Fundamentals of Fluid Mechanics for Chemical and Biomedical Engineers
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Flow Separation and Drag

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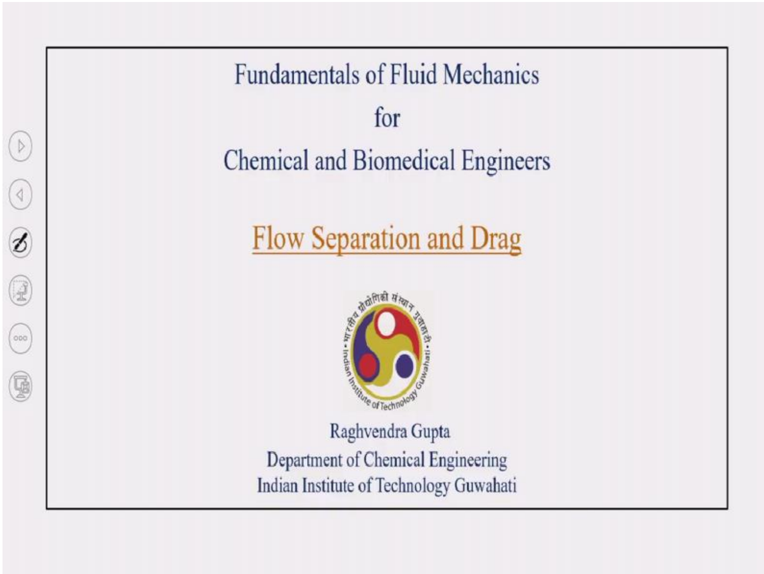

Hello, So, in this module we have been talking about boundary layers. We looked at that how boundary layer can help in predicting the drag where we can have outer solution and solution in the boundary layer and combined together we can find that the drag is non-zero and the D'Alembert's paradox can be explained.

Now, then in the last class we looked at momentum integral equation and we solved the problem of flow, boundary layer flow over a flat plate where we considered that the pressure gradient in the outer flow, along the flow direction. So, that means if the flow direction is along x direction dp/dx =0. So, we considered that 0 pressure gradient, but that was the case of a flat plate.

Now when we look at several engineering problems or several problems of engineering interest, in most of such cases we will not get the case of dp/dx =0. It might be either greater than 0 or it might be less than 0. So, in today's class we will look at that when the gradient is non-zero, when the pressure gradient dp/dx is non-zero, the analysis in such cases becomes very difficult.

So, we will discuss just qualitatively some of these things and when the gradient is adverse that means when dp/dx is positive then we will see that the flow can separate from the boundary and we have been talking about flow separation all along this course in different contexts.

So, we will be looking at flow separation a bit more in detail and then, we will talk about drag and lift and how we can calculate the drag and then finally, we will look at the flow behavior over a sphere and the standard drag curve for drag on a sphere at different Reynolds numbers.

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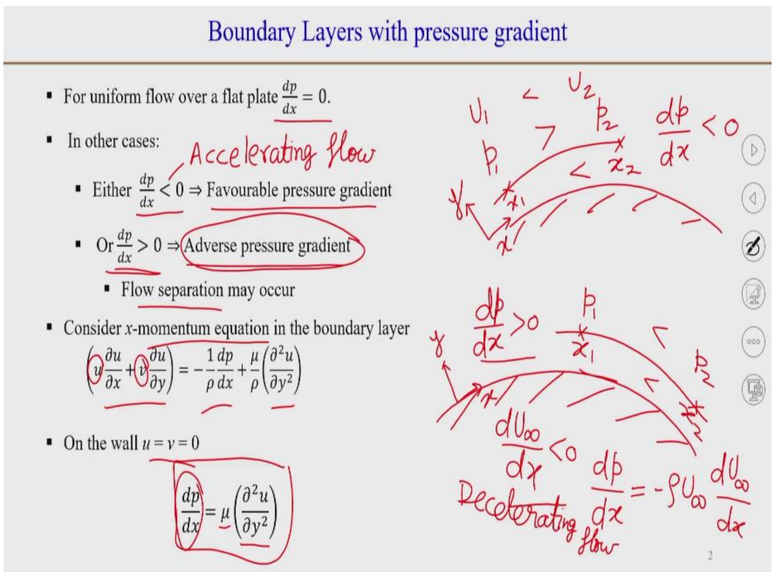

So, we said that when the flow is uniform over a flat plate dp/dx is equals to 0, but if this is not the case then it might be either less than 0 or greater than 0. So, if you have a boundary and on that boundary if the pressure is changing from p1 to p2 and the x direction is along the boundary and y direction is normal to it. So, these points let us say x1 and x2.

Now x2 is greater than x1 and if p1 is greater than p2 then we will have dp/dx =because both of them p1 - p2 and x1 - x2 will be of opposite signs, So, we will have dp/dx is less than 0. So, that means when the pressure is higher here and from outer flow solution we can say that U1 will be less than U2. That means the flow accelerate, So, when you have a dp/dx is less than 0 then you have accelerating flow in the outer region.

Now, in the boundary layer there is a drag, the fluid experience is the viscous drag which basically try to retard the flow and if it the pressure gradient is less than 0 then it helps the fluid to overcome this viscous drag. So, basically that is why it is called it is the favorable pressure gradient it favors the flow.

Now, if on the other hand, we have a dp/dx let us say in this region. We again take two points and p1, p2 but now let us say p2 is greater than p1 point x1 and x2, again the similar coordinates we have. So, in this case, x1 is less than x2 and p1 is less than p2 So, you will have dp/dx is greater than 0 and you will have dU_2/dx is less than 0. Remember from a boundary layer we have dp by or from Bernoulli's theorem we can write $dp/dx = -\rho U_2 \, dU_2/dx$.

So, again, what will happen in this case, the flow is decreasing. The flow velocity in the outer flow as we move along it decreases. So, the flow will be decelerating, that means that in this case, along with the viscous drag there is the flow also need to overcome the deceleration caused by this pressure gradient, which is opposing the flow. So, that is why it is called adverse pressure gradient.

