Fundamentals of Fluid Mechanics for Chemical and Biomedical Engineers
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Lecture 33
Bernoulli's Equation and Flow Measurement

Hello. In the previous lectures we discussed about inviscid flow and from the Euler's equations which are valid for an incompressible or in general for a inviscid flow, not necessarily incompressible, they can be for compressible as well as incompressible fluids. So, we derived those Euler's equations and we from the Euler's equations we also derived Bernoulli's equation.

And when we talk about Bernoulli's equation they can be used in a number of situations, of course, we need to remember the limitations and the assumptions involved and use them judiciously so that they give the correct results. There are some flow meters which are also or their operating principle or the way the flow rate and pressure drop is correlated is based on Bernoulli's equation.

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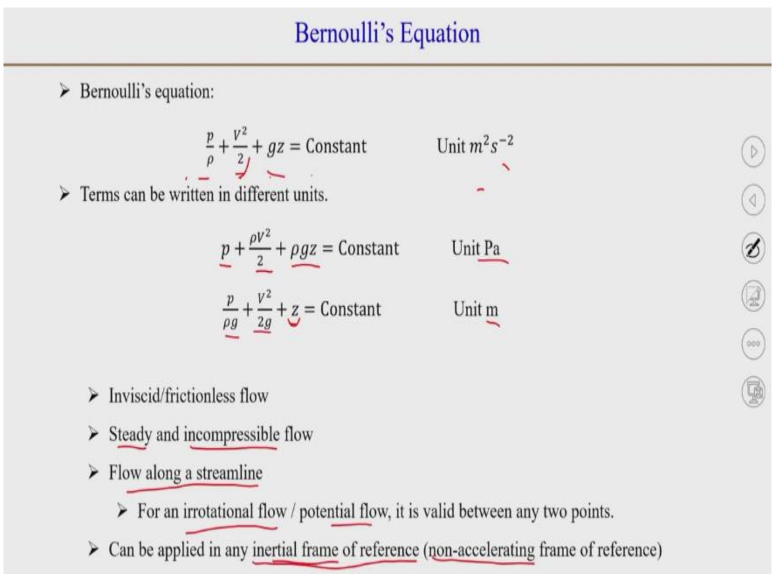

So, in today's lecture we will discuss the Bernoulli's equation, its assumptions, limitations and where can we apply it as well as some application in such flow meters. So, let us look at Bernoulli's equation first and we can write in different forms. For example, if we write it p/ρ $v^2/2$ + gz =constant. So, we have seen it from the momentum perspective. We can also see this in terms of energy that this is the pressure energy, kinetic energy and gz, or the gravitational or potential energy.

Now if we write it in this form, so each term let us say $V^2/2$, so meter per second^2 that means meter^2 per second^2. So, each term is in this form or we can write this in terms of energy per unit volume so to say. So, $p + \rho V^2 + \rho gz$ and you can see the unit here will be the unit of pressure so Pascal or you can write in terms of heads. So, this is what is called pressure head, velocity head and the gravity head or height.

So, the unit here is meter. You can use either of these forms, but we need to remember that when we are using the equation all terms should be consistent. It may not be the common mistake is that sometimes we are using z in this form, whereas we might be using $V^2/2$, not $V^2/2g$ and that causes a mistake. So, we need to remember that all the terms that we are using they are in the same units.

Now once we have looked at the form, we also need to remember or remind ourselves what are the assumptions. So, the flow is inviscid or frictionless flow, so any flow where the viscosity or viscous effects can be important there, we need to use it carefully, either we need to take into account the losses as we will see in number of cases which we solved, for example in pipe flow or we will need to see that the viscous effects are not very important.

So, there should be some way that you do take into account or assess the importance of friction in a particular flow where you are applying Bernoulli's equation. Then the flow is steady and flow is incompressible. So, these two assumptions we need to verify and when we are applying Bernoulli's equation then it is applicable for flow along a streamline except that when flow is irrotational or potential then it is valid between any two points. So, this we should always be careful about.

Then there is another term here, that it can be applied in any inertial frame of reference. Actually, that is also true say Reynolds transport theorem that we have derived, what we have discussed is that it can be applied in any inertial frame of reference and inertial frame of reference means that it is any non-accelerating frame of reference.

So, where the frame of reference is either stationary or if it is moving it is moving with a uniform velocity. So, if you remember when we discussed Reynolds transport theorem, we discussed Reynolds transport theorem when our control volume was fixed or we also discussed Reynolds transport theorem when our control volume was moving with a constant speed and the term V there was relative to the velocity relative to the control volume.

So, we have not discussed accelerating frame of reference in this course and we will not do that, but we need to remember that this Bernoulli’s theorem when we apply it is applicable only for inertial frame of reference and inertial frame of reference means that the frame of reference is not accelerating, so it is either fixed or moving with a constant speed.

