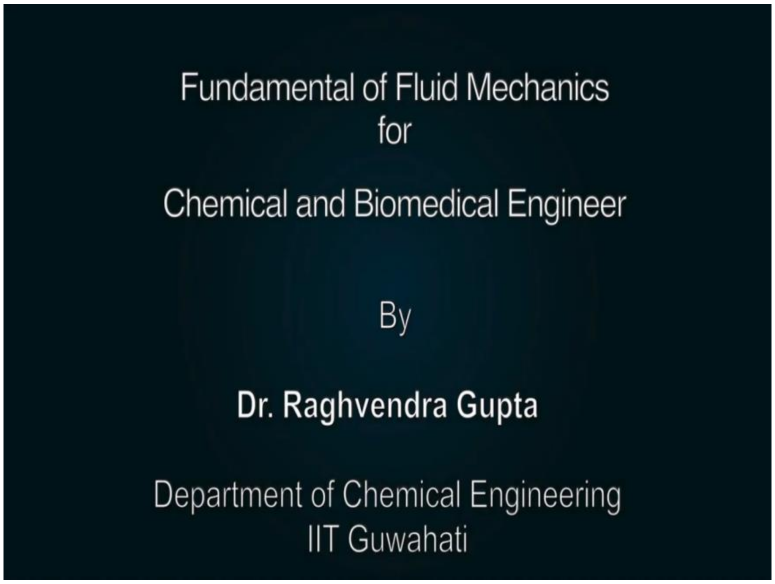
Fundamental of Fluid Mechanics for Chemical and Biomedical Engineer
Professor Raghvendra Gupta
Department of Chemical Engineering
Indian Institute of Technology Guwahati
Lecture 37
Introduction to Turbulence

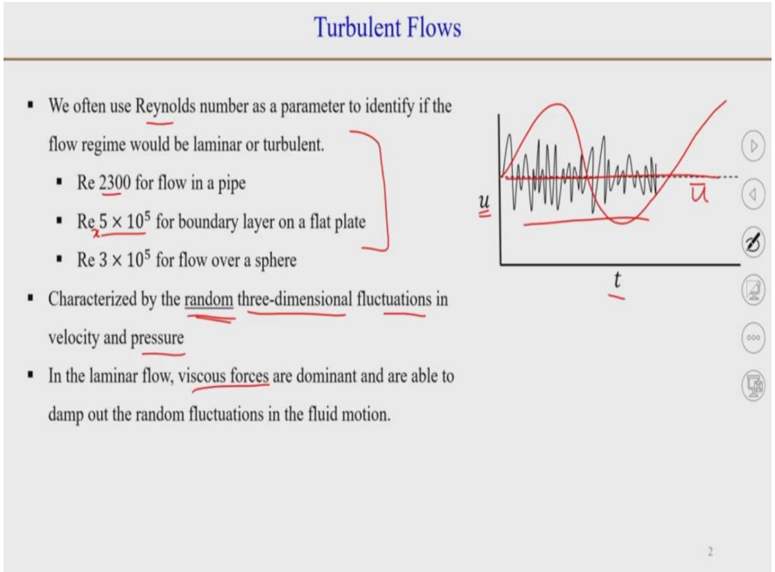
(Refer Slide Time: 00:24)



Hello in this lecture we will be talking about turbulent flows. So, in the previous weeks, we have done lot of analysis and in most of the cases we were looking at laminar flows and we briefly discussed for example when the flow is going to be turbulent. The turbulent flow is very statistical that means it is the flow is very random. So, all the equations that we solve, it might not be possible to obtain such clean solutions for turbulent flows or at least there is no such solution available for turbulent flows.

So, in this module, we will be looking at briefly what is turbulence, what are the important features of turbulence and some important topics for example of what is the velocity profile for turbulent flow in a pipe and then how we can obtain the quantities of engineering interest in a turbulent flow.

(Refer Slide Time: 01:54)



So, when we talked about turbulent flows, we said that if the Reynolds number is greater than a certain value then the flow is observed to be a turbulent flow in So, for example when we flow in a pipe, we somewhere around Reynolds number 2000, 2100 and 2300 the flow becomes turbulent. For a boundary layer over the flat plate where the pressure gradient in the outer flow was 0, we said that somewhere around 5 into 10^5 or 3 into 10^5 of that order Reynolds number where Reynolds number is Rex, the flow is observed to be turbulent.

