Fundamentals of Fluid Mechanics for Chemical and Biomedical Engineer
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Lecture 40
Flow in Pipes: Major Losses

Hello, so, in this last module what we are going to discuss is, flow in pipes. If you go to any chemical process industry, the first thing probably you will notice that the industry has a number of pipes. And when the, say, crude oil needs to be transported from the production site where the oil is being drilled and being brought to the surface, it needs to be transported to the refineries.

And then further on, it needs to be transported to, from the refineries to the distribution side. Sometimes, it may happen via trucks and at other times it may happen, the transportation of oil as well as say, natural gas happens via pipelines. So, flow through pipeline is a very important topic for chemical engineers. Now, when we talk about say biomedical engineer, again the size of the pipes will vary.

It may be of the order of few mm, and few centimeters. But again, the flow in pipes is important, there the flow will be a laminar for example, when we talk about the cardiovascular network, you have different pipes starting from aorta which is about 2.5 centimeter to down to the capillaries which are about say, 10 microns or so. So, in, in each case you have network of pipes.

And one of the requirements, that we need to do say, as an engineer to design the pumping system, so when you need to design the pumping system, you need to know how much pumping power or how much head the pump should be able to provide. When I say head, how much pressure that the pumps should be able to provide, that the losses that happen in the system they are overcome. So, that is a very important calculation, and the focus in this module actually, will be on such things, especially the calculation of head loss in the pipes.

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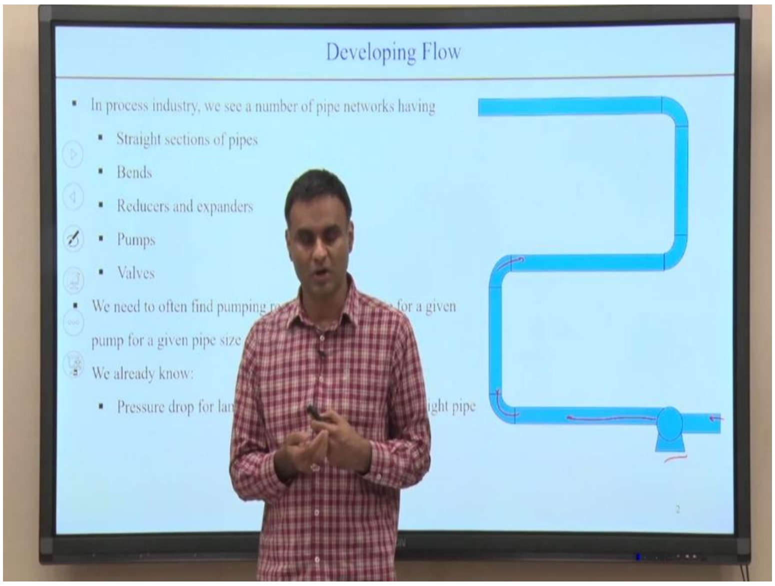

So, if you take a typical piping network, you may have a, a pump which push the fluid through and then it goes through and you have a number of bands etcetera. So, apart from bands you will have say, valves which may control the flow rate, you may have reducers and expanders, where the pipe sizes are changing say from, 1/ 8 inch to you are getting to 1/ 4 inch or other way around.

So, all these elements of the piping system, we need to take into account and see what are the pressure losses in these elements. Now, we have done a bit of flow in pipes, we have looked at laminar fully developed flow in pipe and obtained a relationship for pressure drop in terms of flow rate, what we call Hagen Poiseuille law.

We also looked at turbulent flow in pipes, but for turbulent flow in pipe in the previous module, we looked at the velocity profile, we still do not know the, or we have not discussed what is the pressure drop when the flow is turbulent in a pipe. So, all these things we will discuss. But even when talking about laminar flow, what we discussed is fully developed flow.

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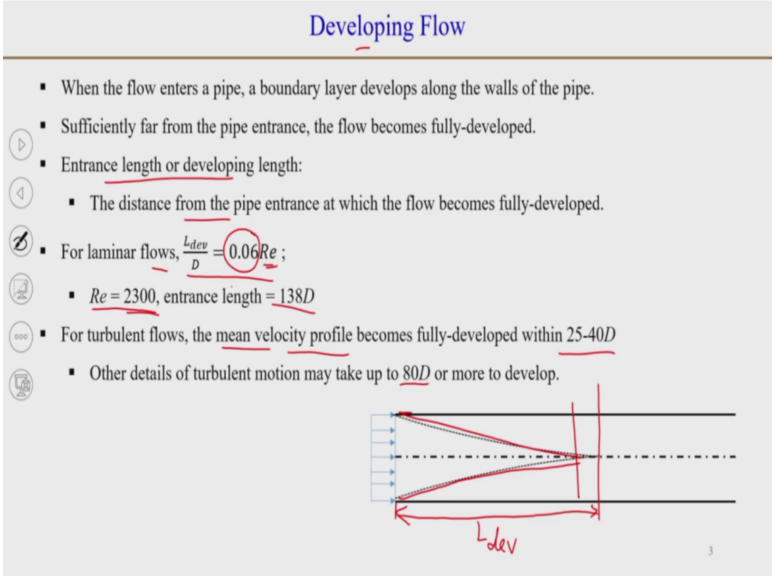

So, before we talk about the pressure loss, we will just touch upon the topic of developing flow. So, if you have say, a pipeline which is say, the pipeline is in a river, then because you can consider the boundaries of the river sufficiently far away, then the velocity in the middle of the river, probably it will be turbulent in most of the cases. And when it is so, then the velocity at the entrance of the pipe will be uniform.

So, the flow enter with a uniform velocity. Now, when, when it encounters the wall and when the fluid encounters the wall, the velocity at the wall needs to be 0 because of no slip boundary condition. And the boundary layer start developing on the wall of the pipe, and eventually what will happen, these boundary layers will grow as we have seen flow over, in the case of flow over a flat plate. And when these boundary layers grow, it will be let us say you have a circular pipe, then boundary layer will grow on at each point on the say, entrance.

Now, these boundary layers will grow and then eventually they will merge. So, the point where the boundary layer cannot grow further, is where you will have a fully developed flow, the velocity profile will be parabolic, for a laminar fully developed flow. But before that, the length that is required, you call that length as developing length or entrance length. So, basically the entrance length or developing length, it is defined as the distance from the entrance of the pipe.

