Fundamental of Fluid Mechanics for Chemical and Biomedical Engineering

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Lecture No. 34

Boundary Layers

In the previous module we talked about inviscid flows. Where we looked at inviscid flows, the irrotational flow. And, and then for an inviscid irrotational flow we we talked about the velocity potential and so on. And we also looked at Bernoulli's equation. So, when the flow is inviscid it becomes easier because you can solve the equations governing the flow it becomes easier to analyse the flow mathematically.

So, there has been a lot of development using potential flow finding out the solutions for different flow problems. However, there are some limitations for these inviscid flows and they cannot for example satisfy the no slip boundary condition on the wall they cannot predict the drag etcetera.

In today's lecture we will look at boundary layer flows and in this module we will be looking at external flows in general. So, we will talk about boundary layers which coupled with a potential flow can be used to analyse the flow over plate, flow over spheres or any kind of flow problems they can be used to solve.

And then we will look at the concept of drag and lift on different bodies which are useful for a number of chemical engineering applications or and as well as in biomedical applications. Because, in any flow we generally encounter there are objects submerged into it. So, what are the forces on such objects?

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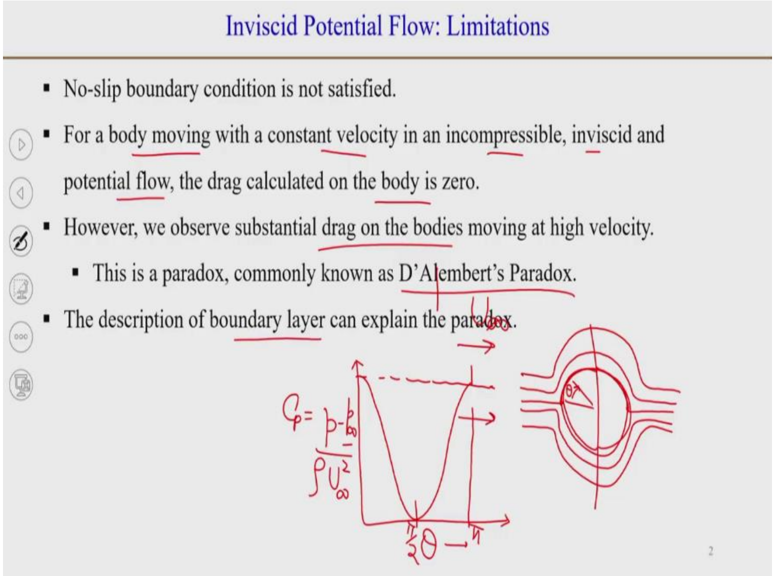

So, in the potential flow when you have a potential flow the boundary condition on the wall which is we specify that the boundary condition on the wall is no slip boundary condition that is not satisfied. For example, if you plot the velocity vectors surrounding this you might get vector something like this and the velocity is non-zero on the wall. So, that is one limitation because the no slip is something which has been observed experimentally.

When you analyse potential flow say around a circular cylinder a cylinder which has a circular cross section so the stream lines are symmetric. And if you plot the pressure so if you plot the pressure that will be say Cp which is $p - p_\infty / \rho U_\infty^2$. So, U_∞ refers to the velocity. The free stream velocity and p_∞ is the free stream pressure far away from the body so it might be in the pressure at $_\infty$ where the effect of body is not there.

And on x axis here so this has to be symmetric. So, and on x axis what we have is angle θ which is the angle along this direction. So, the pressure is symmetric and the potential flow because the flow is inviscid so the viscous drag is 0. And as a result, it can be shown that for a body which is moving with a constant velocity in an incompressible inviscid and potential flow the drag calculated on the body is 0.

So, the drag can come because of two components it can come because of the pressure forces or because of the viscous forces. As the flow is inviscid so viscosity is assumed to be 0 or viscous terms in the Navier stokes equations they are neglected. So, there is no viscous tract.

Now, as we can see here that the pressure on the two sides of the sphere on this side and that side is same. So, this is basically the angle π / 2 which where the Cp value is minimum and then angle π so the pressure has been plotted on either half you can say top half or bottom half in this image. So, the when the pressure on the left and right side or or the upstream and downstream side of the sphere is same so the pressure difference or the drag because of pressure will be 0.

And this is not only true for a circular cylinder but any non-symmetric body also it can be shown that the drag on the body is 0. So, this is a paradox because when we look at or when we experience that any body which is moving in air it maybe it may be in water for example a car moving in air it experiences a drag. So, what we observe in our day-to-day life the the results shown here is counter intuitive or it is not predicted by the theoretical analysis.

So, this is a paradox and this was observed or explained by D'Alembert. He, he said that he can he could prove mathematically that drag on an incompressible inviscid and potential flow or on a body is 0. But and this cannot explain the drag observed on the bodies experimentally or in our day to day life.

So, there was lot of because when the theory could not prove the experimental observations or our common observations then the there was a lot of theoretical development but it was not being used for engineering calculations or for practical purposes. And the only thing people used for engineering calculations was empirical data. Now, somewhere around 1904 Ludwig Prandtl he introduced the concept of boundary layer and tried to explain this paradox.