And similarly, for flow over a sphere at a Reynolds number is of Re 10, 3 into 10^5 flow is observed to be turbulent but with all these limits we always said that if the flow is smooth, if the free if the flow is free from disturbances, So, that means that turbulence depends on the disturbances that are there in the flow.

Now when we talk about any flow which is happening practically, we will see that there are certain disturbances in the flow. For example, if there is flow in pipe there will be disturbances because of the pumping system oscillations in the pumping mechanism, some vibrations in the pipe environment from the exit boundary condition or there can be number of factors that can give rise to fluctuations.

And these fluctuations, if they grow then the flow will eventually become unstable and the unstable flow will give rise to secondary motions and these secondary motions will further give rise to more unstable flows. So, all that can cause or all that causes the flow to be turbulent. Now when the

Reynolds number is low, these disturbances are dissipated or they are damped by the viscous effects.

So, until the viscous effects is dominating, then the flow remains laminar and when these viscous effect is not able to damp out the disturbances in the flow, then the flow starts becoming turbulent. So, if we that what causes or how do we define the flow to be turbulent? We often use the word eddies and with eddies what we or what we relate is recirculation.

So, whenever we when we see a recirculating flow, we that the flow or some of us try to think which is not correct that whenever there is a recirculation in the flow, flow is turbulent or other thing is that when the flow is unsteady, then some people might or there might be a wrong notion that if the flow is unsteady, then the flow is turbulent.

So, the turbulent flow is not characterized by the recirculations or the unsteadiness but what defines turbulence is the turbulence itself. So, if we look at the literal meaning of word turbulence which is basically the randomness the chaos, when we in English that turbulent times, that means that there is nothing in order. There is a lot of chaos, So, there is a lot of randomness.

So, similarly if we talk about a flow and if we take a probe and measure let us , u component or the x component of velocity with time and if we get such a behavior where we cannot anything about the velocity with respect to time, if you have a velocity something like this which appears to be sinusoidal or you can decompose into some sinusoidal components in a definite manner of course any signal you can decompose using Fourier series into sinusoidal components but what I am talking about that flow is periodic, then it is not turbulent.

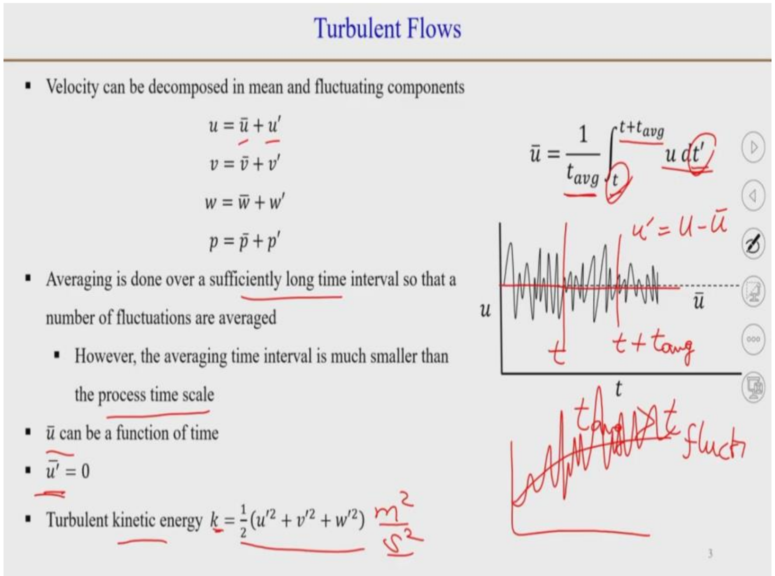
So, turbulent is basically defined by the randomness and when there is randomness, there will be fluctuations. So, your flow is always fluctuating, it might be fluctuating about a mean. So, that will be your mean velocity or mean flow velocity. So, that is the main characteristic, the flow will be when we have these fluctuations So, these fluctuations will give rise to a three-dimensional motion, So, the turbulence is always three dimensional of course when there are fluctuations. Fluctuations means the velocity is changing with time. When velocity changes with time you will have a u component as well as v component w component. So, you might have one mean flow.

