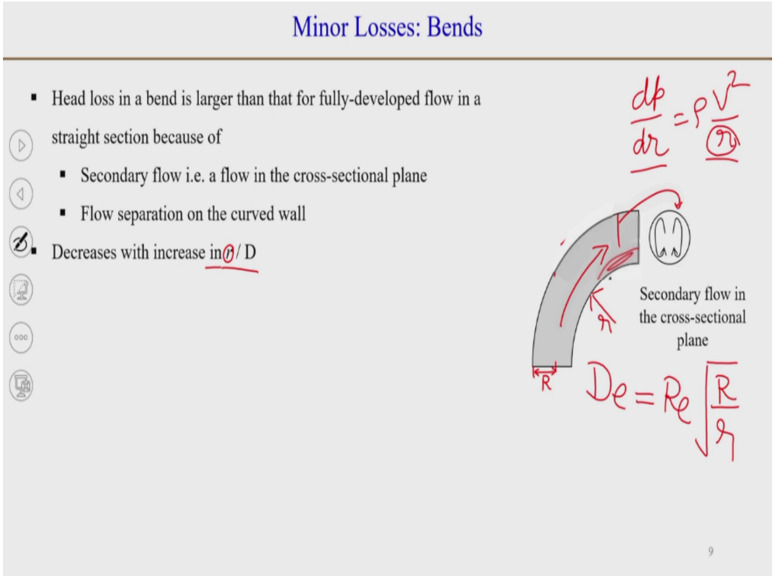

Then other component is bends or elbows. So, in bend and elbow, the flow is turned and when the flow is turned, you could see two things happening here, one is the secondary flow might develop. So, if you take a cross-sectional area as have been shown here, you will see secondary flow in the cross-sectional area of the pipe.

So, by secondary flow I mean that the main flow or the primary flow is along the pipe length. But if you take a cross section and look at it, there are vortices in the pipe cross section, which are called Dean vortices and this is because, when you have a curvature, secondary flow come into picture because of the centrifugal forces. So, when we talked about say inviscid flow then we looked at that dp/dr =ρ V^2/r, flow along a streamline in the streamline coordinates and that was inviscid flow.

So, it was that the centrifugal force could balance the pressure gradient, but here when we talk about real flows, the viscous effects also come into picture. So, it is an interplay with the between the viscous forces, the centrifugal force and the pressure gradient and that gives rise to these vortices here. So, that they call pressure losses + when you have the flow turning you may have also the flow separation happening near the pipe wall, so that will cause further losses.

So, all that cause can cause bend losses, these bends can be the turning maybe 90 degree or 45 degree or say U-bend where you have, 180-degree turn. And this will the strength of these vortices depends on a non-dimensional number, which is called Dean number and this =Reynolds number × the √R/Rc or if we use the nomenclature here. So, this Rc is basically r, where r is the radius of curvature and R is the radius of this pipe here.

So, smaller this radius of curvature higher will be the Dean number and higher will be the secondary flow or the secondary flow decreases when the R value or the radius of curvature increases and the same is true for the flow separation because the turn happens smoothly. So, the R value you can see from here the strength of the centrifugal force will decrease if r is large.

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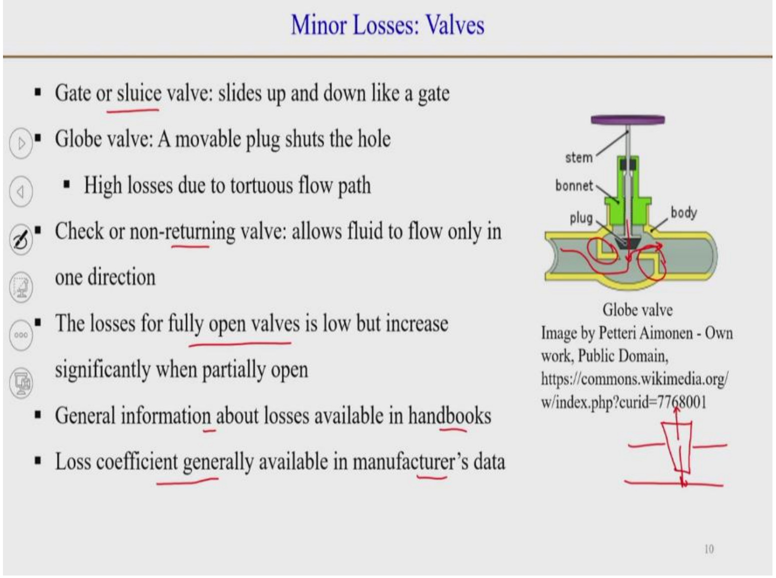

So, you can look at the values in manufacturers or in the handbook for bends. Now, the other component is valves. So, valves are generally used to control the flow. So, you may have valves in the fully open conditions or it can control the flow rate by the partially open valve and when you have partially open valve, they represent the restriction in the flow and these valves can be of different types.