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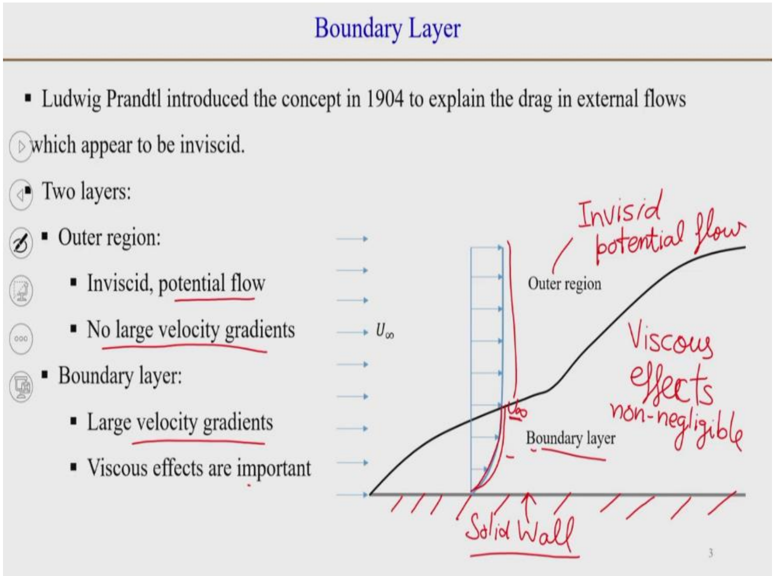

So, we have earlier seen in the introductory lectures that what a boundary layer is. So, Prandtl suggested that even for inviscid flows or the flow that seem to be inviscid because the Reynolds number is very high their edge effect of viscous forces near the near the wall. So, if you consider flow over a flat plate here so this is the a flat plate and this is a solid wall so this plate is solid. And because there is no slip boundary condition here so in a small layer near the wall the viscous effects are supposed to be dominant.

So, he suggested that one can assume or one can consider two layers in the flow the outer region where the flow is considered to be inviscid, potential flow. And the inner region which is boundary layer and where viscous effects are important. So, in the uniform flow if you have a uniform flow approaching a solid plate and you consider only inviscid flow then the flow will be uniform like this. But when we consider the a viscous layer near the wall then there are gradients present.

So, he suggested that in this small or thin layer over the wall the gradients are large and the velocity approaches from 0 to almost 3 stream velocity. Though it may approach asymptotically but for all practical purposes you can assume that at a certain distance from the wall the velocity has approached the free stream velocity. So, the velocity gradients are large in the viscous flows or in the boundary layer flow where the flow is viscous. Whereas, in the potential flow in the outer region the velocity gradients are small and in the boundary layer viscous effects are important.

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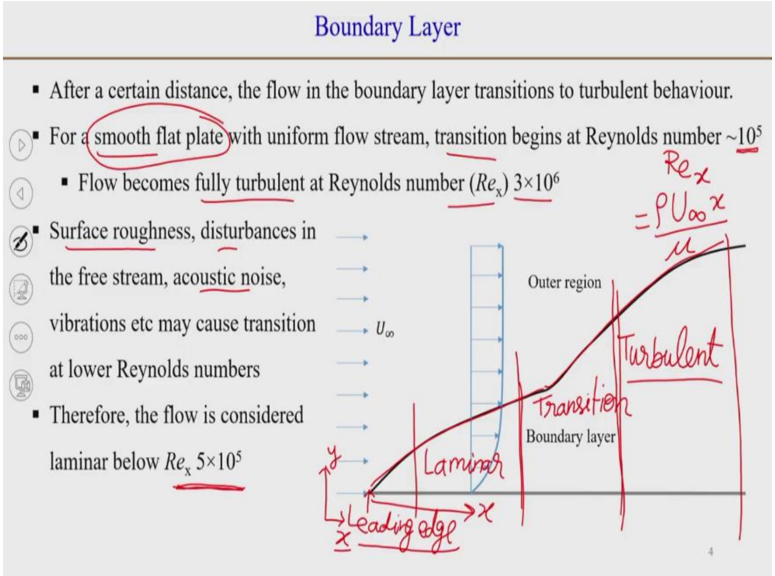

Now, as you can see that there are two different regions here the first one the boundary layer grows slowly and then it there is a sharp increase in boundary layer. So, as the thickness of boundary layer grows the flow becomes turbulent in the boundary layer. So, the flow near the what is called the this point is known as leading edge of this plate. And edge as the flow approaches away from the leading edge, the boundary layer grows and the flow is observed to be turbulent.

So, the flow becomes turbulent when there are disturbances and these disturbances cannot be dissipated by the by viscous effects. And these disturbances to the flow they grow and the they become unstable and then there is a lot of randomness and chaotic behaviour in the flow. So, first the flow transitions so between laminar and fully turbulent flow you will have a transition happening. So, first it slowly transitions to a flow become say from laminar steady flow to it becomes unsteady in in the transition region. And then it further grows to become fully turbulent flow.

Now, as you can see here that it happens after a certain distance. So, if we define a coordinate system let us say in 2D plane x y coordinate system where x is basically the coordinate following the wall and y is the coordinate normal to it. It may be a flat plate which we have shown in all the all the slides here but it can also be a curved plate. Then in that case the coordinate system will be such that that x is following the plate and y will be normal to it or orthogonal to it.

So, it is observed that this transition occurs about a Reynolds number of 10^5. And this Reynolds number is defined in terms of Re x. So generally, when we define Reynolds number we take the transverse direction. When we talk about say flow over a sphere we define that $Re = \rho U d / \mu$ where d is the diameter of the sphere. But, here what we are talking about when we define Reynolds number that $= \rho U_\infty x / \mu$. So, x is this distance from the leading edge.

Now, as we go away from the leading edge the Reynolds number grows. So, the Reynolds number at this point will be small and then it would have grown to a larger value here even higher values here and further higher values away. So, the experiments have shown that if the flow is smooth over a flat plate then the flow remains laminar until the Reynolds number is 10^5 and then at higher values it*ts transitioning to a turbulence.