So, for example, if the flow is happening in a pipe it will be the axial direction in which the flow will be dominant. So, you will have dominant u component or x component of velocity but you will also have fluctuations which will be which will give you the velocity magnitude or some certain magnitude for v and w component that is the r and theta component of velocity in a pipe. So, turbulent is three dimensional, there are fluctuations So, it is of course unsteady and then the randomness which defines the turbulent behavior.

When the velocity is changing, one velocity component is changing with time and you will have other velocity components also changing as well as the pressure will also fluctuate with time. So, what I have plotted here or I have shown here is for u component of velocity but if you take or if you measure v component of velocity or the w component of velocity or if you measure pressure with time, all will give you similar signals.

Now as I said that the turbulence happens because the fluctuations that are there in the flow, they grow they are not dissipated by the viscous forces or they are not damped by the viscous forces or So, when that happens then the flow starts becoming turbulent. So, if you are able to make your system free of fluctuations, then you might observe that all these numbers might be quite bigger. So, for example, people have observed that for a fairly large Reynolds number of the order of 10^5 or 10^5, the flow in a pipe can remain turbulent subject to the condition that you can achieve the flow that is free from all the disturbances.

(Refer Slide Time: 09:44)



So, we talked about that the one of the important properties of turbulence is randomness. Now when you have these fluctuating components, then you can always define a mean flow and then you can define the fluctuations in the flow which is u - u mean and similar things you could do for other components. So, for the analysis because when we are interested for engineering in in the engineering flows, then most of the time our interest is or focus is more into the mean quantities of the flow or the mean velocities, mean pressure not So, much on the fluctuations specifically in the engineering.

So, you can define the mean over a certain time period that t average of 1/t average integral from t to t + t average u dt′. So, t′ is just to differentiate this variable t from the variable in the limit. So, that is why we have the t′ here. So, basically that you can take any time because it is the mean velocity, So, the starting time you could take any time t and t + t average. So, you will get or you will be able to decompose the velocity in mean and fluctuating components same thing you could do for all the velocity components and pressure.

Now the time interval is such that the averaging is done for a sufficiently long time. So, you are averaging the fluctuations over averaging a number of fluctuations. So, the averaging time is sufficiently large when you compare with the time of fluctuations. However, this time is sufficiently or it is smaller than the flow time or the characteristic time of the flow. So, or if you are talking about chemical engineering then the process time scale.

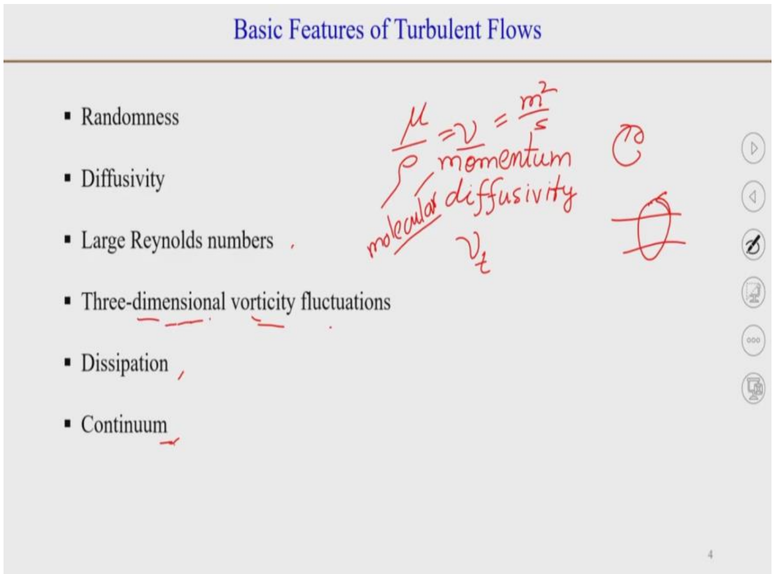
So, the average time is an intermediate time scale which is larger than the fluctuation time scale or sufficiently larger than the fluctuation time scale. So, it actually do represent the mean flow of your system and it should be sufficiently smaller than the flow time. So, if there is unsteadiness in your flow, So, for example if you plot a flow where your fluctuations are something like this, So, your mean might go somewhere like this.