So, in this case, we can say that this length, is what is entrance length or developing length. So, we will write this as L development. Now, if the flow is laminar, then you will have a different entrance length, and if the flow is turbulent, the entrance length will be different. So, for laminar flows, it has been observed that the development length =0.06 Re D. This number you may find varying, you may find it say, in some places it is 0.05, in some places you may find it 0.056.

So, the development length, it depends on Reynolds number, and a non-dimensional term it is say you write as L/D for developing, or you can say if you want to find the length, then it will depend on the flow Reynolds number and the diameter. So, it is L/D =0.06 Re. If you consider, the Reynolds number 2300 which we take as the limit, when the flow above which, the flow does not remain laminar anymore.

So, the highest development length for a laminar flow, is 2000 when we calculate it for Reynolds number 2300. So, if we substitute the values, we will get the entrance length of about 138 D so for Reynolds number, when the flow is laminar, then the development length is about 2000, 138 diameters or 140 diameters. When we talk about turbulent flows, because there is lot of mixing, in the transverse direction you have eddies in turbulent flows. And that causes the flow to develop relatively faster when you compare it with the, laminar flow.

And it has been observed, that the turbulent velocity profile for, for the mean velocity, it develops within 25 to 40 pipe diameters. But other turbulent quantities, such as say fluctuations, turbulent kinetic energy etc, they take longer to develop and it may take up to 80 pipe diameters or more to develop. So, these numbers give us an idea about, how much length will be required for the flow to develop.

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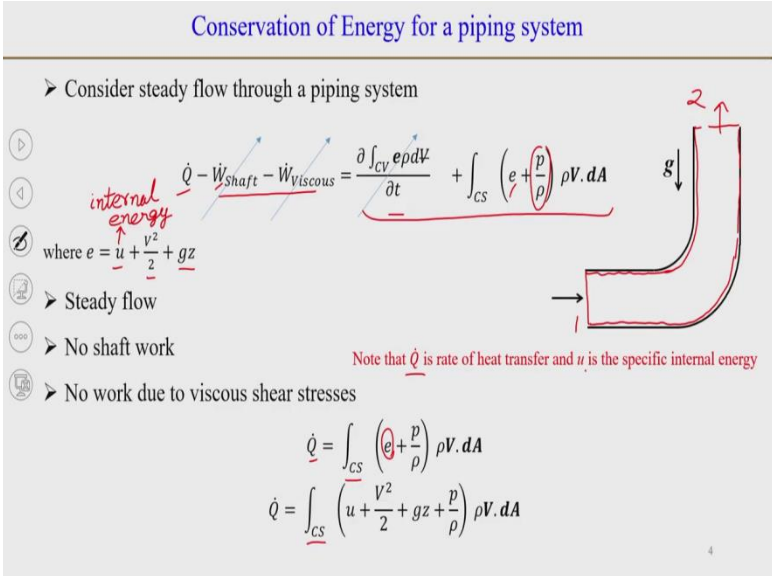

Now, when we discussed Reynolds transport theorem, and the integral analysis, we looked at apart from mass conservation, momentum conservation, we also looked at the energy conservation. So, we will go back to the energy conservation and then, there we applied the energy conservation for the piping system. So, we will do that again or we will recall that. So, if we consider a steady flow through a piping system, we can take a simple pipe with a band.

So, the flow is coming let us set section 1 here, and it is going out at section 2. So, if we write down the energy conservation equation, so this is $\dot{Q}$, which is the rate of heat transfer, heat being given to the system. So, we can consider a control volume here. So, this is my control volume, $\dot{Q}$ is the rate of heat input, $\dot{W}$, you could have shaft work and the work due to the viscous stresses.

So, the rate of work is that, that the work is being done by the fluid. So, you have that, and then you have rate of change in the energy. And when you write this from a system formulation to control volume formulation, you get this, you have the total internal energy of the system + this flow work p/ ρ. So, if we consider the flow to be steady, then this term will go away. And again, no shaft work then, this term will again, will go away.

And because the viscous shear stresses, the velocity at the boundaries of the control volume is 0. So, the viscous, the work due to viscous shear stresses again will be 0. Now, that gives us a simpler relationship, you will have a $\dot{Q}$ which is, rate of heat exchange. That =integral over the contents, control surface e + p/ ρ into ρ V. d A. And we can write e the total energy of the system, in terms of u + kinetic energy + g z.

Remember the e, u here is the internal energy or, or we can say specific internal energy. Because e is the specific total energy of the system. So, we can expand that and then we can consider it for the entire control surface. So, when we do that, the, I would just like to emphasize again, because what we have been doing throughout the course is $\dot{Q}$ is or use Q as, the symbol for flow rate, volumetric flow rate. Whereas, here we use $\dot{Q}$, which is the rate of heat transfer and we have been using small u as the component of velocity along the x direction, whereas u here is, the specific internal energy of the fluid.

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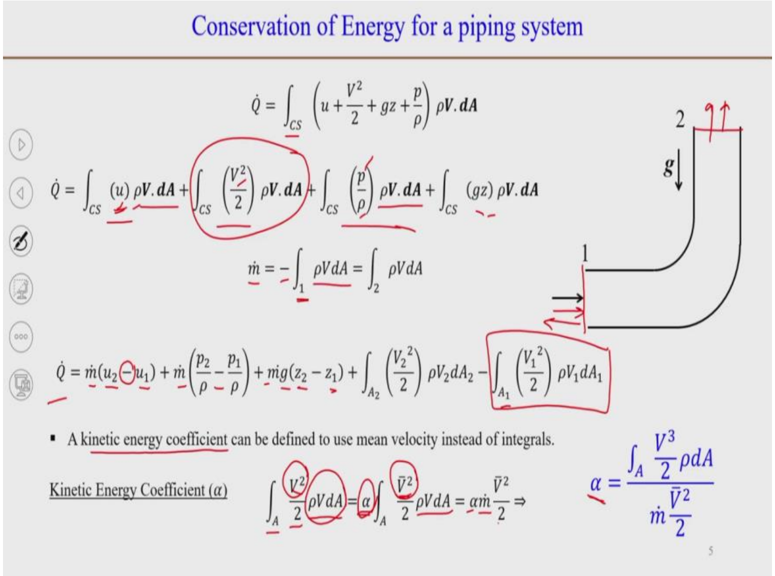

So, if we write down this equation, considering the control surface, then we will have the two surfaces, surface one and surface two, where the velocities are not 0 and we can write down for each term, we can consider the each. So, we have V. dA coming into picture, and then ρ together with it. So, because we want to, or we will know the volumetric flow rate or the mass flow rate. So, we will write this down, in terms of mass flow rate.