Now, at about a Reynolds number of or a transition Reynolds number of 3×10^6 the flow becomes fully turbulent. But that is the case when we consider a smooth flat plate. However, when we consider or in general when we are looking at industrial flows or flow that we encounter in engineering it there might be a roughness on the surface and disturbances to the flow for several regions. There might be some acoustic noise or vibrations so all those factors may cause the earlier transition of the flow to turbulence.

So in general, it is accepted or for calculation purposes it is assumed that the transition occurs or the flow is considered to be laminar below 5×10^5. So, you may see this range that between 10^5 to 3×10^6, the flow edge say from transitioning from laminar to fully turbulent. So, somewhere in between because it is not easy to predict transition flows so what people suggested that somewhere in between we take a number and below which the flow is considered to be laminar and above which flow is considered to be turbulent.

So, while these are not very hard numbers or they are not that the flow is supposed to behave but they act as a useful guide. So, we can consider that Reynolds number 5×10^5 is the number below which we when we can consider the flow over a flat plate to be laminar.

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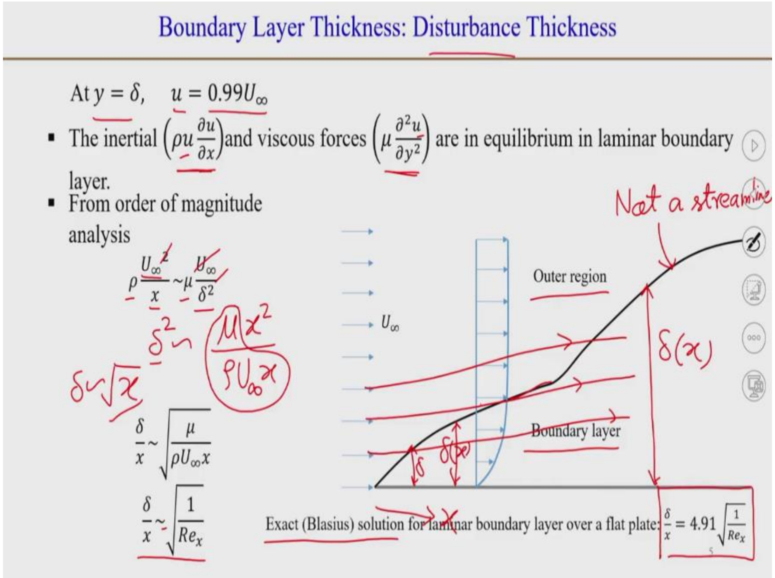

Now, so we have considered the flow over a flat plate and as we can see that as we move along the plate the boundary layer grows. And this boundary layer let us represent the thickness of the boundary layer as δ. And this δ is is a function of x so δ at different places will be different.

Now, one of the things of course we will be interested in to find out what is the thickness of this boundary layer. Because, the goal of boundary layer analysis was that we can solve potential flow in the in the outer region here and we can solve the viscous flows in the boundary layer. We can use some approximations and simplify the Navier stokes equations. So, we will need the thickness as well as the conditions at the region between inner and outer regions. So, we need to identify the thickness of boundary layers.

Now, one thing we should also see here that the boundary layer what we have drawn it is not a stream line. So, if we plot the flow stream line they may well one of the effect will be that as the flow while the stream lines if the flow is uniform the distance between stream lines will be same.

When it approaches the plate but when there is a flow over a flat plate and there is a boundary layer because of the presence of boundary layer these stream lines the distance between the stream lines may change. So, the distance between the stream lines can be slightly more as as it go downstream. So, the stream lines can cut the boundary layer and boundary layer is not a stream line here.

Now, there can be different ways to define a boundary layer thickness so the simplest way to assume it as I said earlier that the velocity will approach to U_∞ exactly only at ∞. Because, it changes asymptotically. So, we can assume that the boundary layer thickness is the thickness at which the velocity becomes 99 percent of free stream velocity. Now, we can also find out or using a simple analysis at least for a laminar flows by considering the importance of inertial and viscous forces.

Because, in the boundary layer the viscous forces are important and you know because the flow Reynolds number is high the inertial forces are also important. So, by considering the equilibrium of 2 we can develop a order of magnitude analysis or we can do an order of magnitude analysis to find out this δ.

So, the inertial term let us say $\rho u \partial u / \partial x$ we can write this in terms of ρ and u we can scale as U_∞. So, because U_∞ is the velocity at the end of boundary layer or at the end of interface between boundary layer and the interface region in in and the outer region. So, $\rho U_\infty^2 d / x$ so x is the distance at any distance from the leading edge and the viscous force forces $\mu \partial^2 U / \partial y^2$ we can write this as μU as again U_∞ and y edge we can take a length scale equal to the boundary layer thickness this edge also called disturbance thickness.

So, $U_\infty / \delta^2 d$. Now, we can simplify this so we will get $\delta^2 d = \mu x / \rho U_\infty$. And we can multiply and divide by x so this becomes Re_x^{-1}. So basically, we can write that as $\delta^2 / x^2 = \mu / \rho U_\infty x$ which is 1 / Re x. And that gives us the δ / x or non-dimensional boundary layer is 1 / √ of Re x.

So, that also shows that from here we can see that δ is proportional to √ of x. So, that means as we go away from the leading edge the boundary layer in the laminar region grows as $x^{1/2}$ and this is valid for the laminar region. Because, we are considering the viscous forces in the laminar region this expression is not valid for a turbulent boundary layers.

And the exact solution for boundary layer was derived by Blasius and the expression that he got was $\delta / x = 4.91 \times 1 / \sqrt{Re\ x}$ or this 4.91 generally is taken as 5. So, δ y $x = 5 / \sqrt{Re\ x}$. So, we could found or we could find the dependence of δ/x / Re x with simple order of magnitude analysis and now we also know that what is the constant between this. So, for a laminar flow $\delta / x = 5 / \sqrt{Re\ x}$.