So, I have listed some of the types here. So, one is called what is called gate or sluice valve. So, basically it has that in your flow you might have a valve, so it may just slide up or down and it can stop or it can open or it can stop the flow. So, it slides up and down like a gate. So, we call it gate or sluice valve, then you have globe valve which looks like this. So, you have a restriction in the flow and the flow even in the fully open condition the flow needs to follow a torturous path or the complicated path.

So, there will be complicated flow structures, possibility of separation. So, even in the completely open conditions you might see that the losses are high in the globe valve and when it is closed, this plug comes down. Then you have a check or non-returning valve. So, these valves are that they allow the flow to happen only in one direction. So, when we talk about cardiovascular flows, all the valves present there between ventricle and atrium and the valve between the aorta and the ventricle and the ventricle and the pulmonary artery, all of them are one-way valve.

So, we use quite often one-way valve in pipelines or they are also called non-returning valves. So, as I said that when you have a valve in the fully open conditions, the losses are relatively low, but when you close them or the more restriction in the flow, the higher losses will be there.

And it is, you could find a more information about the valves in handbooks or the loss coefficient accurate information for a particular valve will be available from manufacturers data sheet.

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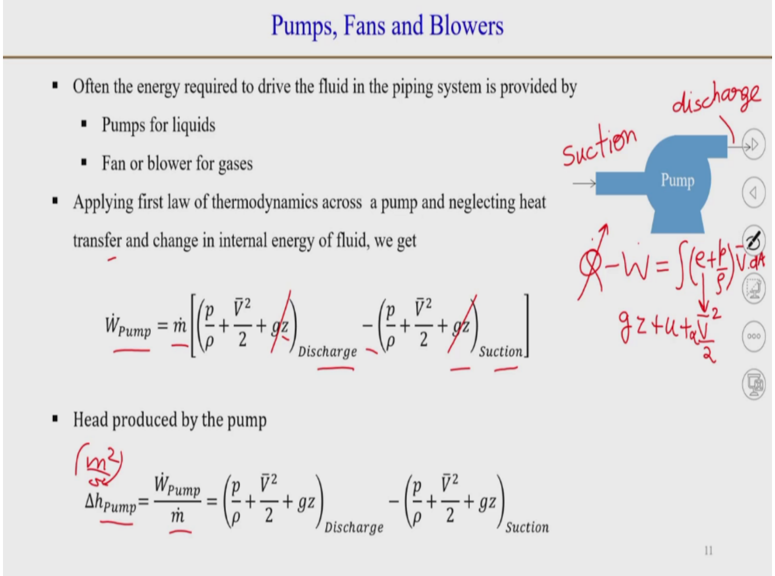

So, we talked about different minor loss components and what causes the minor losses. It can be represented as K V bar^2/2 and this K is the loss coefficient. So, this loss coefficient we can find out or we can look into the data sheets, if we are talking about solving problems, then generally it will be given as the part of the problem. So, we talked about losses and to overcome these losses because the losses happens.

So, that is irreversible loss, the energy is lost. So, that the flow is happening continuously at a particular flow rate, we need to provide energy to the fluid and that is done, that is why we use pumps, for example for liquids or blowers or fans for gases. So, in a piping system when you talk about say pumps, then you could say that they provide energy or they provide a negative head loss.

So, head loss is in your energy equation, they are a kind of sink and pumps provide a source of energy. So, we could write down again the equation for the energy equation. So, if we write down the energy equation and neglect say we had a $\dot{Q}$ - $\dot{W}$, there was an unsteady term which we will not write here and then we had e + p/ρ V. d A. So, the p/ρ was flow work and then we could write this e, in terms of the internal energy + V^2/2 along with an α+ the potential energy g z.

So, if we say that the heat transfer could be neglected then this is neglected and $\dot{W}$ is the work done by the fluid. So, when you have a pump, it does work on the fluids, so this work will become positive, that is pump work, so that is why you have on this side $\dot{W}$ pump and this we can simplify in terms of, we can take the integral out ρ V. d A can become $\dot{m}$ and there will be a - sign at the inlet, which we call suction side.

So, where the flow comes in or where the pump takes the flow, we call it suction side and where the pump discharge the flow, we call it discharge side. So, $p/\rho + V^2/2 + g z$ on the discharge side and on the suction side. Generally, what will happen that the difference between z at the discharge and suction side is very small, so we can neglect this. So, it will be $p/\rho + V^2/2$ as discharge and $p/\rho + V^2/2$ as suction.