At location 1 or at the entrance, the V is pointing inward, the area vector is outward. So, we, as we do usually, it will be - ρ V d A and at the exit, both of them will be pointing outward, so, it will be integral ρ V d A. Now, with this, we can substitute. So, the first term u is internal energy. So, we can say the internal energy at section u - internal energy, at section, section 2 and internal energy at section 1, so u2 - u1, from this we will get and this is $\dot{m}$, because $\dot{m}$ at 1 is negative. So, that is why we have got, this - sign here.

Now, the next term we have this one. So, if we look at this term, again we can write this ρ V. d A the pressure at section 1 and section 2, ρ and p are we can say constant, across the section or we can take the average pressure. So, p 2/ρ - p1/ρ into $\dot{m}$, again the - sign because $\dot{m}$ is negative at, at the entrance or at section 1.

And then we have the third term, which will be $\dot{m}$ into g z2 - z1. So, we have looked at the three terms, and the only term left is this one. So, because it has a V inside it, and when we consider, now we know that the velocity is not uniform inside a pipe. So, we will have the velocity varying across the cross section. So, we will not consider the, we, what we have done is we, we are still having this inside the integral sign.

Because V will be varying across the area. However, this becomes inconvenient, so what we could do? We could write velocity in terms of mean velocity. So, for turbulent flow, it will be the average of mean velocity for the turbulent flows. And average velocity in the laminar flow. So, this can be done, if we defined a factor to take into account, that we are taking the mean velocity in place of the actual velocity.

So, what we could do is, the term of the form $V^2/2$ ρ V dA we can write, because ρ V dA will basically be $\dot{m}$. And we can, in place of V^2, we can write V bar^2. So, when we write V bar, it will be constant throughout A. So, it, we will be able to take it out of the integral sign. So, we can write this as, when we write this as V^2 bar, we have brought in a factor which is called kinetic energy coefficient.

So, we can bring in a kinetic energy coefficient and represent the velocity V in terms of the mean velocity, V^2. So, this term then becomes α $\dot{m}$ into $V^2/2$. So, that gives us the definition of kinetic energy coefficient. So, kinetic energy coefficient α =integral over the area, V^3. So, if we look at here, we can combine this. So, V^3 ρ dA / $\dot{m}$, because ρ V dA is $\dot{m}$ into $V^2/ 2$. So, that gives us kinetic energy coefficient.

Now, if you substitute the values, you will find out because we know what is the velocity profile or how the velocity profile changes for fully developed laminar flow, the parabolic velocity profile. And we can write area in terms of 2 π r dr, dA =2 π r dr. So, we will be able to calculate α for laminar fully developed flow, and you will notice that it comes out to be 2. I will leave that as an exercise for you to do.

If you consider turbulent flow, then the quantity will be very close to 1. So, if you use say, 1/ 7th power law, then the quantity will come somewhere around 1.06 or 1.07. So, we can generally take this value to be α =1. So, you need to remember that when you are solving the problems for turbulent flow, the kinetic energy coefficient is generally your kinetic energy factors or it is called sometimes, it is generally taken to be 1.

But when it is laminar flow, then it will not be 1, and you will need to take the value 2. So, with this we will be able to write, or we will do, we will be able to eliminate this integral sign here. So, let us go back to our energy equation. And we will write this now, by dividing $\dot{m}$. So, we will write this in terms of $\dot{Q}$ / $\dot{m}$.

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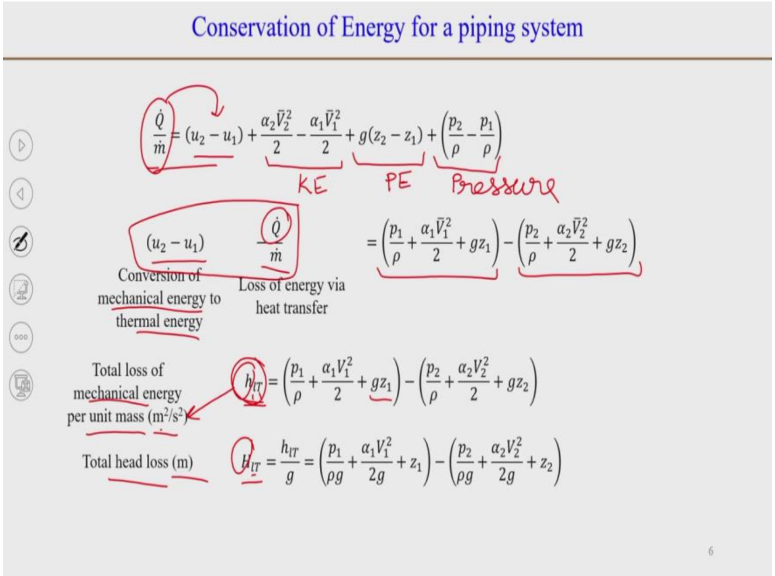

So, $\dot{Q}/\dot{m}$ =u2 - u1, where u2 - u1 is the change in internal energy + α 2 V2^2/ 2 - α 1 V1^2/ 2, which is basically kinetic energy, g z2 - z1, the potential energy due to gravity. And then you have p2 - p1/ ρ, which is basically the pressure or the change in pressure.

So, if you remember, we have three term familiar, the kinetic energy, potential energy and pressure, which we say, that the change, if the sum of these three is constant along a streamline, when we talk about Bernoulli's theorem and assumption was there, that each should have an ideal fluid, the inviscid fluid. Now, if we consider the viscous fluid, so when you have viscosity, there will be friction at the walls. So, the friction comes into play. So, to take into account the losses due to friction, you have these two terms.

So, we will rearrange these systems, these equations. So, we can bring in $\dot{Q}/\dot{m}$ in here, on this side. And this all come on the other side. So, we can write u2 - u1 - $\dot{Q}/\dot{m}$. So, let us look at what is u2 - u1, is basically that is, where u is, when you call it C into t. So, basically when u is changing or when u is increasing, you will have the mechanical energy change into thermal energy. So, because of frictional losses, the mechanical energy is going to change into thermal energy. And some of this energy may be loss because of the heat transfer.

So, that is the total losses, if you consider, this gives you the total loss due to friction. And what is this? This is the mechanical energy we consider, p/ ρ + α V1 bar^2/ 2 + g z1. So, that is the mechanical energy at location 1, at the entrance. And that is the mechanical energy at location 2, at the exit. So, if we would have said that it is an ideal fluid, it is an inviscid fluid, then this term would have become 0, because the losses are negligible and this would be equal to 0. So, you will say that this is the Bernoulli equation.