Now, when we have a favorable pressure gradient our boundary layers are supposed to be thin and they will be hugging to the wall, whereas in case of adverse pressure gradient if the adverse pressure gradient is higher than the flow may separate from the wall. So, that is when flow separation can occur. So, for flow separation to occur the pressure gradient along the flow direction or dp/dx is supposed to be greater than 0.

And by slow separation we mean the flow will become reverse in the direction, the flow direction near the wall will change the fluid particle which were attached to the fluid, they will not be anymore attached or they will have to be go away.

So, to understand this further, we will write down the x momentum equation, the boundary layer differential equation that we wrote. So, we will write these two inertial terms for steady flow and the pressure term, you might notice here we have dp/dx because in boundary layer we had dp/dy =or it can be approximated to be 0 or it can be neglected.

So, that is why we have the normal derivative not the partial derivative here. +, μ/ρ d2 u/dy2, only the velocity gradient in d2 u/dy2, d2 u/dx2 was negligible with respect to this. So, now, when we consider this at the wall, on the wall because of no slip boundary condition the velocity will be 0. So, we will have this equation or the pressure gradient will balance the viscous forces here.

Now, we can see d2 u/dy2 and dp/dx, these two terms here, So, by knowing the sign of dp/dx we can see that because μ is always going to be positive, we can say that what is going to happen d2 u/dy2.

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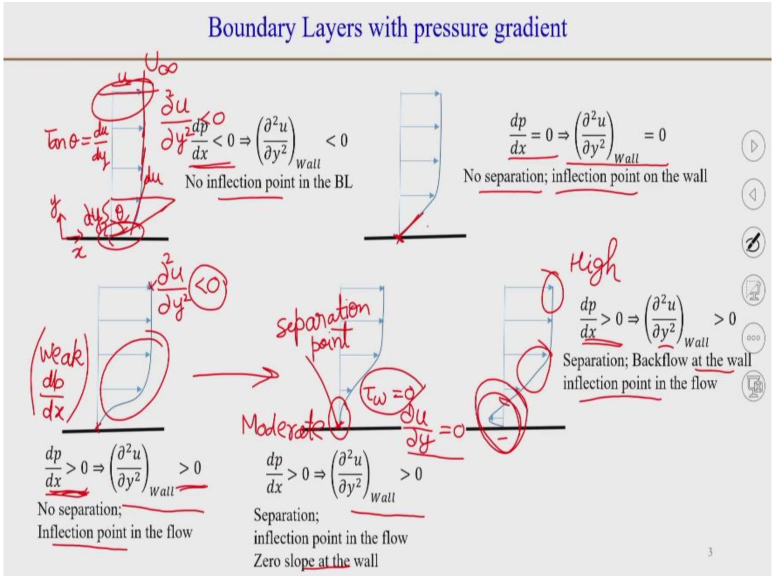

So, let us look at some different cases, if you a boundary layer profile like this. Now see this, that the coordinates there are say x and y coordinate and what these vectors basically on this plot So, that it is the magnitude of u velocity. So, if I take a slope at this point and let us say this angle isθ then I can say that this will be du and this will be dy.

So, we can write tanθ =du/dy. So, higher this value ofθ the greater is my du dy. Now as we go in this profile, as we change the slope, when we approach the outer region where the fluid velocity is free stream velocity, in that case du/dy will be 0. So, here, we will have thisθ will be becoming 0.

So, in this case, what we see that the slope is continuously decreasing as we move away from the wall. So, theθ is continuously decreasing So, that means d 2u/dy2 is negative here and this is the case when we have such velocity profile we observe and dp/dx is is less than 0.

And near the boundary layer region or near the outer and boundary layer interface always have d2 u/dy2 less than 0, because the slope of or this angle will be reaching close to 0, the angleθ will be reaching close to 0. The slope will be changing So, that, So, near here d2 u/dy2 is less than 0.

Now when dp/dx is lass than 0, then we have you can see from this slope here that d 2 u/dy 2 is also less than 0 near the wall. We have this equation that dp/dx =μ delta u/dy2. So, from this equation, if dp/dx is less than 0 then near the wall d2 u/dy2. We are talking about this in the boundary layer, So, when we say, and this equation is valid near the wall only in the, just next to wall that is why we could use the no slip boundary condition there.

So, they are of the same sign. This is negative and this is supposed to be also negative in this region, near the outer region. So, the slope does not change sign or d2 u/dy2 does not change sign when we move from the wall to the boundary layer region. So, there is no inflection point, inflection point where this will change the sign.

Now, when we have dp/dx =0 then again, d2 u/dy2 will be 0. So, that means it will be linear, the slope or d2 u/dy2 is 0. So, that means du/dy is constant. So, that will be linear here or we can sayθ τ w d τ w/dy =0. So, in this case again, we do not see any separation and the inflection point where d2 u/dy2 =0 is at the wall.

In this case, d2 u/dy2 was nowhere 0 in the flow. Here, we have d2 u/dy2 0 at the wall and that is what we call inflection point, the point at which the second derivative of U becomes 0. Now, if we have dp/dx is greater than 0, So, it is a positive, let us say it is weak dp/dx. So, the magnitude of dp/dx is not very high.

Then in such case near the wall from this equation because dp/dx is greater than 0 then d2 u/dy2 is also supposed to be greater than 0 near the wall. Whereas, in this region d2 u/dy2 is less than 0, So, somewhere in this the flow has to change or the second derivative have to change sign from being from positive it will go to negative. So, there will be a value or there will be a inflection point in the flow somewhere here where the slope becomes 0.

Now the next case that if you have moderate dp/dx, So, dp/dx as we go along in this direction let us consider the case is that the dp/dx increasing. So, we can say that it has a moderate value and it is positive. Moderate dp/dx and it is such that this the angleθ has become almost or has become 0 here. So, you have the flow or τ w is becoming 0,θ is 0 that means ∂ u/∂ y =0.