So, that means that your mean flow can also be a function of time, however, if you take mean of fluctuating component then it will be 0. So, that this decomposition that decomposition of the velocity and pressure in mean and fluctuating components this is called Reynolds decomposition after Reynolds. Now when we decompose this and we will see that we can write down the conservation equations for the mean flow.

And how do we characterize the fluctuations? So, that can be done using this quantity which is 1/2 of $u'^2 + v'^2 + w'^2$ and it is known as turbulent kinetic energy. So, know that or just observe that k it is called turbulent kinetic energy but there is no ρ here, So, the unit here will be m^2 per $second^2$.

So, in analogy with the kinetic energy, it has 1/2 $u'^2 + v'^2 + w'^2$ but there is no ρ and that is that defines the intensity of turbulence or turbulent kinetic energy.

(Refer Slide Time: 14:28)



Now, So, let us briefly talk about what are the features of turbulence. So, one feature we just discussed that the randomness is what define a turbulence. So, the flow is regular you cannot find or until now people have not been able to find an order in turbulence. Then diffusivity, So, as a chemical engineer turbulent flows are good news for me because I do not need any extra means of mixing.

So, diffusivity means because the turbulence is associated with lot of eddies, So, you have eddies of all the dimensions and that those eddies are basically the recirculations in the flow. So, when you have recirculations in the flow there is lot of cross stream mixing, So, you have a mixing between the laminas, unlike laminar flow you could have a cross stream mixing in turbulent flow. So, that gives rise to that momentum is diffused, heat is diffused and the mass is diffused.

So, if you want to do a reaction where you want your reactant to mix or reactant and product to mix, if the flow is turbulent then because of turbulence itself the flow will be mixed. Now diffusivity when we talk about for in the laminar flows, it is molecular diffusivity and for momentum we that ν is momentum diffusivity.

The unit if we find m^2 per second, So, if you do not remember what is ν, it is kinematic viscosity and μ/ρ. So, that basically defines that how fast the momentum can be diffused in a laminar flow.

In a similar manner there is another quantity that is defined turbulent diffusivity but that is not something like molecular diffusivity. You can derive the relations for molecular diffusivity from kinetic theory and all but that is not So, for turbulent fluid's property. Turbulent diffusivity is in analogy with the molecular diffusivity. So, we can that this is molecular diffusivity.

So, the turbulent diffusivity is the property of the flow, unlike molecular diffusivity which is the property of the fluid at a particular temperature. Then we already know that it is associated with large Reynolds numbers, So, large Reynolds numbers means the inertial effects are dominating and viscous effects are not So, large when you compare with the inertial effects. So, and that is because when the viscous effects are relatively smaller or relatively lower in magnitude, then they will not be able to damp out the fluctuations in the flow. So, the turbulent flow is always at larger Reynolds numbers.

Now how large and what does this large mean is depends on case to case basis. There are different values for different kind of flows and also depends on how do you define your Reynolds number. Now the other thing is that turbulent flow has three-dimensional vorticity fluctuations. So, when we talk about eddies, there are recirculations the flow edge circulations, So, there is vorticity present in the flow and these fluctuations are three dimensional.