Now, that is one way of representing the thickness or the boundary layer thickness. The another way is what is called displacement thickness.

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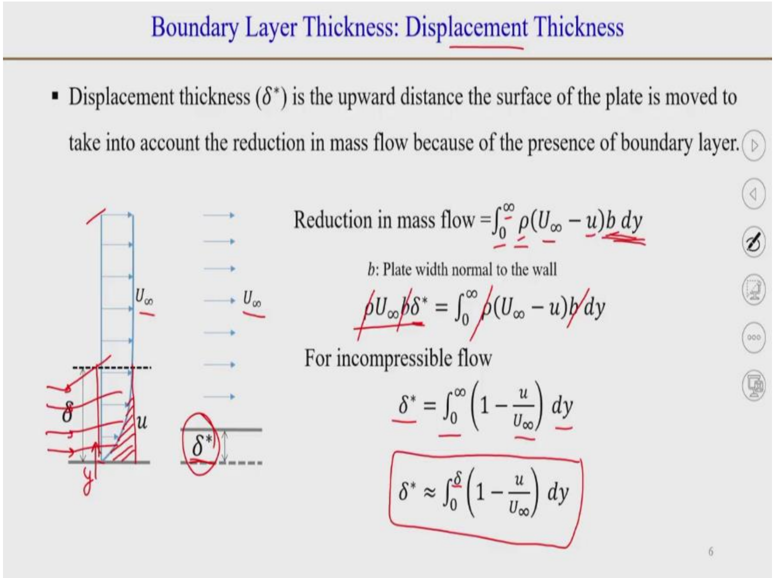

So, displacement thickness because when we have a flow in boundary layer there is a certain gradient in velocity. So, that means the flow that was coming parallel to the wall here it has to turned away. The flow is has to be turned away slightly away from the wall so that it can move into there are gradients in the boundary layers can be established. So, as I said earlier that the stream lines will be moving slightly away from the wall.

Now, that also causes a mass deficit so if you consider that the flow is coming with the uniform velocity U_∞ then you could have simply said that the mass flow rate is $\rho \times U_\infty \times$ the cross sectional area so you can take say integrate from y from 0 to say ∞ and multiplied by the width of the plate say b which is normal to the screen.

But, because of the presence of boundary layer this is not so the there is certain loss in mass because of the presence of boundary layer. So, you can see here that this velocity vector represents the magnitude of U and we have this distance moving as y so u dy this area under this area is where what is causing a loss of loss in mass. So, if we can quantify this loss in mass in terms of thickness that is what is done in terms of δ^* which we call displacement thickness.

So, displacement thickness is basically that if we assume that the flow is uniform and what we have done is that we have moved the plate by a certain distance and that certain distance is δ^* so that the flow everywhere is having a velocity U_∞. So, you can consider that in place of a presence of a boundary layer the plate has become thicker and by what magnitude is δ^*.

So, we can do a simple analysis that reduction in mass flow will be the mass flow rate would have been if there is no boundary layer would have been $\rho \times U_\infty \times b$ dy integrated from 0 to ∞. But, because we have a boundary layer, this will be ρ u b dy so the difference between the two will be integral 0 to ∞ ρ within bracket U_∞ - u b dy. And we know that above boundary layer u = U_∞ so this value will become 0 above say boundary layer thickness δ, b here is the plate width normal to the wall.

Now, we can equate this because we have said that the thickness by which we this mass flow rate can become taken into account by considering a thickness. So, if we consider that the flow is $\rho U_\infty b \delta$ so that will be equal to this mass deficit and ρ and b if the flow is considered to be incompressible they will cancel out. And we will get for an incompressible flow δ^* or displacement thickness = 0 to ∞ within bracket 1 - u / U_∞ dy.

Now, between δ to ∞ u = U_∞ so or we can approximate u = U_∞ so this term becomes 0. So, we can simplify this or approximate it the δ^* = integral 0 to δ where δ is the disturbance thickness or the boundary layer thickness the integration of 1 - u / U_∞ with respect to dy or with respect to y from 0 to δ. So, that is displacement thickness.

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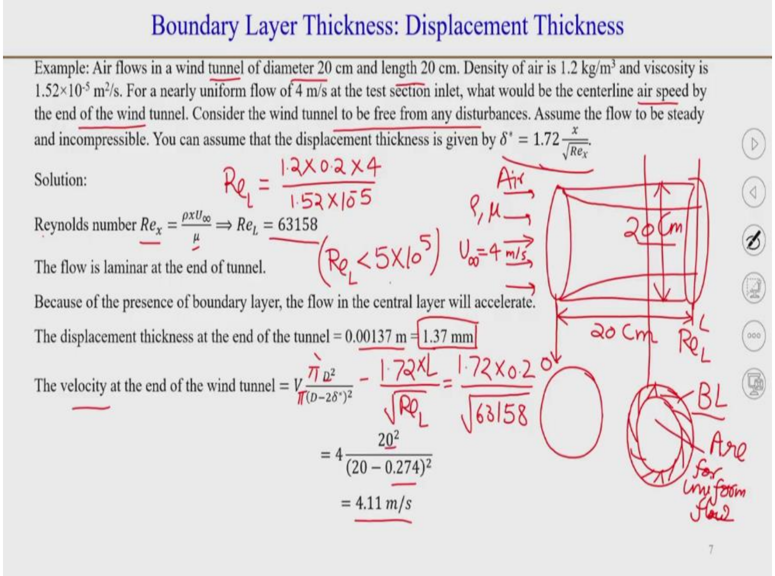

We can look a simple example here that how this can be useful in calculations. So, here a we consider a wind tunnel and the diameter of the wind tunnel and length is given. So, that means wind tunnel is a cylindrical kind of tunnel in this case and the diameter here is 20 centimetre and the length of this tunnel is also 20 centimetres.