So, again, the second derivative is positive du/dy is 0 at this point and that is when you will have the flow separation starting. So, that is called your separation point. So, if you want to calculate or if you want to find out the location of separation point in a flow you can see that where is the location where τ w =0. So, you can find out that. Now, this will again have an inflection point in the flow and the slope is 0 at the wall.

Now if the value is high, it further increased than at high value when it is increased than this critical value then the flow will be reversed because of the pressure gradient, which is stronger in the opposite direction. So, dp/dx is greater than 0, you will have a flow reversal happening near the wall and then, it will change the direction. You will also have an inflection point somewhere in the flow where it changes, d2 u/dy2 changes from positive at the wall to negative here.

So, this separation will cause that there is back flow at the wall. So, as we can see that when you have a flow separation on a surface then the flow will eventually go through in all such stages that the far away from the separation point, you will have dp/dx is less than 0 and then, if the flow starts decelerating then somewhere you will get a point where dp/dx near the boundary layer or in the boundary layer becoming 0 and then it becomes positive.

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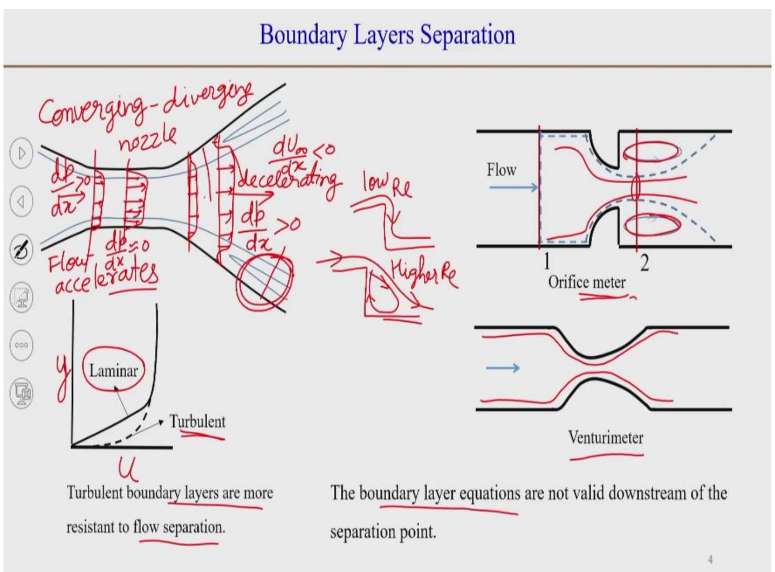

So, there are a number of such cases. One case is for example you have a converging diverging nozzle and you have an incompressible flow. So, in this region where the flow is accelerating because the area available for the flow decreases, So, the velocity will increase and the flow will accelerate as you move along this direction and dp/dx will be positive here. So, you will have the velocity profile for example here that will be something like this.

Now in this region, dp/dx it is closer to flat plate and dp/dx will be approximately 0. The flow might be happening with the uniform velocity. And when you go in this region, when the flow diverges then flow will start decelerating and you will have dp/dx becoming greater than 0 and dU/dx is less than 0.

So, you will see that say somewhere here, the slope might be normal to it and you might have a separation point and then, further down you might see a back flow in this region. So, you will see all such stages, all the stages that we discussed in the previous slide, we can see that all such stages in a flow that separates.

Then, if we remember, when we talked about application of Bernoulli's equation and Orifice meter which we can use to measure the flow rate, we had this picture when the flow accelerates here and there is flow separation happening and the minimum a cross-sectional area of jet that is what we call vena cava. So, here, again, we had flow separation and then we had another flow meter Venturimeter in which the change in area was not sudden.

So, flow does not separate and it is designed in such a manner that the flow does not separate. So, that is why it is called Venturimeter. So, in Orifice meter we had lot of losses, the pressure recovery is higher in such cases, where there is a lot of pressure loss or head loss in the Orifice meter.

Similarly, if we have a sudden change in area or a backward facing step, what we call, then you will see that the high Reynolds number the flow, at low Reynolds number, see when the flow is Stokes flow or the Reynolds number is very low, you will see the attached flow. For example, in a backward facing step you might see the stream lines like this at low Re, but when the Reynolds number becomes higher you will see a flow separation happening in such cases.

Now, if we compare the velocity profile in laminar and turbulent boundary layers then that is a typical profile for laminar flow whereas in the turbulent flow the gradients are the velocity gradients are higher near the wall. So, in the turbulent flow they are more resistant to flow separation because they have more momentum near the wall, So, they can resist the flow separation as compared with laminar boundary layers or laminar flow near the wall or in boundary layers.

Now, when we assumed, when we derived the equations for boundary layer, we did not consider or we did not have the assumption of separated flow and when we have separated flow, then the stream lines in the outer region are also diverged or they go further apart because the boundary layer becomes thicker. So, the boundary layer equations that we derived they are not valid downstream of the separation point.

So, we can still analyze the flow in the region, where the flow separate, before the flow separates. But after the flow separation happens, we cannot use boundary layer equations, they do not remain valid anymore. The outer region, the assumption of outer region stream lines does not hold good and the boundary layer becomes thicker which was not the case.

Because for example, if you have such a separating flow here then you will have u and v component of velocity which are the velocity component along the boundary layer normal to boundary layer. They might be of the equal importance. Whereas we say that in the boundary layer equations we derived that u is less than, sorry, u is significantly greater than the y component of velocity v.

And this came because we could assume that boundary layer is very thin and that will not be valid anymore when the flow separation occurs. So, we cannot use those equations.