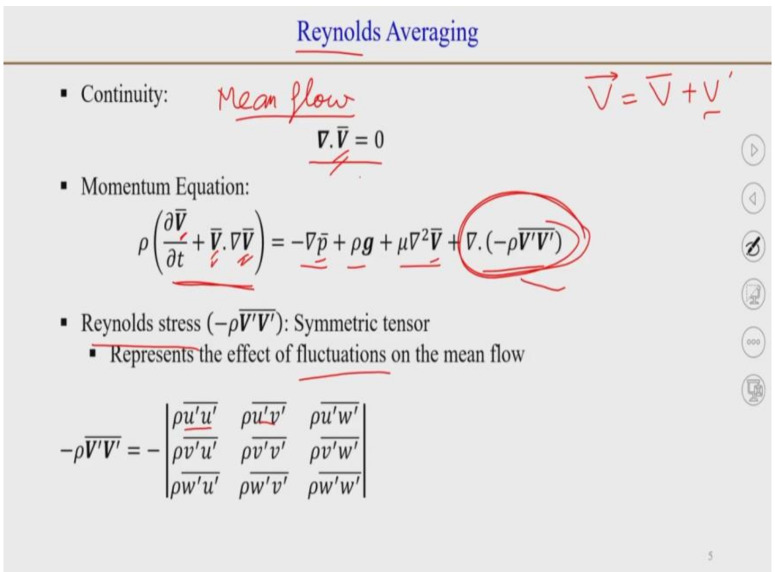
So, it may be possible that in some simple cases you may have the primary flow may have just two-dimensional vorticity and this two dimensional vorticity when the flow becomes unstable it may give rise to secondary flow and the secondary flow will be through three dimensional and the phenomena like vortex stretching etc. happens which make the flow three dimensional.

Then another feature of turbulent flow is dissipation. In the turbulent flows because you have fluctuations and you need energy for this, So, turbulent flow needs a constant source of energy and this energy you have a number of scales present in the flow and the largest scale will be the scale of your system in which you are looking at the flow and the smallest scale what we call Kolmogorov scales.

So, and the energy transfer from the larger scale to smaller scale and at the smallest possible scale which we will discuss in few minutes that the energy is dissipated and this dissipation happens because of the viscous effects and this the energy of the flow is dissipated in terms of heat. So, dissipation is another feature of turbulent flows and the last feature the continuum.

So, that means that the scales it compasses a number of scales but scales the smallest scales are not So, small that you cannot assume that it is not continuum, it is not down to the molecular level. So, the smallest possible scales as we will discuss that they are where the dissipation or viscous dissipation balances the energy or the energy of the flow is dissipated. So, the inertia at the smallest scale is balanced by the viscous effects. So, that is the smallest possible scale, So, you can still consider or that the flow is continuum.

(Refer Slide Time: 20:56)



Now, So, as we said that when we decompose the flow in the mean and fluctuating components, then we can substitute the velocity in terms of So, we can write a velocity vector V = V mean + V´. So, which all of them are vectors, So, if we substitute these in the governing equations of a flow, when we talk about governing equation for a incompressible isothermal flow we talk about continuity and mass conservation equations. So, when we substitute it and then we can average those equations and though that averaging it what is called a Reynolds averaging after Reynolds, So, he obtained the average equations for mean flow. So, after the averaging is done, you obtain such equations.

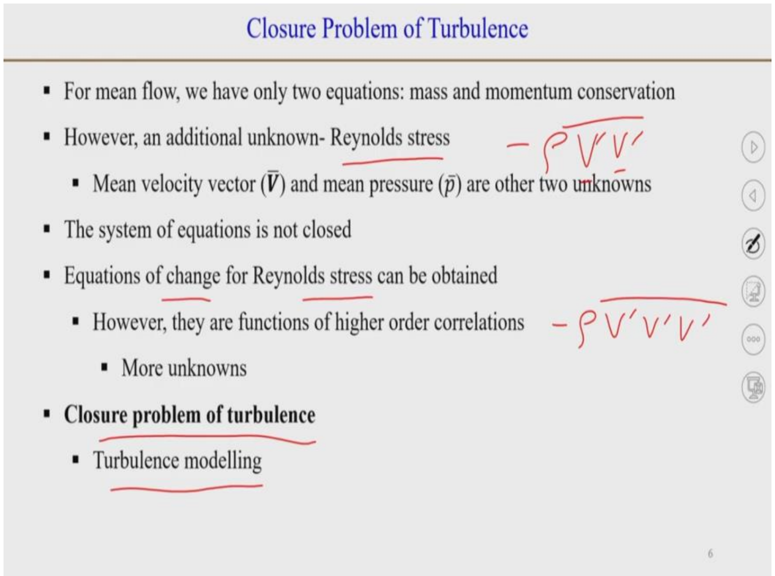
Now first thing you might notice here that we have replaced the velocity V with $\bar{V}$. So, these are all in the vector form, So, the equations are for mean flow. Now because we are interested in the mean velocity and mean pressure components, the continuity equation for an incompressible flow looks similar to what we had for velocity V. So, it is $\nabla.\bar{V}$.