Now, this is the Bernoulli equation, but with consideration or with the taking into account, the frictional losses. So, this term is basically what we call, the total losses or so, total loss of mechanical energy per unit mass, because, the unit that we represent in per unit mass energy, per unit mass it will be meter^2 per second square, you can see it easily from here g is meter per second square into meters. So, you have meters^2 per second^2 and this is total losses.

Now, it also is represented in terms of, where the units are in terms of meter, as we have been looking at, or we have been doing with the Bernoulli's equation. So, we can represent it as capital HLT. So, L represents here loss, and capital T represents total. So, small h, we will use when the units are in meters^2 per second^2. And we call it loss in energy per unit mass. And when we represent it as, in terms of capital H, the unit will be meter and we call it head loss.

So, head HLT is equal to, just this energy or the mechanical energy lost per unit mass / acceleration due to gravity, so that will be our other form. So, now, the question comes, that if we have a flow in pipes, we can use this very simple equation to find out the pressure requirements p2 - p we can calculate, or p1 - p2 we can calculate, that how much pressure or how much head needs to be given, when you use a pump, so which pump you need to choose. But, the problem is that, we do not still know, what is head loss? So, now, the question comes that, what are these head losses.

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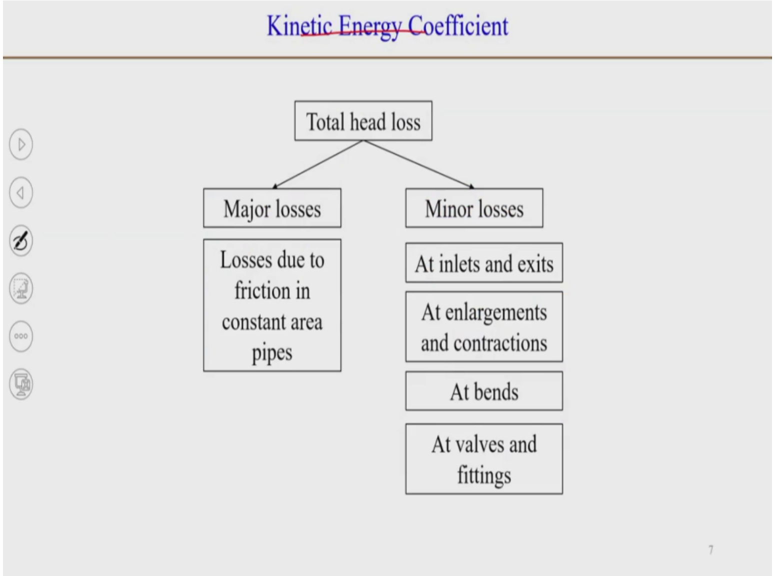

So, we can combine this, this should have been the head losses. So, we can combine this, and say that the total head loss they can be divided into two, major losses and minor losses. It is not necessarily that it is the major losses, which is always greater in the magnitude of major losses is always greater, it is not necessarily. It is the naming convention that we have. And so, major losses is basically the, the losses due to friction when you have a constant area pipes.

So, we will discuss this in detail in the next slide. And then another is, another category of losses is minor losses. So, anything which does not come basically under this category, that losses due to friction in constant area pipes, that will come, that will be covered in minor losses. So, we think about, we talked about developing flow. So, in the developing flow, there will be the pressure losses will not be same as in the fully developed flow.

So, to take into account inlet and exits, it will be in minor losses. Then we have reducers and expanders, when the diameter of the pipe is changing, it may change suddenly or it may change gradually. So, you will have a enlargement in the pipe diameter or a reduction in pipe diameter.

So, the losses at reducers and expanders will again come in minor losses. Then you will have number of bands in your pipelines. So, the losses at the bands, will come in minor losses. And you will have a number of valves and fittings etc. So, those will also come under minor losses.

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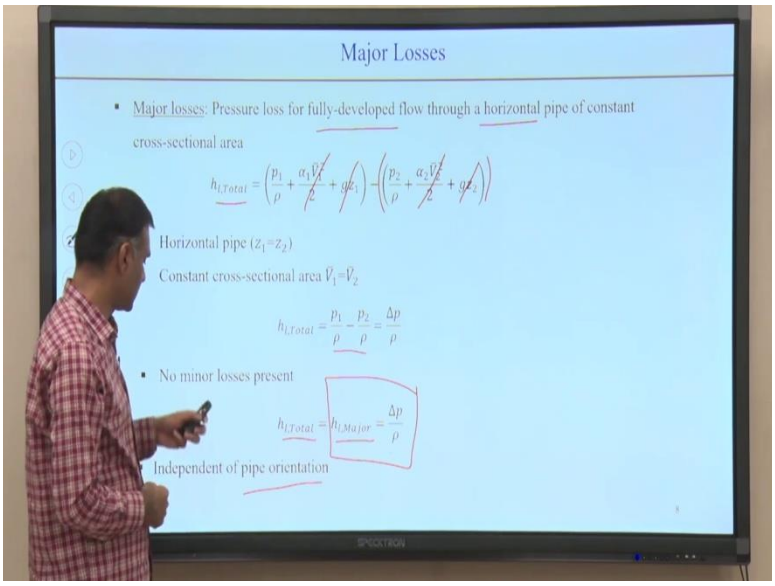

So, let us look at the major losses. So, the major loss can be defined that, if we consider a pipe of uniform cross-sectional area. So, so a straight pipe, the cross-sectional area is constant and we consider that, that this flow or this pipe is horizontal. So, when the pipe is horizontal, with all these assumptions, we can write down the head loss equation. So, HL total =p1/ ρ α 1 $V1^2$/ 2 + g z1 - p2/ ρ + α 2 $V2^2$/ 2 + g z2 in of course, this all in bracket here.

So, when you say that it is a horizontal pipe, then basically z1 =z2 and this will cancel out. When you say, that the pipe is of constant cross-sectional area. So, area everywhere including at section 1 and 2 are equal. So, you will have A1 V1 =A2 V2. And these will also cancel out. So, you will have HL total =p1 - p2/ ρ. So, basically Δ p/ ρ is head loss. So, remember the definition that head loss, even if a pipe network we talk about, is having pipes of different cross-sectional areas, the head loss is Δ p/ ρ.