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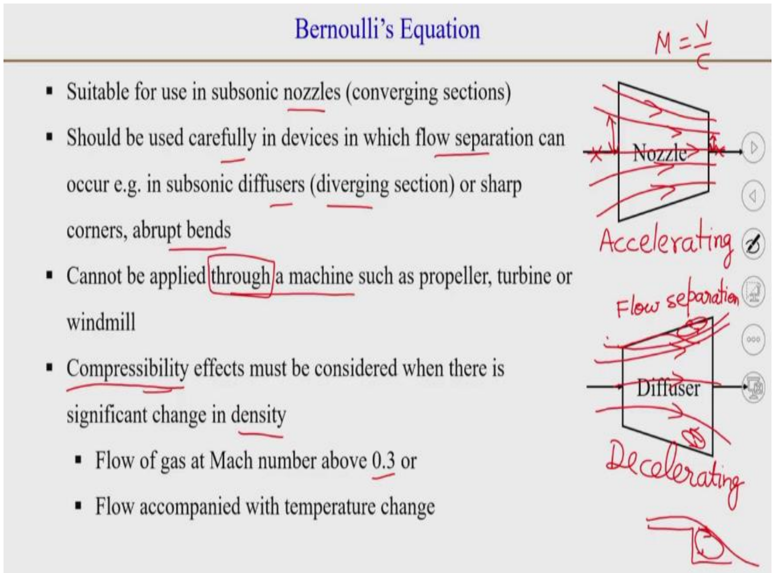

Now one of the common applications we can do it is in nozzles, so nozzles can be when the flow is subsonic. So subsonic means that the Mach number of the flow. If you remember the Mach number is basically M =V/C, where c is speed of the sound. So, if the velocity of the flow is less than velocity of the sound in that medium under the flow conditions at the same pressure and temperature then the flow is subsonic when Mach number is less than 1.

So, in the nozzles, you can apply Bernoulli's theorem when the flow is converging. So basically, that is what nozzle will look like for a subsonic flow and you can see the streamlines will be something like these. So, you can see here that the flow will accelerate and you can also see by the distance between streamlines here, see the streamlines when we discussed the stream function we talked about, the flow rate between two streamlines is a constant.

So, the distance between streamlines is high here or more here and low here. So that means because the flow rate is constant and the velocity is going to increase, and when the velocity increases in the downstream direction then from the Bernoulli’s theorem when you go downstream velocity increases and the pressure will decrease in the downstream direction. So, you can apply the Bernoulli's theorem or you can take say the streamline here and take two points.

Now a similar device like nozzle is diffuser, but it is in the nozzle it is a converging geometry whereas diffuser is a diverging geometry. So, the flow will expand here or the flow will decelerate here. In the nozzle flow will accelerate when we are talking about the subsonic flow is here. So, the flow is accelerating and, in the diffuser, flow will decelerate. So, the velocity will decrease and that means there will be increase in pressure.

So, when there is increase in pressure what may happen that the flow may not always be attached and you may see some recirculation happening near the walls. And so that means the flow separation may occur because because when the flow comes in, in the downstream direction pressure is increasing and at one point the pressure may increase to such a value that the inertia of the flow cannot overcome the pressure gradient and the flow does separate.

So that flow separation is will cause lot of losses and in such cases the the Bernoulli's equation, it is not prudent to use Bernoulli's equation. So, we should use carefully Bernoulli's equation in all cases where flow separation can occur. One, we discussed just now which is say diverging sections or diffuser. Other one, can be when you have sharp corners in the flow.

So, for example you have a step flow around a step and you may have the recirculation happening in this at higher Reynolds numbers. The other thing to remember is that the viscous effects are more important at low Reynolds numbers. So, you should be careful while using Bernoulli's theorem in at low Reynolds number or in laminar flow. So, in turbulent flows you might use it, of course with all these limitations that you need to be careful about sharp corners, bends, diffusers etcetera.

Then you cannot apply Bernoulli's equation through a machine such as a turbines or compressors, propellers, windmills because the Bernoulli' s equation is basically it says the energy of the flow remains constant. So, the energy in terms of pressure and the kinetic energy of the flow + potential energy of the flow, the sum of it is constant. But when you talk, when you are talking about a machine, it might be a propeller, turbine the vanes of turbine or the windmill.

So in in these cases there is some energy is being added or in some other machine energy might be taken out for example, so there is a change in energy because of the machine. So, you cannot use Bernoulli's theorem through the machine at the inlet and outlet. You cannot take one point at the inlet of the machine.

You cannot take another point out at the outlet of machine, because there is some addition or removal of energy in this machine, so we should not be using through the machine. Of course, you could use it, that your both the points are upstream of the machine, or your both the points are downstream of the machines then with by taking care of other points, you could apply Bernoulli's theorem, but you should not use it through a machine.

Now because the assumption is that the flow is incompressible, so when the compressibility effect might come into picture then we should not use it, and when the compressibility effect will come into picture, when the density can change. So that can happen when you have for example a high Mach number flows as we just discussed. So generally typical value that we take that if the Mach number is above 0.3, then the flow becomes compressible. So, Mach number should be less than 0.3.

And if there are significant changes in pressure or temperature that they might change the density significantly because density of a gas, for example, for an ideal gas you have $\rho = PM/RT$, so it is a function of pressure and temperature or for real gases also they are strong function of pressure and temperature. So, if there are significant changes in pressure and temperature then also the flow may be compressible. So, you should take into account all these factors when applying Bernoulli's equation.