What fluid will give is, it will provide the head. So, if the cross-sectional area at the suction side and the discharge side, the pipes that we use at the discharge side and the suction side if their cross-sectional areas are same, and then the flow rate will be same. So, if you talk about liquids which is used, as a pump is used for liquids. So, if we talk about liquids, it is an incompressible fluid. So, for an incompressible fluid, mass flow rate is going to be same and density is constant.

So, the density at the inlet and outlet will be same that means volumetric flow rate at the inlet and outlet of the pump is same. So, when you have a pump, it does not change the velocity of the fluid, what it will change is p/ρ. So, velocities will also be cancelled, as we can see, as we will see. So, this is head produced by the pump that we can write. So, head is in meters^2 per second^2. So, we can rearrange this as $\dot{W}$ pump or work done by pump divided by $\dot{m}$ which is mass flow rate.

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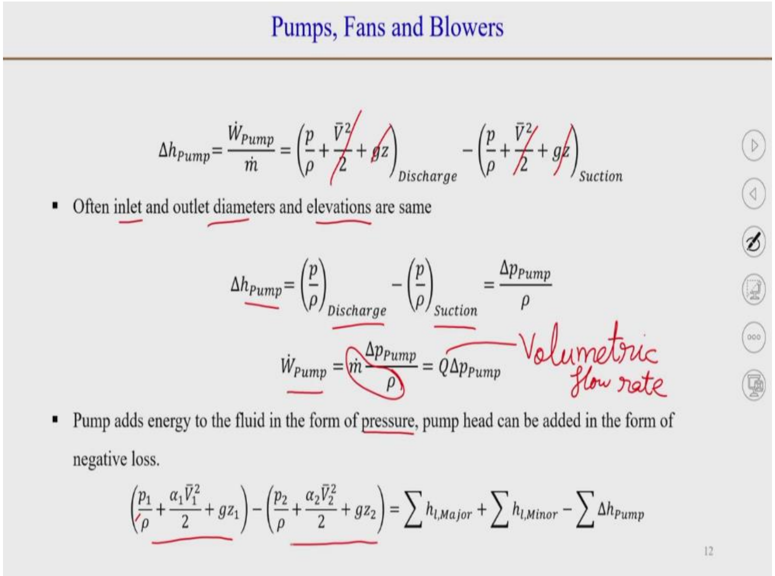

So, in this equation because the elevations are same as we just discussed, and if inlet and outlet diameters are same, then these terms will cancel out. So, the head gained or the head increase by the pump is p/ρ as the discharge side - p/ρ at the suction side. So, that is Δ p/ρ that is the head gained at the pump. And so, we can write this work done =$\dot{m}$ × Δ p/ρ or $\dot{m}$/ρ is what is volumetric flow rate.

So, that means that the pump adds energy to the fluid in the form of pressure, it does not increase the kinetic energy. So, when we are writing the equation for a pipe flow, for a single series where the pipes are connected in series, then we can write the equation as say the total mechanical energy at point 1, p1/ρ + α/2 V1 bar^2 + g z1 - the mechanical energy at point 2 =the major losses + minor losses and we will have - Δ h pump.

So, it is the energy loss between point 1 and point 2 that will be equal to the energy loss because of major losses energy lost because of minor losses and -, so the energy that is gained because of the presence of your pump in your piping system.

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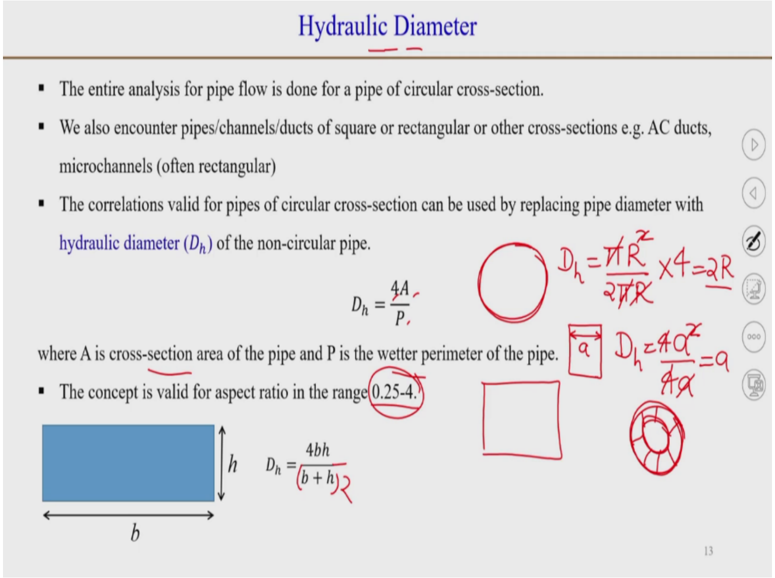

So, if we look at that what we have done in the pipe flow analysis till now is we have looked at the losses and the energy gained because of pump. Now, most of the time we encounter pipes which are circular in nature, but in some places for example, you have AC ducts and you will all know that these AC ducts are generally in rectangular or^2 shaped in cross section or if you talk about say micro fluidics there again the cross-sectional area will be rectangular or say semi-circular or triangular.