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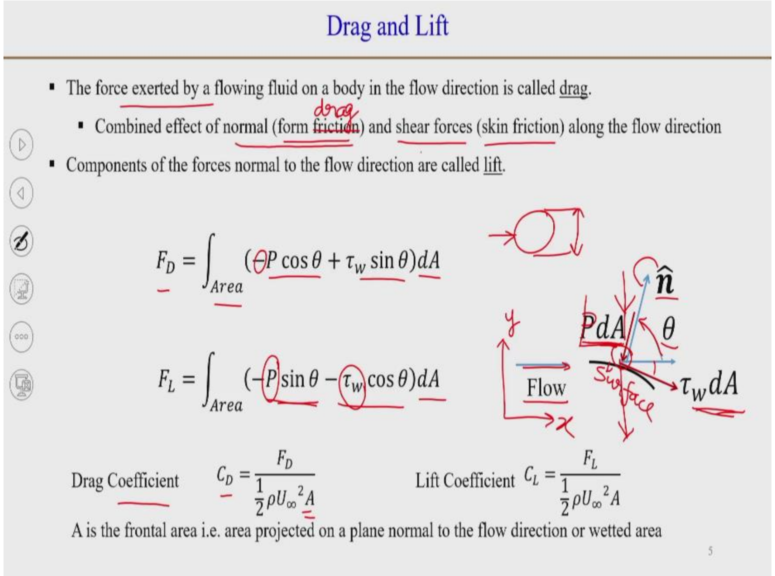

Now, we will define what is drag. So, we have talked about it quite a few times earlier in the course, say for example, when we were talking about dimensional analysis and we talked about or we looked at the problem of drag on a sphere.

So, to remind ourselves that drag is the force that is exerted by a flowing fluid on a body in the flow direction. So, if you have a flow happening and if you are standing in the flow then you experience a force that want to take you along or that want to drag you along. So, the drag name, the term comes from there, that the force that a body which is in the flow it experiences.

Now, there can be several cases, the fluid might be moving, the body only might be moving and fluid is stationary or both are moving. What we consider or when we consider drag it is the relative velocity between the fluid and the body. And the body considers or body experiences the force in the direction of the flow, because if the flow is happening in this direction then we will experience the force in this direction.

Whereas, the fluid which is flowing because of the presence of body, it will experience a force in the opposite direction. So, the fluid will experience the drag in the opposite direction, but body will experience the drag in the direction of flow.

Now, this drag can be because of both the surface forces. It can be because of, because when a flow happens, the flow will come in contact with a body on the surface of the body and so, there will be surface forces on the body because of the fluid and these forces can be in the form of pressure forces or viscous forces.

Now viscous forces can also have normal components So, you can have viscous stress which is normal to the wall and viscous stress which is tangential to the wall. So, you can have a viscous shear stress which we commonly hear about and in some cases, for example, if you talk about Stokes flow around a sphere there the normal component of viscous stress is non-negligible and you might need to consider.

In most of other cases especially at high Reynolds number flow the viscous normal stresses are generally negligible. So, you will have pressure forces contributing to the drag and you will have viscous forces contributing to the drag. Now, the effect of normal forces or pressure forces we can say here that comes under form friction.

So, the drag or the frictional force that the body experiences actually drag should have been a better word here, because friction generally we refer to the force that is tangential to a surface. So, the form drag is caused by the normal stress, which is generally the pressure and the drag caused by viscous forces is called shear force.

So, viscous, the tangential forces which are caused by viscosity or viscous forces they are called skin drag or skin friction and the component of forces which are normal to the flow direction they are called lift. So, we have two-dimensional flow, then the direction along the flow or the force on the body along the flow is called drag and normal to it is called lift.

Now, if we consider a surface here, So, this is a surface and the normal to this surface is this vector, unit normal n and the flow happens along this direction. So, the angle between flow direction and the normal vector isθ, then we can take the components of pressure force which acts normal to the surface and inverts to the surface, So, it will be opposite to the area of vector.

So, that is PdA and the shear stress is τ w dA which acts in the tangential direction at that point, tangential to surface. So, we can write the drag force FD is equal to, if we take the entire area then we can integrate this over the area for this differential surface we can write, the component of pressure force will be PdA cosθ and my x and y coordinates are these. So, PdA cosθ will be the component of force PdA or component of pressure force, So, PdA cosθ will be the contribution of pressure force towards the drag here. So, PdA $\cos\theta$ and the - sign because this acts in the negative direction.

Similarly, the component of τ w dA will be τ w dA into $\sin\theta$. So, that will be the contribution from the shear stress. In the same manner, we can write the lift force. So, lift force will be the force here let us say along the y direction or what we have shown here. So, that will be normal to the flow direction.

So, that will be - P $\sin\theta$ dA because the component of pressure force along this direction will be PdA $\sin\theta$ and it acts in the negative direction. So, - PdA $\sin\theta$ and the component of τ w dA along this direction will be τ w dA $\cos\theta$ with a - sin there. So, that is how we can calculate on any surface the drag and lift forces.

What would we need there? We will need the pressure field. We will need the τ w which we can calculate using, if we know the velocity field from the velocity field we can calculate the shear stress on the wall and we will get value of τ w and if we integrate it over a surface along the flow direction.

So,θ is the angle at a particular location between the flow direction and the area normal. And generally, we define it in non-dimensional terms. So, the drag coefficient or what we call CD is FD/A /1/2 ρ U_2^2, which is the dynamic pressure. And the lift coefficient is CL =FL /1/2 ρ U_2^2 A.

Now for most of the cases, this A will be the frontal area. So, for example, if you have a flow around a sphere then the area is the projected area normal to the flow. So, for A sphere this will be πr^2 where r is the radius of the sphere. But for at least one case, when we talk about a plate which is along the flow direction then we will have weighted area because in those cases if we have a thin plate then the area will be 0. So, let us look at this thin plate.