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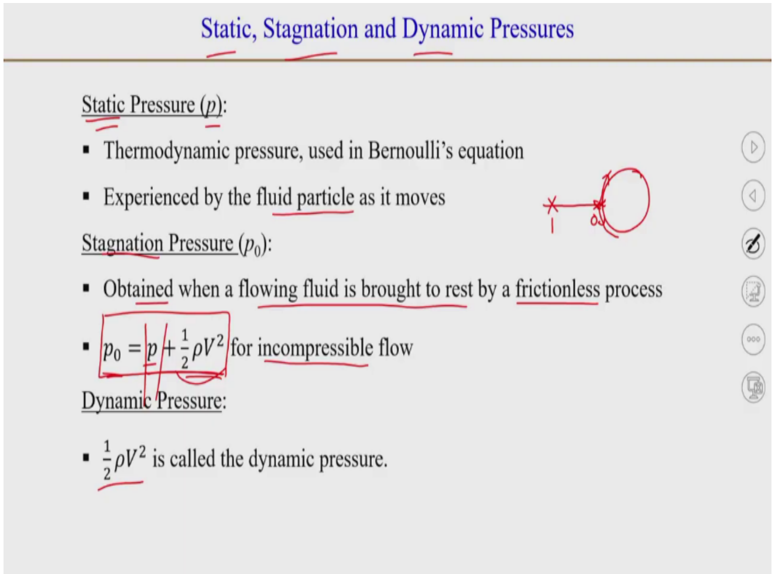

Now we have talked about the pressures, we have been using the term pressure from the very first class of this course and even before it, we used pressure in our colloquial language and then you have different terms here, say static pressure, stagnation pressure and dynamic pressure. So, we will just try to remember or try to remind ourselves what these terms mean?

Static pressure is basically the thermodynamic pressure. So, when we talk about pressure that there is molecular motion of the molecule, the molecules are moving randomly in a fluid depending on if it is liquid, then their movement is relatively restricted, if it is gas then they are free to move, they are more free to move around.

Now if you put a surface there the molecules will be hitting the surface and from that the force per unit area or the normal force per unit area is what you call pressure from thermodynamics. So static pressure represents that pressure and it is called thermodynamics pressure. So static pressure is generally the pressure that we are referring to. Of course, we have also talked about in static pressure that you can talk about absolute pressure and gauge pressure.

So absolute pressure is the real value of pressure and gauge pressure can be or gauge pressure is the pressure with respect to atmosphere. So, the static pressure or absolute pressure - atmospheric pressure is what is you call gauge pressure. In some cases, you might also use the term vacuum pressure, which is how much if the pressure is lower than atmospheric pressure, then the difference between the atmospheric pressure and the absolute pressure that you generally call a vacuum pressure. So static pressure is basically thermodynamic pressure and it is the actual pressure that the particle experience as it moves.

Now there are another terms, so other term is what is called stagnation pressure. So, stagnation pressure, there are actually the term static and stagnation, they have about similar meaning, static also that something becomes stationary. But the static pressure has nothing to do here with static term. The stagnation actually means that if the flow is brought to rest, so stagnation pressure is the pressure when a flowing fluid is brought to rest by a frictionless process.

So, there is no frictional losses and if the flow comes to rest, then the pressure is called stagnation pressure. So, if you consider a flow, let us say flow around a sphere when when the flow approaches at this point and this point is what we call stagnation point, because the flow, the velocity at this point becomes 0, just just next to the wall and then the flow turns here.

So, the fluid is at rest here, the pressure here will be stagnation pressure or the gravity is same. So, if you take point, two points here, let us say point 1 and point 0 here, you can write that p0 + 1/2 ρV^2 and the velocity is 0 there. So that term will be 0. So, you will have the stagnation pressure which is being represented as p0 or p_o, that will be equal to p + 1/2 ρV^2, where p is the static pressure. So, this is for an incompressible flow. So, for an incompressible flow we can define stagnation pressure as sum of p + 1/2 ρV^2.

Now, you have a static pressure which is because of p or p is what is called static pressure and ρV^2 is this term comes because of the motion of the fluid. So, this is known as dynamic pressure. So, 1/2 ρV^2 is known as dynamic pressure. So, stagnation pressure, it is also called total pressure is the sum of static pressure and dynamic pressure. So, we should remember these three terms and know that when to use which term.

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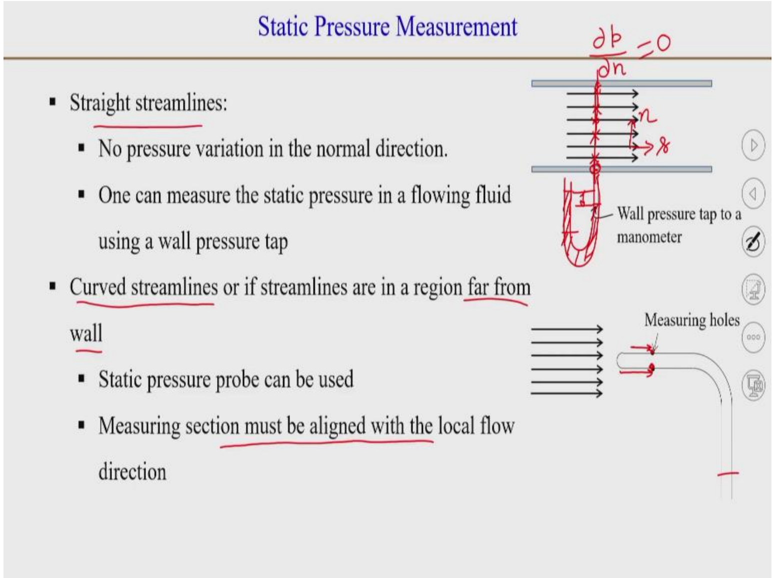

Now as I said that we will discuss some measurements also here. So, one can measure static pressure by simply in a flow where the streamlines are straight. So, as we discussed, when we discussed Euler's equation that if the streamlines are straight then ∂ p/∂ y or ∂ p/∂ n, when we talked about the in the streamline coordinates that was a function of radius of curvature.

So, if radius of curvature is infinite or the curvature of the streamlines is 0, then in such cases ∂ p/∂n will be 0, or ∂ p/∂ y if you want to use Cartesian coordinates. So, remember what we talked about s is the coordinate along the streamline and n is the coordinate normal to the streamline. So, when the streamlines are straight in the flow, then the pressure you take at this section normal to the streamline pressure at all these points are same.