So, in such cases what do we do and the simplification that we could make is we can use the concept of what is called hydraulic diameter. So, the hydraulic diameter is basically defined as 4 × A/P, where A is area of cross section of the pipe and P is wetted perimeter of the pipe. So, wetted perimeter is the perimeter of the pipe, which wets the fluid. So, if you talk about a circular cross-sectional area, then this perimeter is wetted perimeter and you can calculate Dh =πR^2/ 2 π R, π and π will cancel out and this multiplied by 4.

So, you will have Dh =2 R, if you talk about a rectangular cross section, then you will have let us say this is of side a, then Dh =4 multiplied by cross-sectional area which will be a^2 divided by 4 a. So, you could simply cancel out everything and you will get Dh =a. If you have a co-annular system then your cross-sectional area will be this area, whereas, the wetted perimeter will be the perimeter of the inside pipe + perimeter of the outside pipe.

And this concept is valid when you have this hydraulic diameter sorry, the aspect ratio. So, aspect ratio is, if you talk about say rectangular channel the aspect ratio is between 0.25 to 4. So, the ratio of one dimension to another, so you could well say that it is 1 to 4, so the other dimension will be 1 to 0.25. So, in either case it is until the ratio is 4, ratio of the larger and the smaller dimension is 4 then we can use these concepts. So, this is the rectangular cross section. For rectangular cross section, it will be 4 times b × h/ b + h × 2 here.

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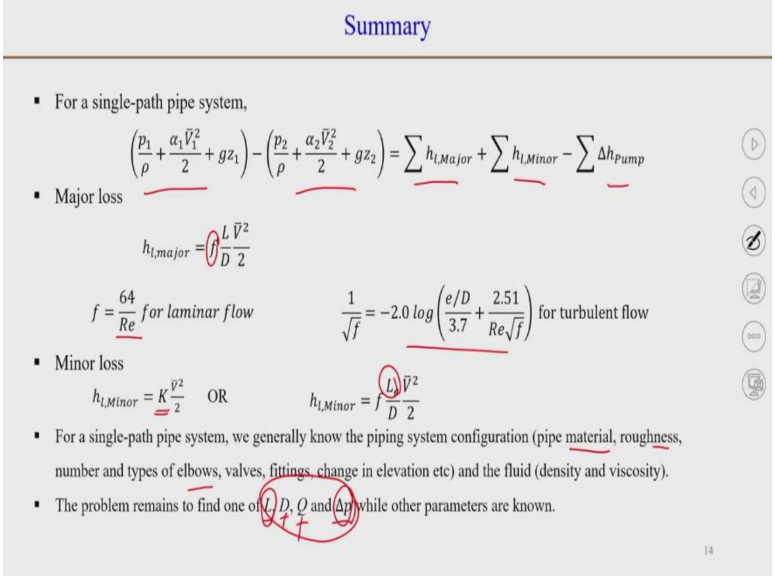

So, if we summarize that for a piping system where it is a single pipe or the pipes in series and it has bends, pumps, valves, fitting etcetera, then we can write down the energy equation which can be used to calculate things, the mechanical energy at point 1 - mechanical energy at point 2 + all major losses in all the straight pipe sections + all the minor losses - the head increase at the pump.

The major losses we can define and we can obtain using this formula and we need friction factor. So, friction factor is 64 Re for laminar flow and you can use Colebrook relationship for turbulent flow. Minor losses, we can define in terms of K minor loss coefficient or in terms of equivalent length. So, we can combine all this. And for a piping system because we will know that what is the pipe material, what is the roughness of the pipe, how many elbows, how many valves, fitting etcetera do we have, what is the elevation.

So, z 1, z 2 at different places and the fluid properties. So, it will reduce to that, we might need to calculate the length of the pipe, if we are designing a piping system, we might need to design it for that, what diameter should be required and we might need to look at the flow rate for a given piping system or the pressure drop over the entire length. So, we may be working with these four factors and the problems can be that you have been given three of them and you need to calculate one.

So, in some cases, you might need to. For example, when you are talking about diameter and Q because in all these definitions you need to calculate Reynolds number and the Reynolds number is a function of diameter and velocity, and velocity is Q by cross-sectional area. So, these numbers are D and Q will come in the definition of Reynolds number. So, you might need to solve the problem when you need to calculate D or Q iteratively.

First you might need to assume one diameter or one flow rate and then do the calculations. And you do it again until you find a converged solution. Whereas in the case of L and Δ p, the problems will be straightforward, you can form a equation using this and solve the problems. So, we will discuss this further and solve some examples in the next class. Thanks.