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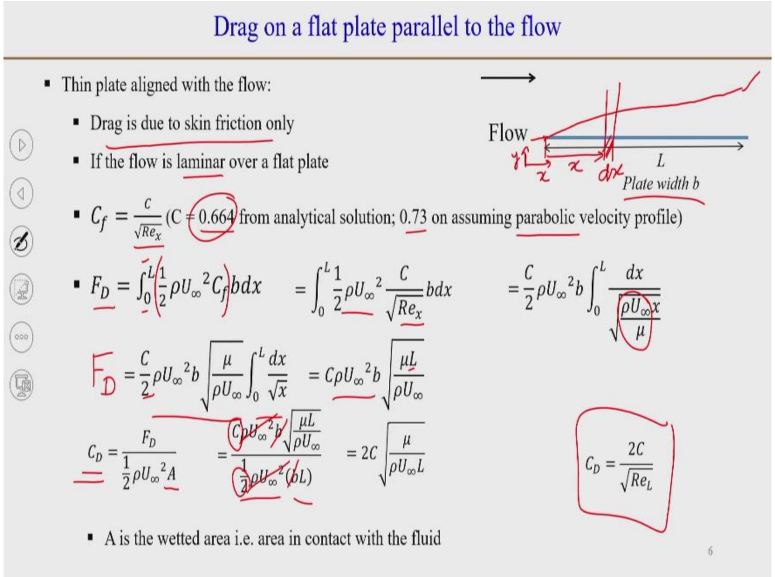

If you have a flat plate parallel to the flow, in this case because the plate is very thin So, the frontal area actually negligible and in this case the drag because of the pressure will be negligible because the pressure forces along the x direction will be 0.

So, this is one such case where the drag is only due to skin friction. In most of other cases we will see that the contribution comes from both, contribution comes from the form drag and skin friction. If the Reynolds number is low then the viscous effects are dominating, but when the Reynolds number becomes high it is predominantly the contribution from the form drag or from the pressure.

So, we know that when we have a flat plate then you will have boundary layer developing into it and then it becomes a turbulent. So, what we will consider here is the laminar that the flow over flat plate is laminar because we know the solution for laminar flow over a flat plate. We looked at in the last class and we obtained the relationship for Cf the other day and Cf is defined as τ w/1/2 ρ U_2^2.

So, you have another Cf here which is a skin friction coefficient and it varies with x, whereas when we talk about drag, drag is not a local quantity where Cf is local quantity. It depends on x CD will be an integral quantity. So, we can write down the drag of force =τ w which will be basically Cf multiplied by 1/2 ρ U_2^2 multiplied by the dA.

So, b here is the plate width which is the plate width normal to the screen and the length of this plate is let us say L. So, we can integrate over the plate area if we take a differential element at a distance dx and the distance x if we take a small element of length dx, then we can integrate it from 0 to L.

Now, we can substitute the value of Cf here. Cf is a constant $/\sqrt{Rex}$ and if we use the analytical solution from the Blasius solution then this value of this constant is 0.664. When we use in momentum integral analysis assuming a parabolic velocity profile then we obtained this value of C is 0.73.

So, we will just consider this as a constant C and because $1/2\ \rho\ U_\infty^2$ is a constant remember for a flow over a flat plate when the flow is uniform U_2 is a constant, it does not depend on x. So, we can just take $1/2\ \rho\ U_2^2$ C and b all out of the integral and we can expand or we can write Rex, we can expand it So, we can write it as $\rho\ U_2\ x/\mu$.

Now, again, $\rho\ U_2/\mu$ is constant, So, we can bring all this outside the integral sign and in the inside the integral we will have $dx/\sqrt{x}$, So, which is a basically integration of $x^{-1/2}$ which when you integrate it, it will be $x^{1/2}$ or $2\sqrt{x}$ and when you put the integral limits then you will get $2\sqrt{L}$ So, we will use that.

So, it will be $C\ \rho\ U_2^2$ b and $2\sqrt{L}$ when we used, So, this 2 will be canceled out. So, you will have $C\ \rho\ U_2^2$ b into U μ and multiplied by L here and under the $\sqrt{}/\rho\ U_2$. So, that is the drag force.

Now we can find out the drag coefficient. So, drag coefficient is $FD/1/2\ \rho_2^2/A$. As we said earlier, that in this case A will be the weighted area of the plate. So, if the flow happens only on one side of the plate, which we are considering now then area will be bL. If the flow happens on both side of the plate then it will be bL at the top and bL at the bottom.

So, we will consider only one side of the plate, So, that is why we call it weighted area, the area over which the flow is happening. So, FD is $C\ \rho_2^2$ b into $\sqrt{\mu}\ L/\rho\ U_2$ and this $/1/2\ \rho\ U_2^2$ bL.

Now $\rho\ U_2^2$ will be canceled out, b will also cancel and you will get 2 multiplied by C one L or $\sqrt{L}$ will cancel, So, you will have under the $\sqrt{\mu}\ /\rho\ U_2/L$ which is nothing but ReL.

So, we can write that CD =2 times C $/\sqrt{ReL}$ and we know the value of C, it can substitute the value 0.664. So, that is, if the plate width it is such that or the plate length L is such that the flow is laminar over this small plate then you will have this drag coefficient.

If the plate is longer then of course the boundary layer will grow and become turbulent in the downstream region and then, we will need the expression for friction coefficient in the turbulent boundary layer region also and then, we can use the same procedure to find the drag on the plate.

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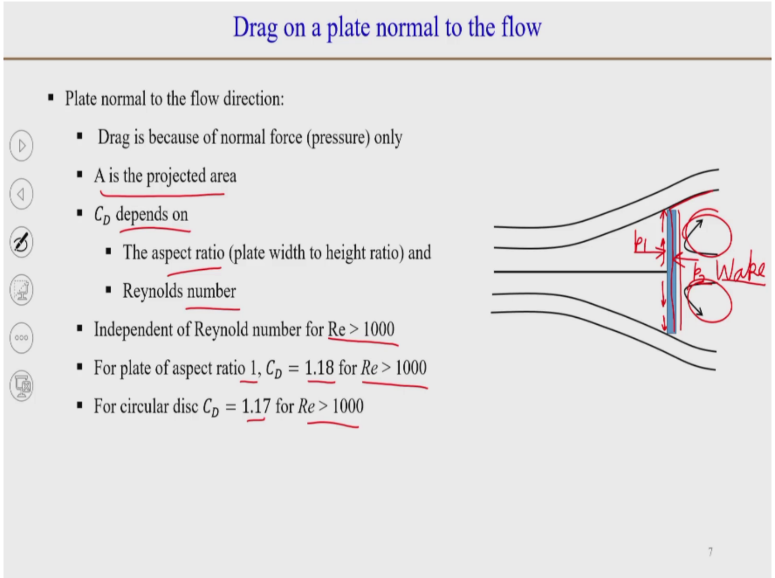

Now, we can consider the other case where the plate is placed normal to the flow. So, in this case there will be viscous drag or there will be viscous force on the plate, but that will not be along the flow direction, So, that will not be called drag. So, when we talk about drag that will be only because of the pressure difference say p1 on this side and p2 on this side.