So, if we can make a small hole and put a pressure tap here in the wall and connect it with a manometer, for example, so by measuring the pressure differential, we can measure the by measuring the difference in pressure here we can actually find what is the pressure at any point, because if you want to find pressure at this point, you can very well measure it at the wall, because the pressure at the wall and anywhere normal to the wall at this section everywhere it is same. So, we can do and the pressure, static pressure in a flow can be measured. So there one can use a wall pressure tap.

On the other hand, if the streamlines are curved, then or the flow is in a big region where there are no walls present, so flow is far away from the wall or the streamlines are curved then we will not have this luxury that the normal to the streamlines, this will not be constant then we can use what is called static pressure probe.

So, you can use a probe which is aligned with the flow and you can measure pressure. So, the flow is happening just parallel to, so probe is put in such a manner that the flow is parallel to this probe and you have the taps here or here. So, from these taps, you can connect it to manometer, and you can measure the pressure or static pressure there.

Now the thing here is that this probe, it will because it is being put in the flow, so it will affect the flow. So, your stem should be slightly away from the measurement points, so that it does not interfere with the flow. The other point is that your measurement section should be aligned with the local flow direction, so this measurement section should be aligned with the flow direction, then only you will be able to measure the correct values, because then only you will be measuring the static pressure. So that is the care that you need to take.

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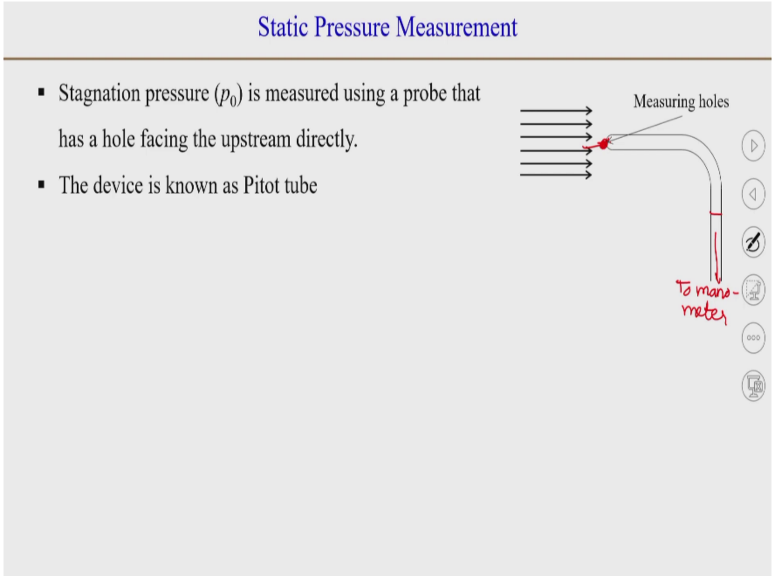

Now you can measure stagnation pressure. So, to measure stagnation pressure, you can put a probe in the flow and if you measure the pressure at the point at the front, so which will act as a stagnation point because this the flow may enter from here, but remember that this is connected to a manometer or to the hands of a manometer.

So, this is not going a flow, I mean this will one can sense the pressure here. So, the flow need to come to stop at this particular point and this device is known as Pitot tube, and if you observe this is found in, they use also it to measure the wind speed or in aeroplanes to measure the speed of the aeroplane. So, this measures, the stagnation pressure directly.

So, if your measurement port or the measuring hole from where you get in touch or your device, your manometer is in touch with the flow or is in contact with the flow, if it is at this point then the flow will come or flow will come to a halt or flow will become stagnated at this particular point.

Now when we talked about in the previous slide here, because the streamlines you have to put it in such a manner that the flow is parallel to streamlines. So, there is no flow through this. So that means it will measure the static pressure at that particular point. It does not interfere the velocity. That is why it is important that it should be placed parallel, your probe, when you are measuring static pressure using this static pressure probe, it should be aligned with the flow, that it does not interfere or does not alter the streamlines. It should be parallel to the streamlines, the the probe.

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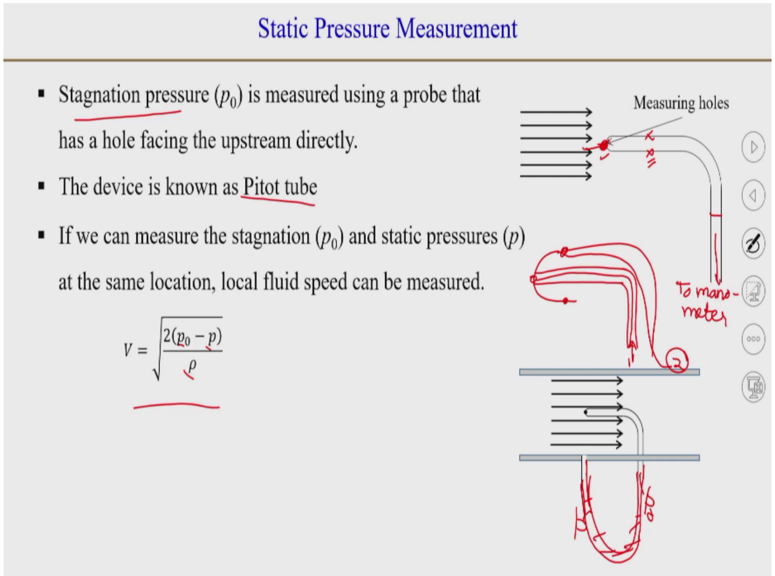

In this case, you can use the manometer again. It should be aligned so that the flow which is coming here it can be stopped and you can measure the stagnation pressure. And this device is known as Pitot tube. So, what we have seen is that you can measure stagnation pressure using the Pitot tube.