Now, the air approaches this flow it approaches with say uniform velocity we have been given ρ and μ of air and the velocity here U_∞ you can say is given as 4 meters per second. And then we need to find that what would be the central line air speed by the end of internal. So, when the flow approaches this wind tunnel on the walls of this wind tunnel the boundary layer will grow from all sides.

So, if you look at a cross section this will be your cross section at entry and if you take a typical region or away from the in T section you will have a region where there is boundary layer in the flow. And the boundary layer will keep growing. So, first thing we can look into it that if the flow is laminar, if the flow is laminar, then we can use some concepts from that we have just learnt. So, we can first calculate the Reynolds number of the flow.

Now, the Reynolds number Re x is defined ρ x U_∞ /μ and the Reynolds number will be highest at the end of tunnel. So, we can calculate Re L and if we calculate Re L which = ρ is 1.2 × L is 20 centimetres so 0.2 meters × U_∞ which is 4 meters per second / 1.52 × 10^{-5}. And that number will probably come around 63000. And if we remember that Reynolds number less than 5 10^5 we can consider flow to be laminar.

So, it has also been suggested that we can consider that wind tunnel is free from any disturbances. So, if the flow is smooth then the flow is going to remain laminar in the wind tunnel. Now, the question is that when there is a boundary layer the area available for the uniform flow will keep decreasing as we go away from entry to the exit of this tunnel. And how this can be calculated easily is using displacement thickness because if we consider displacement thickness which again will be a function of x.

So, if we know the displacement formula and we have been given the formula for displacement thickness we could actually obtain it using the formula of δ which is for a laminar case δ / x = 5 / √ Re x and if we could substitute in the formula that we just write for displacement thickness we could obtain it. But, we have been given it directly so we could not use it here.

Now, we need to calculate that what is the centre line airspeed at the end of a tunnel. So, if we can calculate that what is the displacement thickness at the end of tunnel and from that we can calculate what is the area available for flow. Let us say this is area for uniform flow or or say potential flow in this outer region. Then we can calculate from mass balance that what is the how by how much the flow will be accelerated.

Now, we can calculate the displacement thickness and the displacement thickness δ* at L will be equal to 1.72 × L / √ Re L. So, we can substitute 1.72 × L is 0.2 √ of Re L which is 63158 and it comes out to be 0.00137 meter or 1.37 mm. So, you can compare or you in terms of number you can see here that this value is 1.37 mm which is displacement thickness. And if you look at this formula boundary layer is about thrice of it 5 / 5x / √ Re x.

So, boundary layer will be 3 times of it. So, in in this flow where the diameter is 20 centimetre and the length where the plate plate length is about 20 centimetres the boundary layer has grown by a thickness of 1.37 × 3. So, 4 1.4 × 3 about 4.2 mm. Now, we can calculate what is the area at the end of the wind tunnel and use A1 V1 = A2 V2 to find the velocity at the of at the end of wind tunnel.

So, that will be V × π D ^{2}d which is A1 V1 which is the mass not the mass but volumetric flow rate at the entry. So, V which is 4 meters per second × π D ^{2}d / π D - 2 δ ^{2}d. So, if we consider that the flow is uniform but it the wall has become thicker by a distance D - 2 δ is the diameter as we move away from the from the entry in the wind tunnel or is move in the wind tunnel.

So, when we substitute the numbers we can write this D is 20 because these numbers are to be cancelled out so I have written these in terms of centimetres itself so this is 20 centimetres and δ will be 0.137. But, you will have δ* and δ* so this will be 2 δ* so 0.137 centimetre × 2.274. So, when you solve it the velocity comes out to be 4.11 meters per second. So, that will be the velocity at the center line at the end of wind tunnel. So, this is a simple example where we see that how the displacement thickness can be useful.

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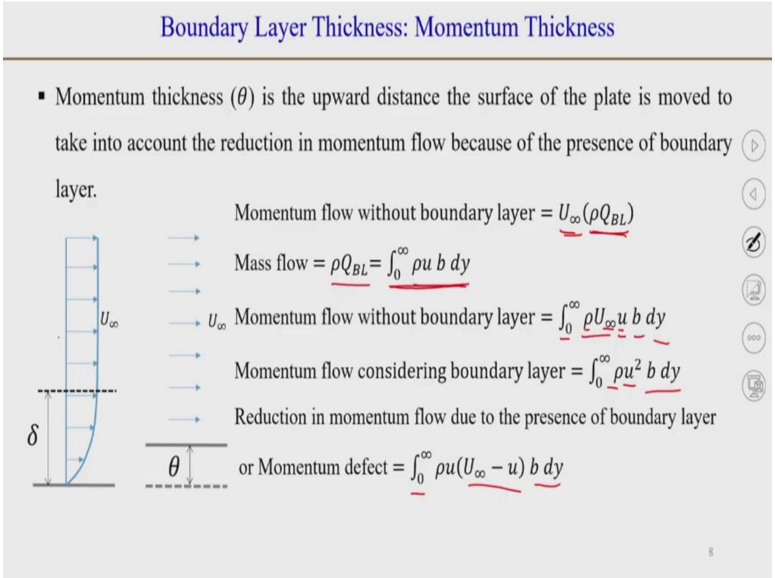

Now, along the same lines there is another thickness that is defined which is called momentum thickness. So, as we discussed that displacement thickness is the thickness by which or the distance by which the plate need to be moved up so that the momentum deficit because of the presence of boundary layer is taken into account and the flow above is uniform.