Because all the other things for example, the change in cross-sectional area will be taken into account, in the minor losses. So, that is my head loss, but when you look at the head loss, so say, head loss in 1 inch diameter pipe and 2 inch diameter pipe needs to be calculated separately. Because this will be dependent upon the diameter of the pipe. So, when we are talking about here, because it does not have, we consider that the flow is fully developed.

So, there is no inlet or exit losses, we are not looking at here any valves, any fittings, any reducers and expanders because their cross-sectional area is constant. So, there are no minor losses present. So, basically our total head loss in such case will be =the major losses Δ p/ ρ.

So, major loss is Δ p/ ρ, which is the pressure drop per unit density. And so, this is the definition of head loss.

And it does not include the gravity term. So, it is independent of pipe orientation. So, when, when you are looking at, even if the pipe is say vertical or inclined, it does not change, the major loss value does not change. Because when you will be writing the energy equation, you will have a different term say, z1 - z2 you have to take into account for that. So, the major losses that, they do not depend on the pipe orientation.

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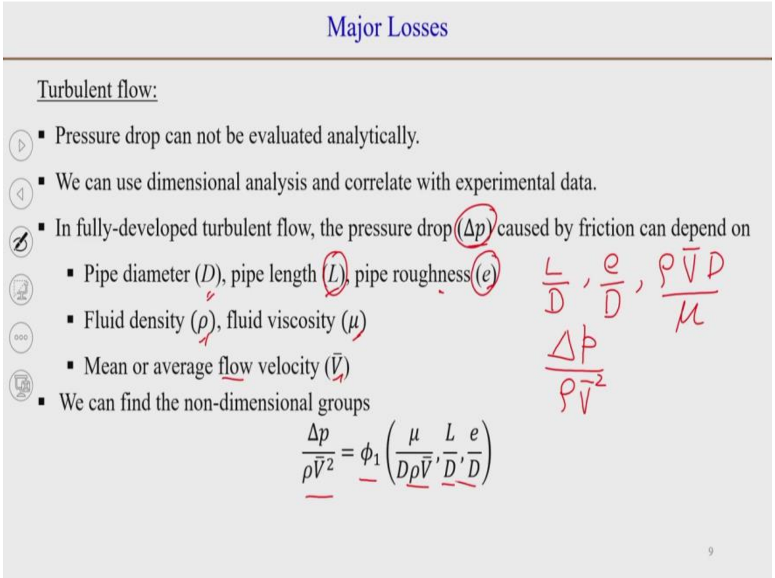

Now, we sort of know now, that if the flow is laminar, pipe is of constant cross-sectional area and the flow is fully developed, then in the laminar flow, we can calculate major losses, we have this Hagen-Poiseuille equation. So, we will be able to do that. But what we do not know is still, how to do it for turbulent flows. So, let us discuss for turbulent flow. Now, because, nothing we have learned until now for turbulent flows, there is no analytical solution that we could derive.

And in the same way, we will not be able to find the pressure drop also analytically like, what we did for laminar flow. So, it is not possible and when we could not do anything, analytically the best way is to look at the, the powerful tool of dimensional analysis. So, we could go back and see that, what are the factors on this pressure drop through a pipe in turbulent flow might depend?

And so, we can list down, that the pressure drop may depend on the pipe geometry. So, pipe geometry when we talk about, it may depend on the diameter of the pipe as you see here, it may depend on the length of the pipe, as well as it may depend on the roughness of the pipe. So, we can write this roughness of the pipe in terms of, small e here. Now, it can also depend or it will depend on the properties of the fluid that is being used.

So, it may depend on the viscosity of the fluid, it will depend on the density of the fluid. So, ρ and μ come into picture. And then it will depend on the flow velocity. So, we will take a mean or average flow velocity, let us say V bar. So, now, we will have this, that Δ p is a function of all the six quantities. And we can group them in dimensionless groups. What we could do is, we can do the Buckingham pi theorem or use Gibson's method or just by looking at the variables we can define, the non-dimensional groups.

So, if we take let us say, as ρ V bar and diameter as our repeating variables, then we can just see that what are the non-dimensional groups, we can form. And we see here that there are two more quantities, which had the dimensions of length. So, we can simply write them as L/D and e/ D. So, two dimensionless groups we have already formed. Now, we have been doing dimensional analysis and we are very familiar dimensionless group, we have as Reynolds number.

So, we can define it as say, ρ V bar D/ μ or it is reverse, depend on how do you do it, if you do Buckingham pi theorem analysis, then you will get is μ into ρ - 1 V bar - 1 d - 1. But what we are looking at is dimensionless group, it is 1/Re or Re does not matter. Now, the last thing is, we have covered all the dimensionless, all the dimensional quantities except Δ p. So, where you, non-dimensionalize Δ p, you will see that you will have a Δ p/ ρ V bar^2.

So, that is the dimensionless groups, because we have been seeing or we have been looking at, we can non-dimensionalize with the dynamic pressure. Pressure with, so pressure or Δ p is non-dimensionalize by ρ V^2. So, we can write down that Δ p/ ρ V^2 is a function of these other three non-dimensional groups or dimensionless groups.

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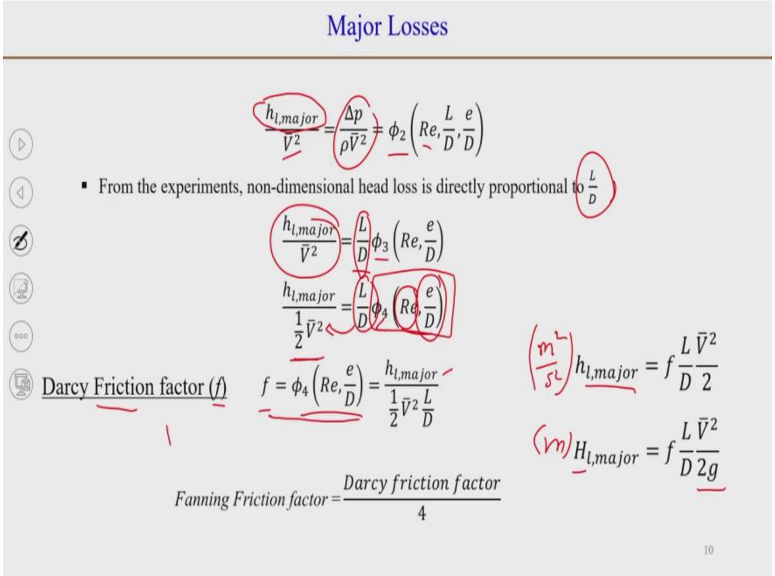