If we measure, so if in our tube we have both the holes or both the points here, that we have measurement holes here and you can have holes here, and if you can connect both of them with a differential manometer then you will be able to measure the static pressure and stagnation pressure and you know that they are related with p0 =p + 1/2 ρ V^2. So, if you know p0 and p, you can find V knowing the density.

So that is the formula if you do a little bit of algebra that V =√ 2, difference of stagnation pressure and static pressure divided by the density. And you can measure the local flow velocity. So, if you have an arrangement where the flow is or the streamlines are parallel, you can have a wall pressure tap here, where you measure static pressure and stagnation pressure or Pitot tube where you can measure the p0 and you can connect the two using a manometer.

So, from that you will be able to find out that the the pressure difference. Once you get the pressure difference, you can calculate V or you can have in the same tube, you can make an arrangement for example, if you have a tube and you can put a probe which is going to one arm of manometer and you can have the holes and this is going to other arm of the manometer. So, you can connect the two with two arms of manometer. And you can measure the difference between the two and you can find the pressure difference between stagnation and static pressure. So, either the, either way it can be used.

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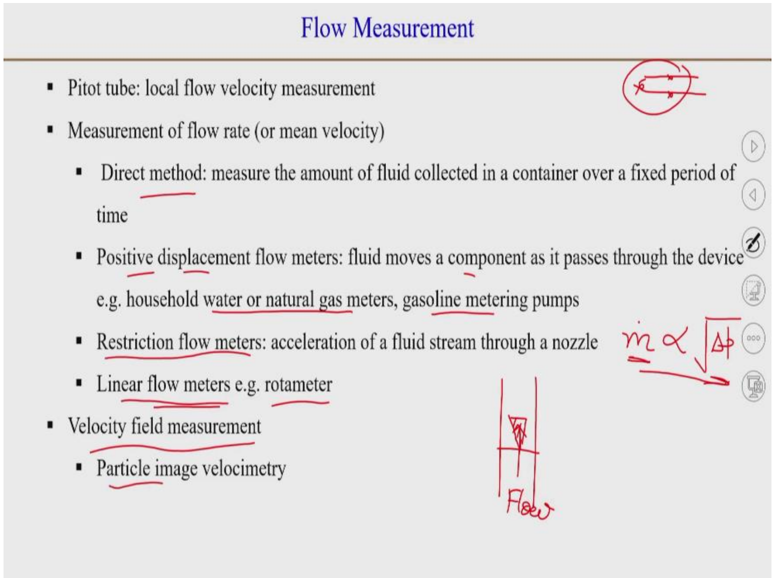

Now using a Pitot tube, we can measure the flow velocity. Now apart from that, so Pitot tube what it gives is, because you are looking at the flow velocity in Pitot tube at a particular point. So, you will get the flow velocity at this local point. So, Pitot tube give you the local flow velocity. If you want to measure flow rate, which is the general requirement, whenever you want to do any experiment or any flow, the general requirement is that you want to measure the flow rate.

So, a flow rate measurement is quite important, and the simplest thing one can think of that, if I have a vessel and I can collect the fluid flowing let us say for three or four minutes and I can see that how much fluid has been collected and by measuring the volume of the fluid collected, because the flow is incompressible, all the fluids that we are talking about here are incompressible fluids.

So, for incompressible fluid, we can measure the volume and we can know the time, so in a known time the volume is known V/t is what we will know as Q. The assumption here is that the flow is steady, so the flow rate is constant with respect to time, it does not change. So, we can measure it directly and the better we can measure, the measurement is free from errors, then it is direct, you can measuring actually the flow rate.

Now there is another set of flow meters which are called positive displacement flow meters, in which the fluid moves one of the components of the device, for example, in our household water, so the movement of flow is is moving a device. So, the flow rate is proportional to the device movement and that is what is used or the same principle is used in water or natural gas flow meters or in petrol pumps, the same principle is used in gasoline measurement or the petrol flow measurement.

Now what we are going to discuss here, or the focus that we have here is what is called restriction flow meters. So, we can by putting a restriction in a flow which is moving steadily, we can put a restriction in the flow. So, the flow may accelerate first, because the area of the flow will decrease and then when it moves away from this restriction, it will again expand and it will decelerate.

And that will cause, because there is a change in velocity, so from the Bernoulli's principle that will cause the change in pressure. So, we can measure that change in pressure and relate it with a change in flow rate or mass flow rate. So, in restriction flow meter as we will see in few minutes that the mass flow rate is generally proportional to $\sqrt{\Delta p}$, and they are quite commonly used.

You can use different type of restriction depending on your requirements, depending on the cost of the equipment and depending, the running cost. So, you can use different elements for a restriction flow meter. And their principle is dependent on as I said on Bernoulli's principle, so we will discuss them in detail.

Here the point is that the mass flow rate is proportional to $\sqrt{\Delta p}$, so you will not be able to measure it over a wide range, because you will be restricted, that the large range of flow cannot be measured by using one manometer, you cannot have a very, very large range. Generally, it is 1 is to 4 range that these flow meters can provide.