Here, what we consider that it is momentum deficit which is taken into account and the thickness or the distance by which the plate needs to be moved up so that the momentum deficit because of the presence of boundary layer is is taken into account the reduction in momentum is taken into account. So, again a similar schematic picture. Now, if there would have been no boundary layer us then the flow in this case would have been $U_\infty \times \rho \times Q$. So, this Q boundary layer it just refers to the flow in this region the flow we consider here is this.

So, basically ρ Q BL is the mass flow in this entire region from 0 to ∞ ρ u b dy. So, the thing to consider here is or thing to note here is that it is not ρU_∞ b dy we have considered the mass flow assuming that there is a boundary layer. So, the actual flow that is there over the plate is being taken into account actual mass flow so that is integral 0 to ∞ ρ u b dy.

Now, if all this mass has been moving with a velocity U_∞ that is the momentum flow without boundary layer. We can also substitute this ρ Q BL by this value so we can write U_∞ as a constant so this will be integral 0 to ∞ ρU_∞ u b dy. Now, if we consider boundary layer then in place of U_∞ what we will have is u.

So, this will be ρ u^{2}d b dy that will be the momentum flow considering boundary layer. And the defect momentum deficit or what is called momentum defect will be difference of these two. So, integral 0 to $_\infty$ ρ u U $_\infty$ - u × b dy so that is the momentum defect.

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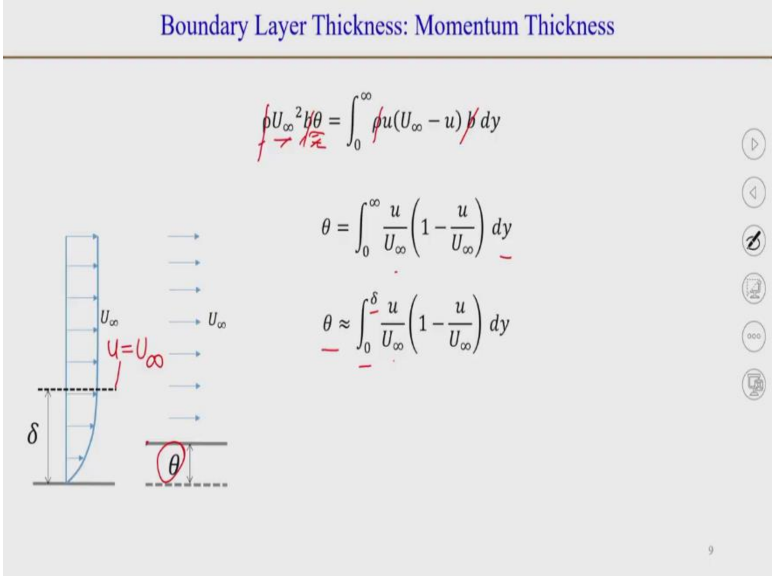

Now, we can equate this momentum defect by considering a thickness θ by which the plate needs to be move and this will be equal to θ into because when we will consider this if the flow is moving with the velocity U $_\infty$. So, ρ U $_\infty$ ^{2}d × b × θ and we can cancel out the terms such as ρ and b. So, this gives us θ = integral 0 to $_\infty$ u / U $_\infty$ × 1 - u / U $_\infty$ dy.

And above this boundary layer when we have a boundary layer beyond boundary layer in the outer region u = U $_\infty$ so we can again consider that θ = integral 0 to δ u / U $_\infty$ 1 - u / U $_\infty$ dy. So, momentum thickness is basically the distance by which the plate needs to be moved up so that the momentum deficit is taken into account or reduction in the momentum deficit is taken into account.

The important point to note in this is that we consider when we consider this mass flow it is not ρ U $_\infty$ ^{2}d b dy. It is ρ U $_\infty$ u b dy and this considers the actual mass flow that is there in the boundary layer it is only that this mass flow if it would have been moving with a velocity U $_\infty$.

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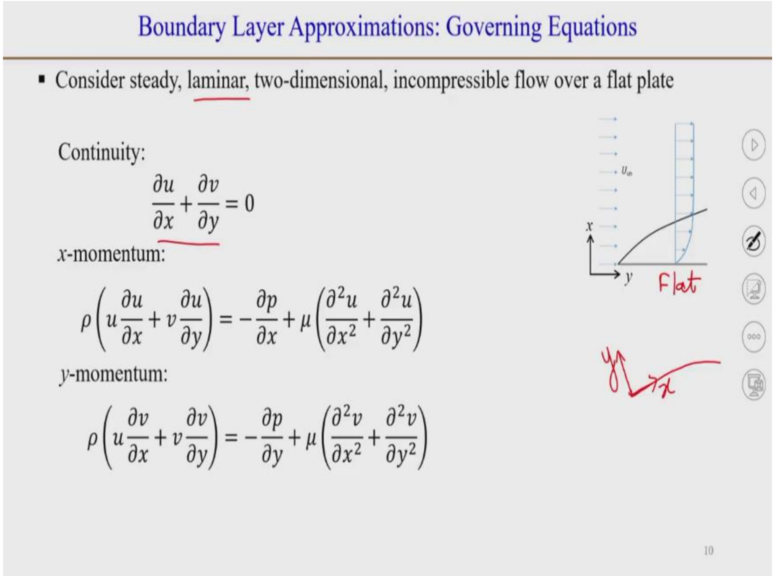

Now, we will see that we write down the Navier stokes equation considering a two-dimensional flow over a flat plate we have considered here a flat plate and the flow is laminar. So, these equations what we are going to write new stokes equations they will be valid for a laminar flow.