Now if you look at the momentum conservation equation, there is one more term or one extra term that has appeared here, you have the acceleration term on the left hand side and then you have a pressure gradient again where you have a mean pressure and the gravity term and the viscous term but an additional term has appeared here which is - ρ V′ V′ and V′ you can see here that V′ is the fluctuations.

So, if you look at this, it is like ρ into V into V that means it is kind of momentum flux and it will have units of Pascal which is units of stress also. So, this is known as turbulent stress or there is a specific name for it after Reynolds, it is called Reynolds stress and it is a symmetric tensor you can expand it in the components ρ u′ u′ ρ u′ v′ and So, on. So, it basically if you look at these two equations, in these two equations everywhere we have only mean quantities $\bar{V}$ and $\bar{P}$.

The only place where we have the fluctuations coming into in this term which is the Reynolds stress. So, it basically represents that what is the effects of these fluctuations in the mean flow. So, because this terms looks like a stress, So, it is termed as a mean or it is termed as Reynolds stress or the turbulent stress and it causes an additional stress in the mean flow. That is your equations for the mean flow after be done after doing Reynolds averaging.

(Refer Slide Time: 24:40)



Now, if you want to solve this system of equations, we had two equations. If you write the velocity in the vector form So, you have continuity equation for the mean flow and the momentum conservation equation for the mean flow and you have two unknowns, $\bar{u}$ or $\bar{v}$ which is a velocity vector and $\bar{p}$ the pressure. So, $\bar{p}$ is mean pressure and $\bar{v}$ is mean velocity but you also have an additional term that comes in - ρ V′ V′ averaged. So, the bar over the quantity represents the average as we saw earlier. Now when this comes into picture, then we do not know the fluctuations.

So, if we do not know the fluctuating velocity field, how to find this that is the question because this is an additional unknown and we have only two systems of equations for mean flow. Now this system of equations is not closed, So, like we could or people have written the equation of change for this Reynolds stress also but they are further functions of ρ V′ V′ V′. So, the components like - ρ u′ v′ w′ and ρ the average of u′ u′ w′ and So, on. So, you will have more unknowns coming into picture and you have I mean to eliminate one unknown you have brought into another unknown and this will keep going on.

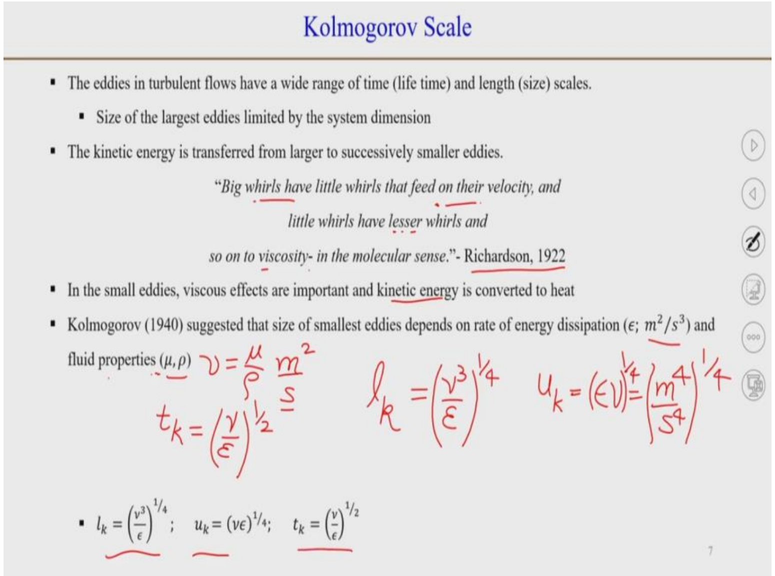
So, finally we have to cut down somewhere and find a closure relationship. Why we are talking about or why do we you closure relationship? Because we had two equations, continuity for mean flow and the momentum conservation for the mean flow but we had three unknowns the mean velocity, mean pressure and the Reynolds stress.