So, another set of flow meters, which you find very commonly in chemical engineering or even in biomedical engineering applications which are called rotameters and they are linear flow meters. So generally, in rotameters you will have a small device or a small cone or a spherical ball, which will act as a indicator of the flow and the flow comes in and because of the flow this restriction to the flow or this small sphere or cylinder or conical device, it will be moving with the flow, and because this will be being dragged, so let us say the flow is coming in from this direction or from bottom to top and it is installed vertically.

So, there will be a drag and there will be buoyancy and gravity force. So, from the balance one can find, because drag is proportional to velocity or drag is a function of velocity. So, the movement of this body will indicate the flow measurement and here the movement of this body is proportional to the flow rate. So that is why it is, they are linearly related. So, it is called linear flow meter. And one common example is rotameter there are other examples also, say vortex flow meters which are linear type of flow meters.

Now when we talk about flow measurements, pitot tube I said that it gives us the local velocity or we can measure the local velocity using Pitot tubes. Using these flow meters, we will be able to measure flow rate, volumetric flow rate or mass flow rate for an incompressible, so they are related by density which is a constant, so you can measure either.

Now in some cases especially, let us say, if you want to develop or troubleshoot something then you will need to get the detailed velocity field, for example. So, if you have solved your system of equations using computational fluid dynamics and then you need to verify your computational fluid dynamics or CFD simulation, then how do you do that? Because the flow rate will give you a macroscopic parameter, just the one quantity. You will not get what is my velocity profile or what is local velocity at a particular region of interest.

Similarly, pitot tube will give you velocity at a particular location only, but if you want to measure the velocity in the entire domain or in specific region in the domain then what you could use is the techniques which are relatively new and you can use these techniques for velocity field measurements. So, the idea here is this is one technique which is called particle image velocimetry.

So, idea in particle image velocimetry is that you introduce some particles in the flow and these particles are so small that they are, the flow around them is Stokes flow and they they relax immediately. That means they take the velocity of the flow very quickly. So, when you compare the flow timescale and the timescale of the relaxation of these particles, by relaxation, I mean that how much time it takes for the particle to adjust to the change in fluid velocity.

So, if the relaxation time is very small, so the particles can quickly assume the velocity of the fluid then if I measure the velocity of the particles which are basically the tracer particles which you call that they can trace the velocity of the fluid, then by measuring the velocity of these particles, I can measure the velocity of the fluid.

Now once we need to do it, so how will we do it? We can introduce the particles in the fluid and take images of the flow at two instance. So, you can take one image at time t another image at time $t + \Delta t$ and if you Δ, if your Δt is very small of the order of few microseconds or few nanoseconds depends in, depending on what velocity you want to measure, then your by measuring the or by taking the images at time t at time $t + \Delta t$, you can find out the displacement of these particles.

Now displacement of particle, it will depend that what is the particle density that how many particles are suspended in the fluid in your, in your image area, you can use different techniques which we are not going to talk about. But the idea is that you take the images of the flow at two time instance at time t time $t + \Delta t$, Δt being very small. And you measure the displacement of these particles which follow the fluid faithfully in time Δt. So, let us say this is Δx, and $\Delta x/\Delta t$ will give you the local velocity at that particular point, at that particular time instance. So, this is something which is being used very widely.

The other technique, the same technique, but there the advantage is that when you want to take pictures, your the channel in which you take the image of the flow, it needs to be transparent. Then only you will be able to take the pictures if your pipe generally is curved, then you need to put in some technique so that the pipe curvature effect is nullified or minimized. So, all the challenges come into particle image velocimetry.

What if your pipe is opaque and you cannot make it of a transparent material say glass or PDMS or other polymers, then in such cases you can use say, what is called radioactive particle tracking, because radioactive particles, they can or the radio radiation radioactive radiation, they can pass through the non-transparent materials or the materials which are not optically transparent, they can be transparent for radioactive radiations.

So, in such cases, one can use a radioactive particle and this particle is follow supposed to follow the flow faithfully. Of course, you will not use number of particles, but you will take one radioactive particle and flow it for a long enough time that it will cover the entire flow domain, and from that you will be able to find the velocity field. And this technique is called radioactive particle tracking.

There are other techniques for example Laser Doppler or Anemometry and Laser Doppler, velocimetry from which using which you can measure the particle or you can measure the velocity of the flow. So, these such techniques particle image velocimetry etcetera they provide the detailed information. So, for example, they can provide you the solution of the Navier-Stokes equation or you can compare the solution of Navier-Stokes equation from these. Whereas the flow meters, for example, they will provide you the macroscopic quantities such as flow rates.

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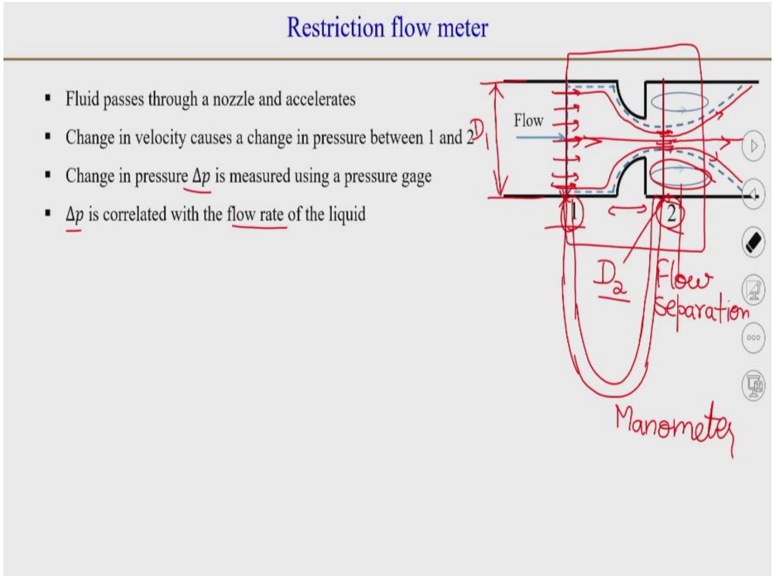