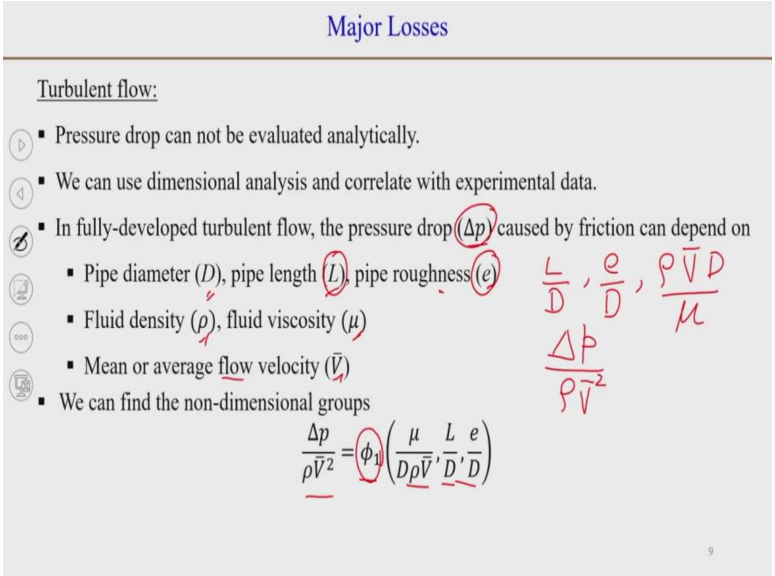

We said that, Δ p/ ρ is h l, major or we, we found that the major losses =Δ p/ ρ. So, this term Δ p/ ρ V^2 will be major losses / V bar^2. Now, the experiments have shown, that this is the head loss or major loss is directly proportional to L/D, because the dimensionless and dimensional analysis can give us the information that, what are the relevant dimensionless or dimensionless groups, but we cannot find out from the dimensional analysis, that what is the function or in what way they are related.

So, now we need to go to the experimental data and try to see what relationship could be there. So, the experimental data has shown this, that L/D is directly proportional to head losses. So, we can write this, the functional dependence in this form. You might see that we have changed this to, in the previous slide it was φ 1. Because it was Δ p/ ρ v^2, then we could write this in terms of, because here we had, 1/Re.

So, we have changed it to say φ 2, because we have changed to Re here. And then again we have changed it to φ 3 because the function will change. But, this is just to differentiate the functions, we still have that it is a function of Re/ D. So, we will write this h l, major / 1/2 ρ V^2, because 1/2 ρ V^2 is, is what we have called kinetic energy. And so, kinetic energy per unit mass, we can write as 1/2 of V^2.

So, h l, major/ 1/2, 1/2 V^2 =L/D φ 4 into Re e/ D. Again, because we have / 1/ 2, so a different function. Now, this functional dependence that, this function depends on Reynolds number and the roughness factor e. So, this is group under a factor, which we call friction factor or Darcy friction factor. So, Darcy friction factor we define as, this = φ4 e/ D and that is h l, major / 1/2 ρ V^2/ L/D.

So, we can bring in L/D in the denominator here. So, that is the definition of Darcy friction factor. And Darcy friction factor is a function of Reynolds number and L/D. So, we can write down or we can represent the major losses in terms of Darcy friction factor, taking into account that Darcy friction factor is the ratio of h l, major / or / 1/2 V/^{2} L/D. So, if we write this in terms of h l, major, where it is in $meter^2$ per $second^2$, then our definition will be this.

If we write into capital H where the unit is in meter, then we will need to divide by g. So, V y, V bar^2 / 2 g. So, that is our definition of Darcy friction factor. Now, there is another friction factor, that is defined which is called Fanning friction factor. And it is just one fourth of Darcy friction factor. The, the commonly used friction factor we have is Darcy friction factor, but every time we need to be careful that, the friction factor that has been given is Darcy friction factor or Fanning friction factor.

So, now, we have defined, what we have done is, we looked at that on what factors can Δ p depend upon. And then we grouped all those in dimensionless groups. We used a bit of information from experimental analysis and said that, these major losses is dependent, is proportional to L/D. And then remaining function we have defined as, Darcy friction factor, which is a function of Re and L/D.

So, what now we need is, what is f as a function of Re and L/D. So, if we know Re and L/D, if we know the pipe length, pipe diameter, fluid, so which will give us the density and viscosity and the velocity. So, we will, if we know the Reynolds number, L/D then can we calculate f?

That is the question, we will ask ourselves. Of course, we could do it for laminar flow, and we can rearrange the equation that is obtained for pressure drop in laminar flow.

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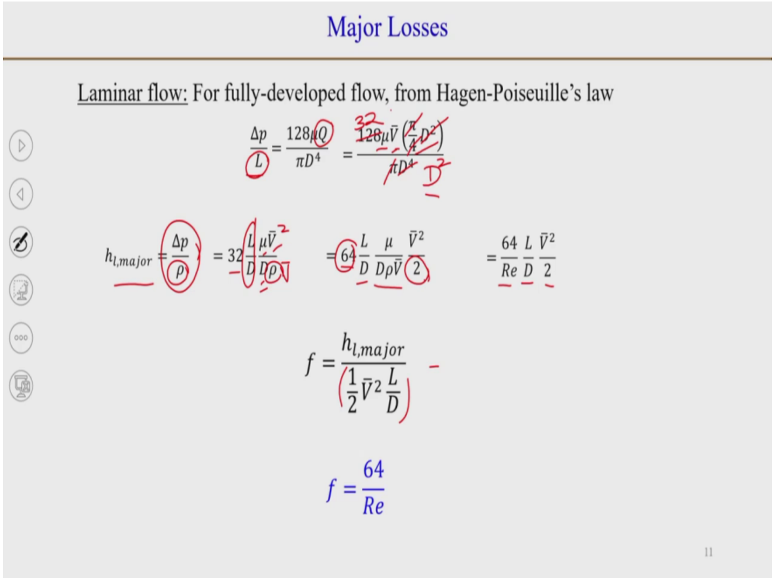

So, we know for laminar fully developed flow, that the Hagen-Poiseuille law gives us that the pressure drop per unit length =128 μ Q into π D^2/ 4, where Q is volumetric flow rate. So, we can expand it or we can write Q in terms of mean velocity V bar into cross-sectional area π/ 4 D^2. So, we will see there, that π and π will cancel out, and D^2 will cancel out. So, we will have D^2 remaining here. This will give us 32.