So, there is an additional variable and we need one more equation to close the system of equations So, that the system of equations can be solved and there does not appear to be any fundamental relationship which by which we can represent, for example, the same problem came when we talk about laminar flows or when we derive Navier-Stokes equation and then the constitutive equation for a Newtonian fluid or for a new Non-Newtonian fluid came into picture where we related the shear stress or the stress with the velocity gradient.

So, stress was represented in terms of velocity gradient for the Newtonian fluid we wrote tau = - μ du/dy or its expanded form for three-dimensional flow. Now there is no such thing here fundamentally. So, this system of equations is not closed. As a result, there has been lot of effort to close the system of equations and this problem is known as closure problem of turbulence and the models that have been developed to close this system of equations, all that comes under turbulence modeling.

So, we will not go into detail of turbulence modeling there are number of books and material available on in literature where you could go and look at what are the different turbulence models for example k ε model, k omega model which are present to solve such kind of or to find out the mean flow.

(Refer Slide Time: 28:53)



So, the other thing that we will talk about today is the Kolmogorov Scales. So, as I said that the energy transfers in a cascaded manner in the turbulent flows. So, the turbulent flows have a number of time and length scales and that is why when you are solving for turbulent flows, if you are using CFD then it will not be possible to capture all the scales that are present in the flow until you have very small grid size and very small time steps.

So, when because you solve it in a discrete manner, So, grid size is basically when you divide your control volume or your region of interest in number of smaller volumes and then solve the system of equations there and then solve those algebraic equations and if it the flow is unsteady, then you solve in a unsteady manner.

So, the smallest volume that you need to resolve all the scales in the flow, all the gradients that are present in the flow + the smallest time scale by which you can smallest time step by which you can capture all the possible time scales in the flow is very large. So, that is why we use in most of the engineering applications that mean flow equations and the turbulence modeling is used to close the system of equations.

Now the question comes that can we identify or can we find that what are the scales present or at least can we find out what is the largest possible scale and what is the smallest possible scale So, we can know that what is the range of scale present in a particular flow. So, the size of largest eddy or largest scale present in the flow, the eddies which are of different sizes, the largest eddy can be of the size of your system itself. So, that is determined by the system size.

Now from these largest eddies, you will have the energy being transferred which is called energy cascading, energy is being transferred from the largest eddies to the smaller eddies. Now So, when that happens, there is this quote from Richardson about a century ago which s that, “Big whirls have little whirls that feed onto their velocity, and little whirls have lesser whirls and So, on to viscosity.”

So, that suggests that the energy goes from largest vortices or from the largest eddies to the smaller eddies and So, on that goes on until the energy is dissipated. So, when it goes to smallest possible scale and at the smaller scale viscosity is dominant enough that it can dissipate the energy that is there in the flow. So, in the smaller eddies these viscous effects become important and this kinetic energy is converted to heat.

And based on dimensional arguments Kolmogorov suggested that the rate of energy dissipation, we can relate that what is the smallest size of eddies with the properties of the fluid which are for a isothermal flow μ and ρ which is viscosity and density. So, he suggested that the size of the smallest eddies, it depends on the rate of energy dissipation which is represented by ε and energy we just talked about kinetic energy, h m^2 per $second^2$.

The fluctuations 1/2 $u'^2 + v'^2 + w'^2$ and the rate of this energy dissipation will be m^2 per $second^2$/second, So, this is m^2 per second ^{3}. And it depends on the fluid properties, So, two fluid properties we can combine them together in the form of ν which is μ/ρ. So, the unit of this will be m^2 per $second^2$.

So, using dimensional arguments, he said that we can construct or we can find the size of the smallest eddies using these two quantities, using k and ε. So, now basically what he did that using ε and ν he constructed the length scale of the eddies, time scale of the eddies and from that you can also find the velocity scale of the eddies. So, if you look at the units, ν is m^2 per $second^2$ and this is m^2 per second ^{3}.

So, we can try to construct a length scale, let us , lk now if we look at this and we want a unit which is in the units of m, here we have m^2 per $second^2$, m^2 per second ^{3}. So, the unit of ν is m^2 per second. Now what we could do? We could eliminate second and then we will be able to find an expression in terms of only m^n and then we can do 1/n and find the length scale.