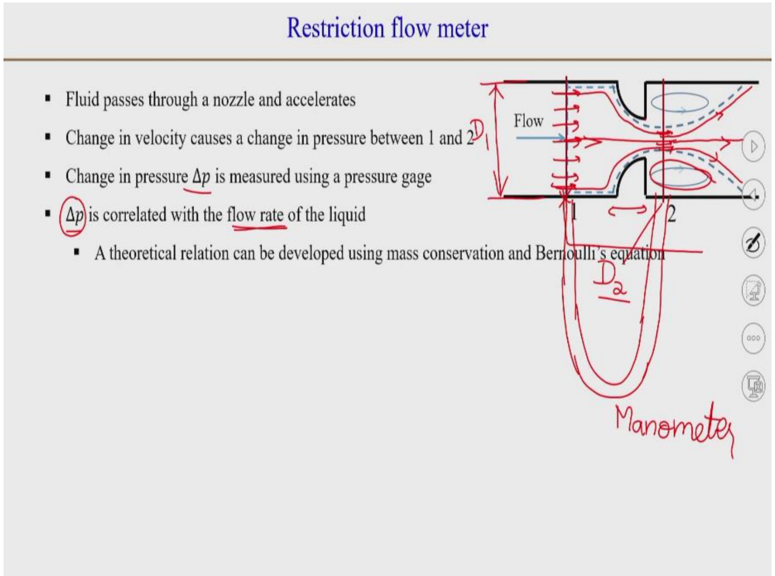

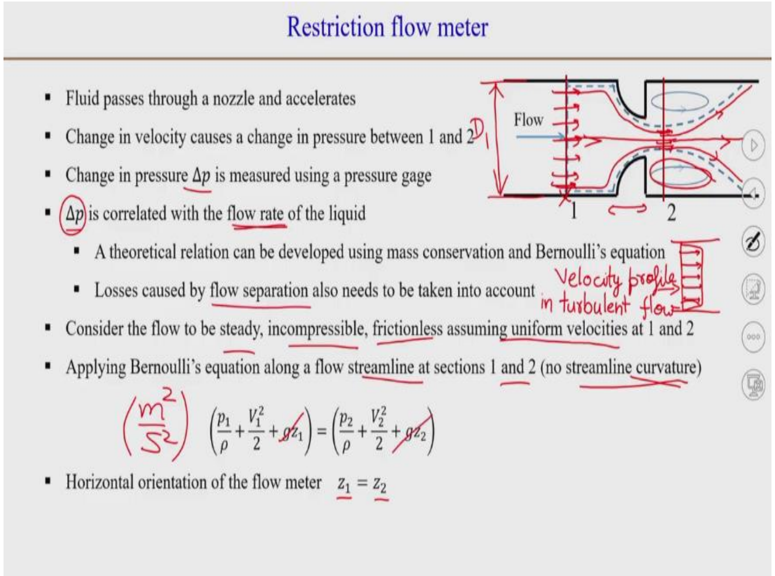

So let us come to this restriction flow meter. So, in the restriction flow meter, basically the idea is that this is your pipe where the flow is approaching and you install a restriction in this region here. So, this restriction here you have a nozzle kind of thing here. So, the flow will, as you can see from here that the flow streamlines will be passing through and the and the flow will accelerate, the streamlines will come closer, when when the flow goes through here. And then the flow when then the flow will expand again.

So, if your flow velocity is high, then you will also have or you may have what is called flow separation at this point. So, we need to remember here that we are using or we will be using Bernoulli's equation and we will need to take into account somehow that what is the loss in energy that is being done by flow separation and that loss has to be taken into account in some manner.

Now when the flow passes through this nozzle, you can see that the flow area it decreases, and depending on the Reynolds number and this area ratio so if let us say if this is a diameter D1 at this point and this point where the diameter is minimal, we call, let us call it diameter D2 and when a flow streamline, the streamline that is passing through the center will be straight. So, we can use the flow here or we can we can measure the pressure at these two points, point 1 and point 2.

The advantage of this that at this point the streamlines will be parallel. Similarly, the throat where the flow will be passing the streamlines will be parallel to each other. So, if you can measure the pressure at point 1 and point 2, let us say you put a wall tap or tap here, a tap here and connect it through a manometer. So, you will be able to measure p1 - p2 here.

Now we can apply Bernoulli's equation between point 1 and point 2 and assume or when we install such a restriction flow meter, we will install in a horizontal manner. So z1 and z2, they are equal. So, when the flow accelerates and because of that there will be change in velocity between point 1 and point 2, and that change in velocity will also cause a change in pressure, what we are going to measure is change in pressure Δ p.

And if we can measure or we can correlate this Δ p with the flow rate of the fluid then we can obtain, because as I showed that we can we can measure pressure between point 1 and point 2 or the pressure difference between point 1 and point 2. So, by measuring the pressure difference between point 1 and point 2, we will be able to measure the mass flow rate or volumetric flow rate.

Now we can develop a theoretical relation using Bernoulli's equation coupled with mass conservation or the continuity equation A1, V1 =A2, V2. So let us do this, we can apply the Bernoulli's equation between point 1 and point 2 along a streamline which is passing through the axis. So, and as I said that there is flow separation. So, once we derived the relationship, we will have a coefficient which will take into account the non-idealities or in the flow.