However, the plate we have considered flat here but it may well be a slightly curved plate and in that case our coordinate system will be that the x coordinate is along it and y coordinate is normal to it. And these equations will be valid until our radius of curvature of this curve curved plate is significantly higher than δ which is the boundary layer thickness.

So, we can write down the usual continuity equation which is for an incompressible flow ∇.V= 0 x momentum and y momentum equations.

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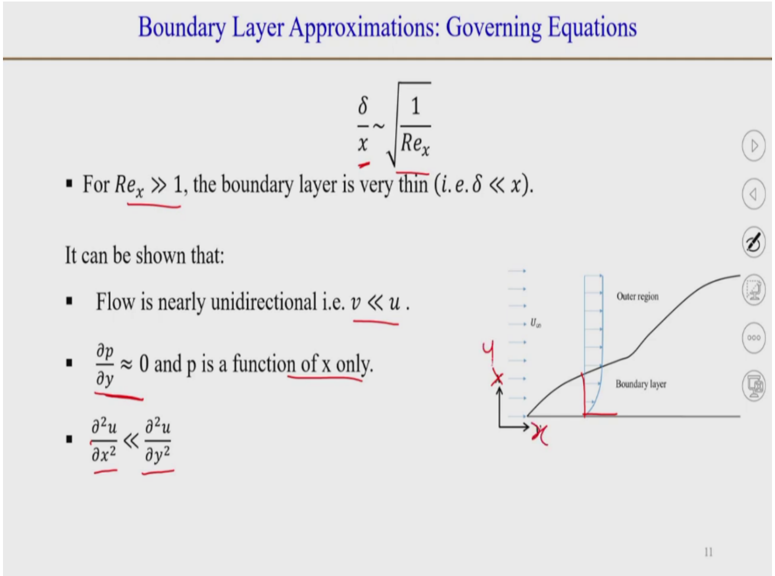

Now, as we saw that the boundary layer thickness is proportional to 1 / √ Re x. So, when the Reynolds number is high then the boundary layer is very thin. That means the y-coordinate is very small compared to the flow along the x direction. So, we can use this approximation and using this approximation we can so like we did for the lubrication approximation.

That the flow will be nearly unidirectional from continuity equation we can show that v is very very small when you compare it with u. And using y momentum equation it can be shown that all the terms in the y momentum equation are of the magnitude δ / L or less than it or δ / x and less than it.

So, δ ∂ p / ∂ y is negligible so that means that p is not a function of y so p is a function of x only. So, in place of ∂ p / ∂ x in the x momentum equation we can write dp / dx. The other thing to note here is that ∂ 2 u / ∂ x 2 is is significantly less than ∂ 2 u / ∂ y 2.

So, in the viscous term both the second derivative they they come in the viscous terms. So, the gradient or the ∂ 2 u / ∂ x 2 the gradients or the viscous term caused by this ∂ 2 u / ∂ x 2 is significantly less than ∂ 2 u / ∂ y 2. So, gradients in this direction in the y direction is significantly large than in these direction along the x direction this is, sorry, this should have been x and this is y.

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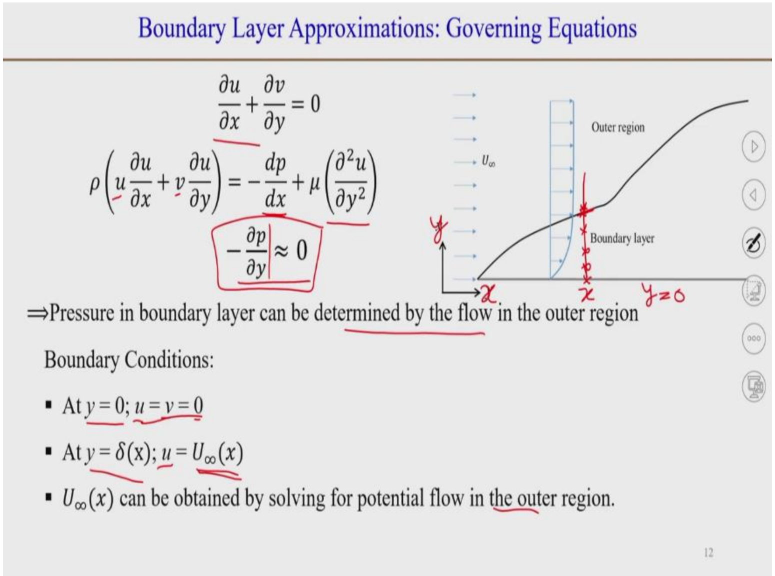

Again, we have the same mistake here so let us just correct it. Now, when we use these simplifications our equations become ∂ u / ∂ x plus ∂ y ∂ v / ∂ y = 0. So, continuity equation as it is the y momentum equation will reduce to ∂ p / ∂ y = 0. And as a result we can write this dp / dx = 0. You might notice that we have only one term in the viscous term because we could neglect ∂ 2 u / ∂ x 2 with respect to ∂ 2 u / ∂ y 2 and all this can be shown using order of magnitude analysis like we did for the lubrication approximation.

Now, so we see here that ∂ p / ∂ y = 0. So, that means that if we take a cross section at any section in in the boundary layer the pressure along y is constant at or same at each point. So, the pressure in the outer region is same as the pressure in the boundary layer. So, if we could solve the flow in the outer region then that pressure field that we obtained from the outer region that can be directly used for in this equation. So, the pressure field can be obtained from the flow solution for the outer region.