And when the flow passes around it there will be flow separation from these boundaries and you will have recirculation in this region, which is also called wake region. So, for such cases for this case the projected area will be or the area of the plate will be the A in the coefficient of drag coefficient, in the definition of drag coefficient. And this CD depends on the aspect ratio, which is the length and width.

So, when we talk about the length or height of the plate and width normal to it, the plate is of course assumed to be thin again, you might have seen some thickness here, but that is only just to demonstrate or So, the plate here. So, it depends on the aspect ratio and depends on the Reynolds number.

However, it has been observed that at higher Reynolds number or Reynolds number greater than thousand it becomes independent of Reynolds number. So, some numbers here we see that if the plate is^2, So, for a^2 plate the length and width are equal So, aspect ratio 1 and CD =1.18 when the Reynolds number is greater than 1,000.

And if the disc is circular 1.17, So, the values are very close to each other and the Reynolds number is greater than 1,000. So, for such cases, the value for a disc you can say for a circular disc or and a^2 disc is about 1.2.

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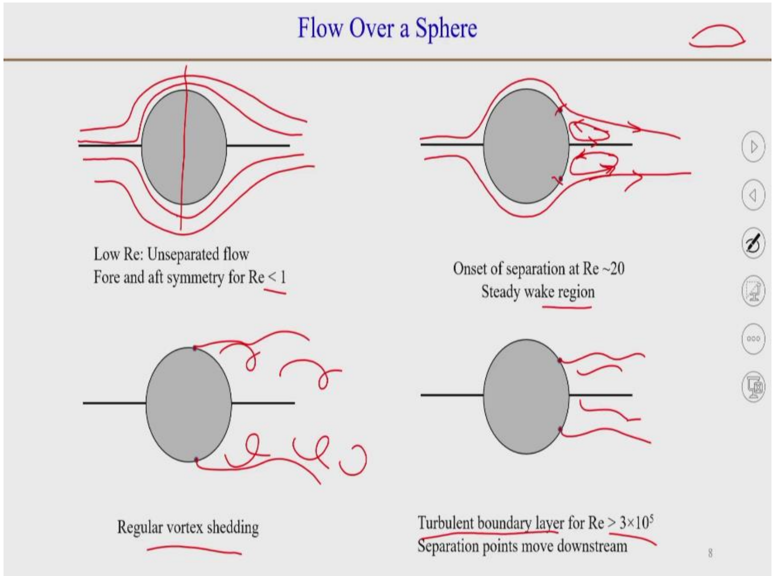

So, now we will just briefly, because for a number of applications when we talk about boundary layers, we talk about boundary layers in the external flow as well as internal flow where we will have, the flow becomes fully developed because boundary layers they eventually the merge and viscous effects are very important in the internal flows.

So, when we have external flows in the say the flow over a car, flow over an airplane, we have boundary layers everywhere. In the chemical engineering applications, you have number of multiphase reactors and these reactors will have a continuous phase and have a dispersed phase.

For example, if you talk about fluidized bed reactor or a packed bed reactor then the flow happens through a bed of particles and these particles which might be stationary or may be moving with a certain velocity in case of fluidized bed.

So, these particles will have boundary layer. These particles, if we considered them to be spherical, they will have boundary layer over them. So, we need to understand the flow behavior and the drag on the spherical particle. One, because they are everywhere; second, because that is the simplest idealization we can make.

Another example, say for, when we look at bubble column reactor. So, bubble column reactor is basically a column of liquid in which a gas is just passed and when gas is just passed it will be because of surface tension, it will be not coming as a continuous stream, it will have bubbles.

And these bubbles if they are very small they will be spherical and if their size is larger then they will be, might be a say, what is called this spherical cap shaped kind of bubble or ellipsoidal bubble, when you develop models for such cases you need the drag information on the bubble. If you want to turn need, if you want to calculate terminal velocity of the bubble then also you need the information of drag.

So, we need to understand the flow behavior around the sphere at least qualitatively. We have seen or when the flow is Stokes flow then flow around a bubble or flow around a sphere is symmetric. If we have a slightly higher flow, So, a Reynolds number is greater than 1. If there, the Reynolds numbers less than 1, then of course, we will have attached flow and there will be 4 and F So, symmetry about this line.

But at higher Reynolds number you will have this asymmetry slightly breaking up, but flow will remain attached. As the Reynolds number somewhere around Reynolds number 20 or 25, you will see that there is a flow separation happening and there are steady vortices forming here. So, because the stream lines or the flow is happening in this direction, these vortices will have flow direction in such manner.

So, flow is separated on this sphere, So, it is back flow and here they will have the same direction. So, between say 20 to 400 you will have steady wakes behind this, when you have say Reynolds number 20 - 25, the size of these wakes will be smaller and as the Reynolds number increases the wake sizes, it increase but wake pretty much sits there, say Reynolds number 130 or 150 until that.

Then slowly vortex shedding starts and the separation point shifts. So, you might see the separation points are somewhere here, but at high Reynolds number the separation points shift somewhere closer to 90 degree and the vortexes start shedding from the sphere. So, this vortex shedding happens and that goes on say about Reynolds number 3 into 10^5 for a smooth sphere and the flow which is free from disturbances.

And at about Reynolds number 3 into 10^5 the flow for higher Reynolds number flow becomes turbulent and the separation points comes downstream. So, you see, the wake region becomes smaller So, this is where the boundary layer becomes turbulent.