So, we can write because, the major losses Δ p/ ρ, so, we can find major losses for laminar flow, that will be 32 and this L will go on the other side, on the right hand side. So, 32 and you have 2 D here. So, we will write 32 into L/D μ into V bar /, one more D is remaining, so D and V / Δ p / ρ. So, this ρ comes in here. Now, we can rearrange it further, because we have μ V bar and D here and row as well.

So, we can try to arrange it in terms of a Reynolds number. So, what we have is a V bar missing. So, we can multiply by V bar here, and in the denominator and numerator. So, what we get is 64 because we have / 2 here. So, we will get this in terms of friction factor. So, there we have in the definition V^2/ 2. So, when we divide by 2, we will have this at 64 into L/D μ/ D ρ V bar into V^2/ 2.

So, we can write this as 64/ Re into L/D into V^2/ 2. And then now, we bring in the definition of Darcy friction factor, which is h l, major / V^2/ 2 into L/D. So, we substitute this L/D V^2/ 2 and the friction factor we will get is, that f =64/ Re. So, for a laminar flow f =64/ Re.

When we talked about or when we derive this relationship, we did not consider roughness factor or we did not take a reference factor into consideration there. But the experiment show that for laminar flow roughness does not come into picture and, and f is 64/ Re even for rough pipes. So, roughness does not have an effect in the laminar flow in pipes.

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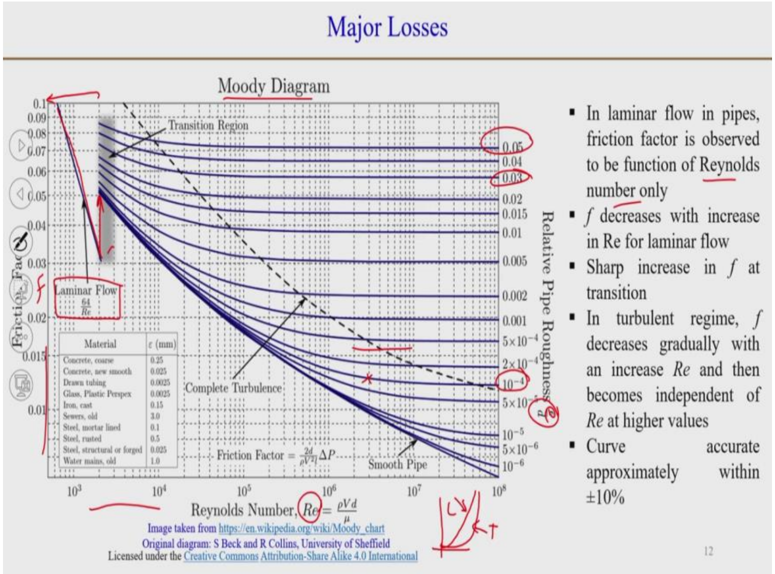

Now, the next question comes that, what if the flow is turbulent. So, there is no functional dependence that one could obtain except by the experimental data. So, a very standard chart has been prepared, which is called moody's diagram. And what it represent is, the functional dependence of friction factor f, as a function of Reynolds number and as a function of roughness factor. So, you can see this roughness factor is e/ D here.

Now, when the flow is laminar in this range, then you see a straight line because the graph is log log plot plot as you can see from the axis. So, the x axis is log log and the y axis is also a log log plot here. So, you see here that, this is a straight line and for laminar flow you have 64/ Re, it is independent of roughness factor. Suddenly as the Reynolds number increases, so you see a sudden jump in the friction factor.

Now, with the increase in Reynolds number, in the turbulent regime, what you see is there are different lines and you have the relative pipe roughness or e/ D value is given here. So, depending on the roughness factor, you will need to follow different curves. You can see that, at higher values of roughness, the friction factor becomes independent of Reynolds number very early. Even at low values of roughness, say of roughness of 10^{-4}, eventually at the Reynolds number higher than 10^3 10^6 here.

It becomes independent of Reynolds number. And so, so, the question comes, why do we have the friction factor having a jump here? And the question or the answer lies in the velocity profile. So, if we define the velocity profile on a laminar flow and if turbulent flow. So, this is for laminar flow and this is for turbulent flow. So, you have a very thin viscous sub layer, and the velocity profile is fuller. So, the gradient is larger as well, so in the turbulent flows, so the wall shear stress is going to be high, and that is why you have high major losses and as a result friction factors are larger in the turbulent flows.

Now, the next question, that why we have roughness effect in turbulent flows, but not in laminar flows? So, if we look at, in the turbulent flows, we discussed that in the turbulent flow near the wall, we have a viscous sub layer. And these viscous sub layers will be very, very thin, higher the Reynolds number, thinner the viscous sub layer is going to be.

So, if that is the case, then when you have the rough pipes, then the roughness factor or this might be the viscous sub layer can be within the, of the order of roughness. So, this could cause lot of disruptions. So, that is why the roughness has an effect in the turbulent flows. Now, if we summarize the Moody's plot, so we can say that laminar flow in pipes, it is a function of Reynolds number only and not on the roughness factor.

It decreases with increase in Re for laminar flow, 64/ Re we have seen. And then there is a sharp increase here in transition. And in turbulent flows depending on the roughness is decreases gradually, and then it becomes constant at the higher Reynolds number values. And these curves has been prepared with a lot of experimental data, appears to be quite accurate within ± 10 percent. So, that this curve could be used to find or to calculate, but if you want to code it, if you want to say, write a code for major losses in the pipeline, then it will be easier to use some algebraic relationship. So, few algebraic relationship, we already have for laminar flow.