Now, we will have unknowns in this u and v because we can obtain pressure from the outer region. And so for these two unknowns and this equation basically is reduced we do not need it anymore because we have used it as dp / dx. So, we will have two equations and two unknowns in terms of u and v. And we will need the boundary conditions so the boundary condition at y = 0 u = v = 0. So, y = 0 is basically the wall so because of no slip boundary condition here the u and v component of velocity both of them are 0.

And at y = δ x so when we consider any x at the boundary layer or at y = δ x which is the interface between outer region and boundary layer u = U ∞ x. So, now we need what is U ∞ x? U ∞ x is the velocity in the outer region. So, this could again be obtained by potential flow solving in the outer region. So, p ∞ x or and U ∞ x can be used to obtain the solution of boundary layer flow and this way we can also couple the two solutions.

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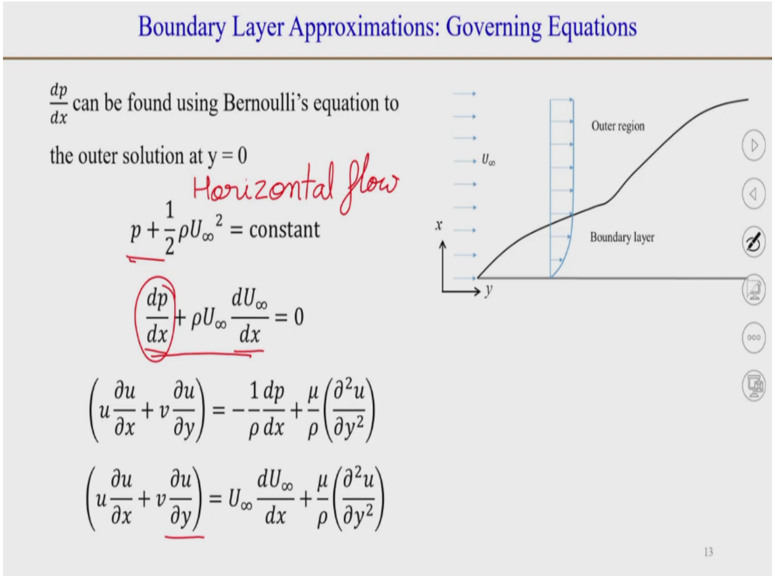

And if we can use Bernoulli's equation for a flat for the case of a flat plate so we can write this as for a horizontal flow where z is not changing. We can use Bernoulli's equation to write p plus 1/2ρ $U_\infty{}^2$d which is constant from Bernoulli. So, we can differentiate it with respect to x this will give us dp / dx plus 1/2 ρ to $U_\infty \times dU_\infty / \partial\, dU_\infty / dx$.

And 2 and 2 will cancel out so you will have this formula. Now, we can substitute dp / dx = -ρ $U_\infty\, dU_\infty / dx$ so we can do that and this will become our simplified equation. And now this along with continuity equation we can use to find the solution in the flow in boundary layers.

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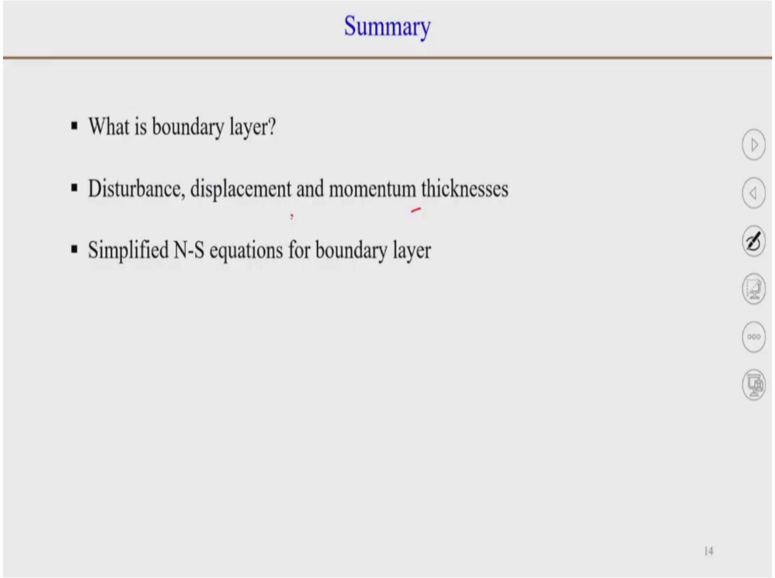

Now, with the lot of development in CFD and advances in computational power the use of boundary layer solutions or the use of the technique that one can decompose the flow in outer potential flow and the inner boundary layer flow has become limited. But, the technique gives us lot of physical insight in the flow we can see that if my solution is able to capture the flow over the boundary layer plus it can also help us in some quick calculations from back of the envelope calculations.

So, what we have studied in today's class is we looked at what is boundary layer and how it is important to explain sub some phenomena for example the absence of no slip boundary condition or or D'Alembert's numbers paradox when we just take boundary layer flow into account along with the outer potential flow.

We also looked at different ways to represent or estimate the thickness of boundary layers. So, disturbance thickness which was simply that at the distance at which u becomes U_∞, then displacement thickness which = when we take into account the mass deficit and momentum thickness when we take into account or by which the distance by which the plate can be moved to take into account the reduction in momentum defect or momentum deficit.

We also looked at the simplified Navier stokes equations that how we can simplify two dimensional Navier stokes equations in cardiac cartesian coordinates to find the solution for boundary layer flow. Blasius, he obtained the solution of these equations exact solution and from that he could calculate the value of δ which we saw earlier in the lecture that $\delta / x = 5 / \sqrt{Re_x}$ or 4.91 to be exact / $\sqrt{Re_x}$. So, we will stop here, thank you.