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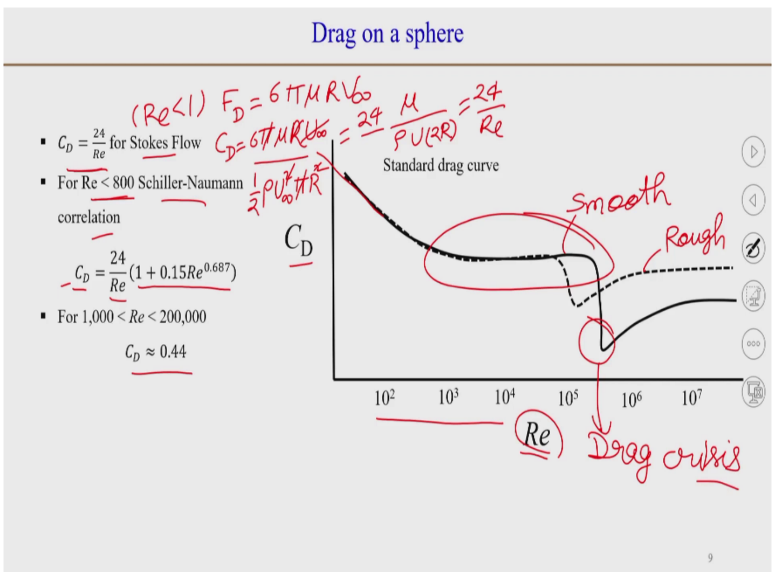

And we can also see that how the drag on a smooth sphere varies with Reynolds number. So, this is drag on a smooth sphere and then a qualitative representation for a sphere, which is rough, but it will depend also on the roughness of the sphere. So, you can see that at very low Reynolds number it will follow the Stokes law which is CD =24/Re.

You know that F drag for Stokes flow =6 π μ R into V and from that you can actually calculate CD =6 π μ R into, let us say this is U_2, So, U_2 /1/2 ρ U_2^2 and projected area which is π R^2. So, this

U_2, U_2 will cancel out you will have a 6 into 2, R will cancel out, π will cancel out, So, that will eventually give you 12 or 24 μ/ρ U into 2 R. So, I have multiplied by 2 in the denominator and the numerator that is why it has become 24 and this is basically 1 over Re So, we can write this as 24/Re.

And this drag also we can, the formulas that we discussed for drag and lift forces, we can see what are the velocity profiles in the or in the Stokes flow over a sphere, there are analytical solution available and using the Vr Vθ and V π and pressure relationship, we can actually find FD =6 π μ R V. But we are not going to do it, because it involves spherical coordinate system and that will be quite a bit involved. So, we will not do in this course.

But you can now nonetheless look at the solution which say for example in book by transport phenomena by Bird, Stewart and Lightfoot the solution is there. So, that is for low Reynolds number. Reynolds number is less than 1 then CD =24/Re this is a log, log plot as you can see from here. So, you will have a linear variation ln CD =ln - Re.

So, at higher Reynolds numbers you will have CD =24/Re into 1 + 0.15 $Re^{0.687}$ and this is called Schiller-Naumann correlation. So, this is one of the correlation which is pretty good for Reynolds number less than 800.

There are number of correlations available for CD less than Re because in such cases the flow, boundary layer flow separation happens, So, it is not possible to find say analytical solution using the boundary layer and outer solution or outer region solution approach, but people have done experiments and measured the drag force and they also have done the numerical simulations and this standard drag curve has been verified.

So, one can use, Schiller-Naumann correlation quite confidently up to Reynolds number less than 800. Then as you can see from this graph here this that is in this region the values of CD it becomes independent of Reynolds number So, that is about 0.5 or say number 0.44 CD =about 0.44 there.

And then, when you increase it further, when the flow becomes turbulent there is a sudden drop in the drag and that is where and it is called drag crisis that is because the flow has changed from laminar to turbulent or from transition to turbulent is there.

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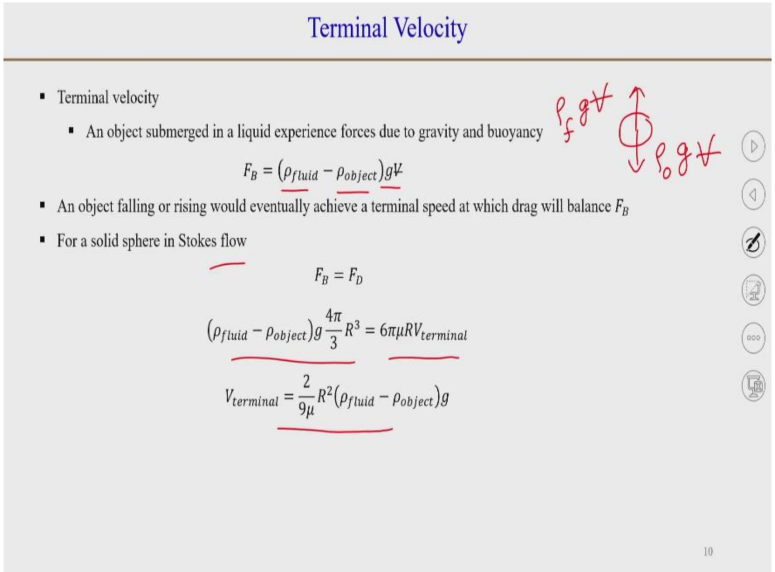

So, that was about flow around a rigid sphere. Now if you have a fluid sphere then you will have a few more things but before we look at the fluid sphere we will just talk about what is called terminal velocity.

So, if you have a sphere moving in a fluid it may accelerate or it may decelerate, it may be a solid sphere, which is going down or a sphere which is lighter than liquid, it might be coming up say for example a bubble. Now in both the cases the sphere will have two forces acting on it, one is the force because of buoyancy.