So, if we look at these two, how we can cancel to find a length scale because this is second and here we have second ^{3} So, what we could do is we could write ν/ε. So, ν is m^2 per second but we want second to be cancelled, So, we can ν ^{3}. So, the units will become m^4. So, if this is m^4, then we can that $m^{1/4}$. So, lk is basically ν ^{3} upon $\varepsilon^{1/4}$.

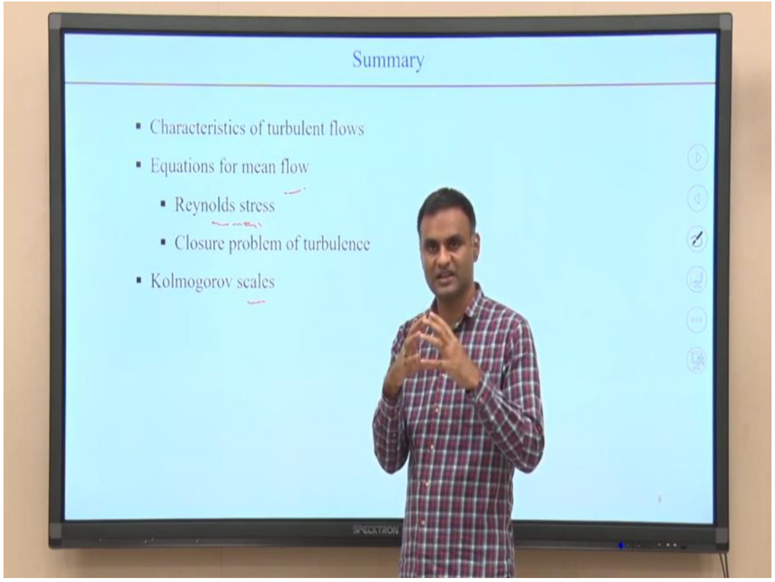
Now we can similarly try to find a time scale, So, to find the time scale what we will need to do is we can eliminate m^2 and then see that what do we get in terms of seconds and then change the exponent. So, we need second, So, what we could do is we can write down ν/ε. So, the unit of ν is m^2/second and ε is m^2/second ^{3}, So, when you do that m^2 will cancel out and what you will end up it with is sec^2.

So, the time scale will be this raise to the power 1/2, So, my time scale will be tk = $\nu/\varepsilon^{1/2}$. Now we can also construct a velocity scale, So, for constructing a velocity scale let us we call it uk and we need that the power of m and s are same and opposite, So, m^n and second is to the power - n and that can be done if you look at a m^2 per second ^{3} and m^2 per second.

So, if you multiply them, So, ε into ν the unit will be m^2/second into m^2/sec^3. So, m^2 So, that will become m^4 sec^4. So, we can simply change this to m^4, sec ^{4} and we need m per second So, this raised to the power 1/4. So, this is our velocity scale.

So, these are the smallest possible scales, the length, velocity and time scales in a turbulent flow and you can see that they depends on ε that is rate of dissipation of turbulent energy and the fluid property is ν. So, ν will be a constant for a fluid at a particular temperature and the ε will depend on your flow, So, that gives you an indication or that tells you what is the smallest possible scale in the flow.

(Refer Slide Time: 39:25)



So, if we summarize what we have discussed in this lecture that we looked at that what is turbulent flow and it is correct what characterizes the turbulent flow. How we can identify if the flow is turbulent and the main characteristic we discussed that it is randomness behavior or the randomness that characterizes turbulent. Then we looked at that we can decompose the velocity and pressure in terms of mean and fluctuating quantities.

And then we can write down or we can use averaging So, that we can find the continuity and momentum equations for mean flow and these equations for mean flow has an additional term, the momentum equation had an additional term which is like a momentum flux or stress So, which is called Reynolds stress and that becomes that defining this Reynolds stress in terms of mean quantities that is known as closure problem of turbulence.

And then we talked about that what are the range of scales in a flow and the largest possible scale is the size of the system and the smallest possible scale we saw that the Kolmogorov scales which are the smallest possible scale where the energy is dissipated energy of the smallest scale can be dissipated by the viscous effects or by the viscosity. So, we will stop here, thank you.