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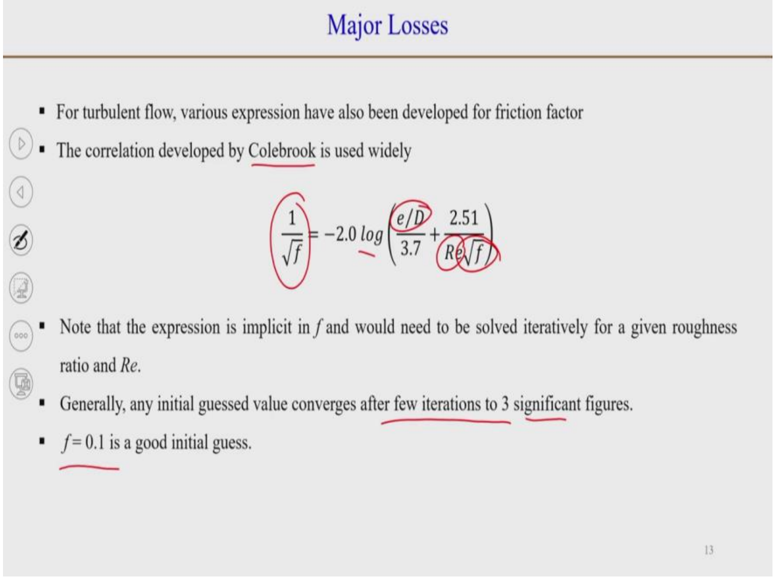

For turbulent flow few algebraic relationships have been developed, which represent moody chart or friction factor. So, one that is developed by Colebrook is, used quite generally. So, that is 1/√friction factor =- 2 log it is, log base e. So, this is e/ D / 3.7 + 2.51 / Re √f. Now, if we know Re, if we know the flow Reynolds number, if we know the roughness of the pipe, then we can see that this equation is explicit.

So, by explicit we mean, that you have f in the left hand side and you also have f on the right hand side. And you cannot separate those that, you can bring both of them on one side because you have a log here. So, what we will need to do? We need to solve it, iteratively. We will need to assume a value initially, that let us say, f =0.1. And then find what is the new value of f and do this again and again until we reach the value of f, that the value we guessed that, that start of calculation and the value that we find, after the calculation they are not very different.

So, it has been shown or if you do the calculations using different values, that even in any case you reach to a fairly accurate value, which is up to three significant figures, up to three places of decimal, within few iterations you get this value accurately. And as a guide, you could use f =0.1, as an initial guess which can help you in converging faster.

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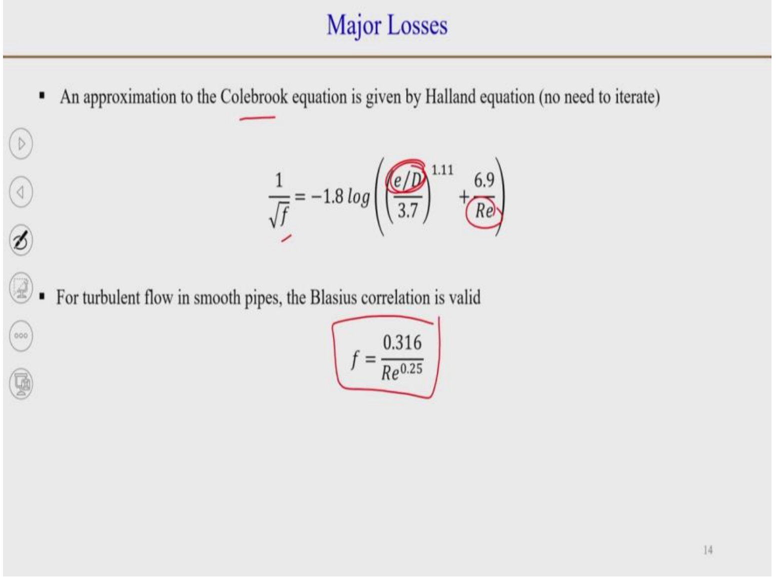

So, the Colebrook relationship, it is slightly difficult in terms of that we need to solve it iteratively. And so Halland equation is another one. And in this it is, it is an approximation of Colebrook equation, but there is no implicit behavior like we do not, we do not need to solve it iteratively. So, we could calculate, if we know the value of Reynolds number and if we know the value of e/ D, then we can calculate it directly using this equation.

All these take into account the, e/ D roughness factor, but if the pipe is smooth, then using the relationships from the Blasius correlation, which is f =0.316 / Reynolds to the power 0.25. So, this is a simpler co-relationship and it can be used for smooth pipes.

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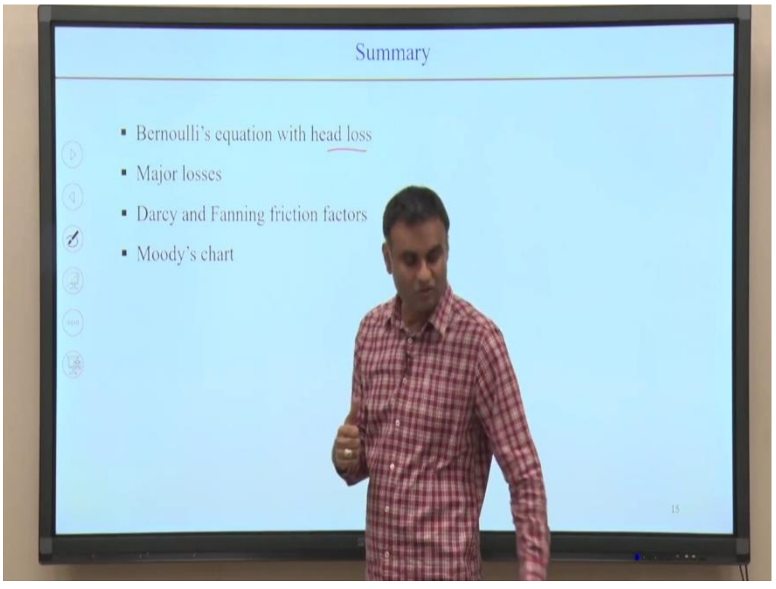

So, what we have discussed in summary today is that, we have looked at the developing flow first, that what is developing flow and what are the development lengths for laminar as well as turbulent pipes for laminar flows, we said that L/D =0.6 Re. And for turbulent flows, the flow tends to develop the mean velocity profiles tends to develop between 25 to 40 D.

And the other turbulent quantities may take up to 80 diameter or so. Then, we looked at the head losses, that how we can calculate. So, using the energy equation and Reynolds transport theorem, we derived the Bernoulli's equation in the form, where we also have the frictional losses or the head losses. And then we define that, what could be the different types of losses, major losses and minor losses.

And we looked at then, what is major loss, so we should, said that major loss is basically the loss for fully developed flow in a pipe of constant cross-sectional area and horizontal pipe. So, that way we define that h l, major, or the major loss which $=\Delta p/ \rho$, and from that we defined Darcy friction factor or Fanning friction factor. And the values of friction factor, f =64/ Re for laminar flow. And, and the relationships or the moody chart for f in turbulent flow. So, we will stop here, and then in the next class we will discuss the minor losses etc. Thank you.