So, buoyancy force and gravity which is how we can write it? That ρ fluid - ρ object into g into V. So, that will come into picture the force due to buoyancy and this is when we consider if an object which is where you have an object which experience is force due to gravity, So, ρ object g into the volume of this object and buoyancy force will be ρ fluid into g into V

Now, the net force or net buoyancy force, combined force because of gravity and buoyancy that will be the force and it will be rising or going down depending on the balance of these. But eventually, it will also experience when it is flowing, it also experiences a drag force or the viscous drag and pressure drag both combined.

So, at certain point of time, it will achieve a steady velocity and that is where, when the forces, the FB or the force due to gravity and buoyancy and the drag forces they will balance each other. So, if we talk about say stokes flow which we have done earlier also So, we can do the buoyancy and drag forces for a Stokes flow, they will balance each other and from that we can find the terminal velocity. But we can do a similar relationship for a sphere at higher velocities using the drag correlations that we just saw.

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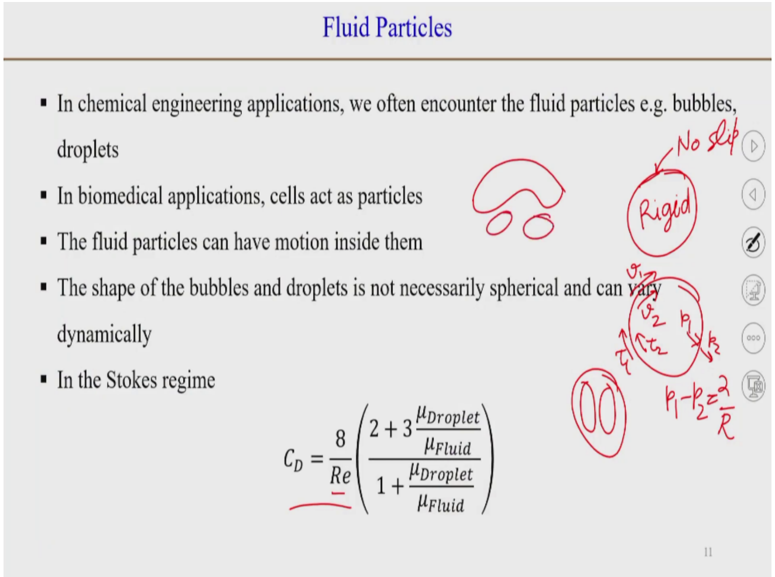

So, when you have fluid particles for example bubbles or droplets or when we talk about biochemical applications they might be cells. So, in these cases, when we talk about a rigid sphere, the boundary condition at the surface of a rigid sphere is no slip, whereas, if it is rigid, but if you have a fluid sphere then you have a continuity of velocity and the continuity of shear stress and the normal stress. So, p1 - p2 =say if it is spherical 2 σ/R.

So, all that and because there is a motion inside the fluid also So, you will see that there are vortices developing in the fluid and that causes or that help in reducing the drag on such fluids.

So, for Stokes flow Hadamard-Rybczynski found a solution for droplets in the Stokes regime So, this is the correlation for the drag coefficient, actually they have found the analytical solution for flow inside the droplets and outside the droplet and from that one can derive the value of CD. You can see that this is a function of apart from Reynolds number this is a function of the viscosity of the fluid inside the droplet and the fluid surrounding it.

Now if it is a bubble then you can say that this value is almost 0. So, you can have the relationship for CD for a bubble. If you say that this is a solid then you can say that the μ droplet is very high with respect to μ fluid or you can say μ fluid/μ droplet approaches 0 and when you use you will get 24/Re.

But the bubbles and droplets they will not necessarily be spherical their shape changes dynamically, especially when the Reynolds number that you calculate based on the bubble diameter that is large.

So, at high Reynolds number you might have a bubble shape like this and in the back of it there will be vortices and these vortices interact with the bubble, So, the shape changes dynamically and when the bubble shape changes again the flow changes, So, it is a very transient phenomena.

And these are some of the things that one might be looking into if one want to look into a problems say or the interaction of between bubbles, So, there are numbers of such problems that one can look into as a research problem.

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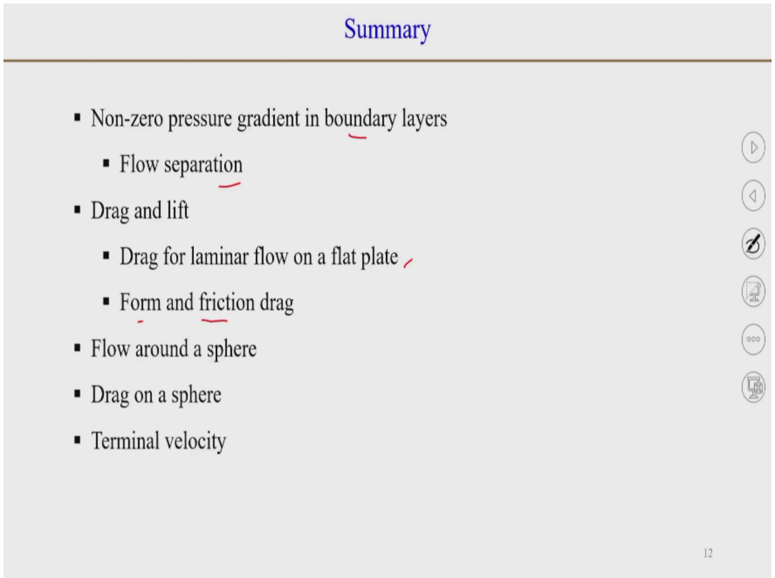

So, let us summarize, what we have looked into today is we talked about that what happens when there are non-zero pressure gradients in boundary layers and then, we saw that if it is adverse pressure gradient, then it may result in flow separation we also explained it or we also try to understand it by writing down the boundary layer, equation near the wall and then we defined drag and lift forces and derived the drag coefficient for laminar flow on a flat plate.

We should also remember what is form drag and friction drag or form drag and skin drag caused by pressure and viscous forces and then, we briefly looked at flow around a sphere qualitatively and then drag on a sphere and then terminal velocity flowing.

So, with that, we will stop here. Thank you.