So let us consider that the flow is steady, flow is incompressible and it is frictionless. So, we are assuming flow to be frictionless, we are also assuming that the flow velocity is uniform. So now remember that when we talk about flow in pipe, because this will be installed, the restriction flow meter will be generally installed when you have flow in pipes. So, if the flow is laminar, then the velocity profile is parabolic and the assumption is of uniform velocity is a gross assumption.

Most of the time when you talk about chemical engineering applications in the plants, the flow, pipe is larger, few inches and the velocity is high. So, the flow is often turbulent. Now when you have flow to be turbulent as we will discuss later on, the velocity profile in turbulent flows is something like this. So that means there is some effect of wall near the wall, so you have the gradients larger near the wall, so this is, this is the velocity profile in turbulent flow.

So, the assumption of uniform velocity is quite valid in the turbulent flows because the velocity is uniform except near the walls. And we will apply Bernoulli's equation along a, along a flow streamline at section 1 and 2, and there is streamline curvature, remember that at point 1 we have assumed the uniform velocity. So, there is no flow streamline, there is no curvature in the streamlines.

And here we are considering the throat area where the area is minimum. So even if the streamlines will be curved when they are coming in or when we are going out in the throat region, in the smallest area region. But at the throat they will be parallel to each other. So, there is no curvature in the streamlines.

And we can use Bernoulli's equation, $p1/\rho + V1^2\ 2 + gz1$, so you see here the unit here is $meter^2$ per $second^2$. $p1/\rho + V1y^2$, $V1^2/2 + gz1 = p2/\rho + V2^2/2 + gz2$. Now if the orientation of this flow meter is horizontal, so that z1 =z2. And these two terms will cancel out and we will simply have $p1/\rho + V1^2/2 = p2/\rho\ V2^2/2$.

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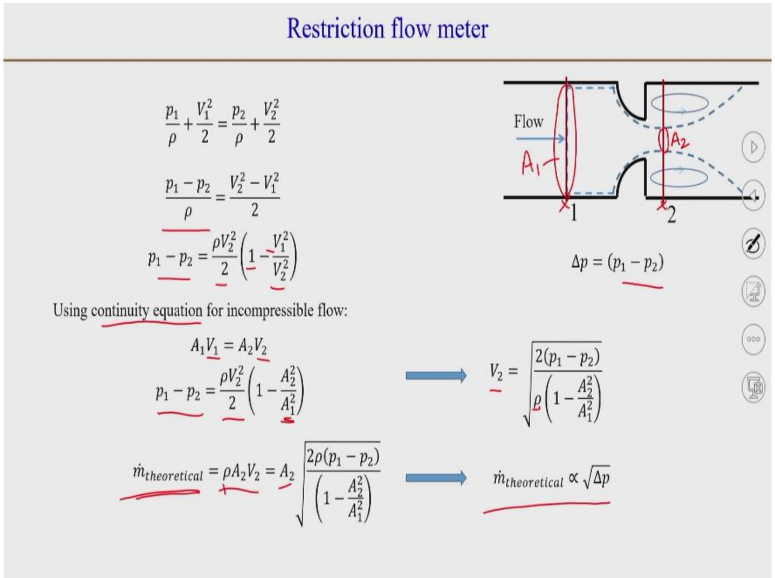

Now what we need to find is is the flow rate, so we can relate it that the pressure difference is already known to us. So, we can rearrange this equation in terms of pressure difference, p1 - p2, which is what we will measure Δ p, p1 - p2. We will put a pressure tap here, a pressure tap here and connect it with a manometer. So, we will be able to measure the pressure difference.

Now we have $V2^2$ and $V1^2$ and we can relate those with A1V1 =A2V2, which will be simplified form of continuity equation for this incompressible flow. So, we can write say, V1 in terms of V2. So p1 - p2 from here is $\rho\ V2^2$, $V2^2$ comes out of the bracket. So, we will have $1 - V1^2 / V2^2$. And we can write V1/V2 =A2/A1.

So, it becomes p1 - p2 =ρ $V2^2$ × 1 - $A2^2$ /$A1^2$. So A2 is the area, cross-sectional area here. This is, sorry A1, and at the throat region, this is the cross-sectional area A2. So now these are, A1 is known to us because it is a geometrical parameter, it is the cross-sectional area of the pipe.

Now A2 is a tricky business, again, because the location of this point will change if you change the Reynolds number of the flow, then the location of this point will change, it will of course also depend on what kind of restriction are we using, and that will also cause that you might be, because you will be using pressure, you will be measuring pressure at one particular point. So, if this location of A2 changes then you might also not be measuring pressure at point p2. So, this is another limitation of such flow meters.

So, we can measure A1, or we know A1 from the geometry or the dimension of the pipe, A2 is something that we need to relate somehow with a geometrical parameter. So, we can write this in terms of V2, so V2 is the velocity here, √2, Δ p /ρ × 1 - $A2^2$ /$A1^2$. And if we want to measure the mass flow rate, so mass flow rate will be equal to ρ A2 × V2.

So, you can substitute value of V2 here and this when multiplied by ρ, ρ will cancel out and you will have a √ρ coming in in the numerator. So that will be A2 × √2 ρ p1 - p2 /1 - $A2^2$ /$A1^2$. So that is theoretical mass mass flow rate. So, this gives us that the mass flow rate is proportional to √p1 - p2 or √Δ p. So, as I said earlier, that in restriction flow meters, the mass flow rate is proportional to √pressure difference between these two points.

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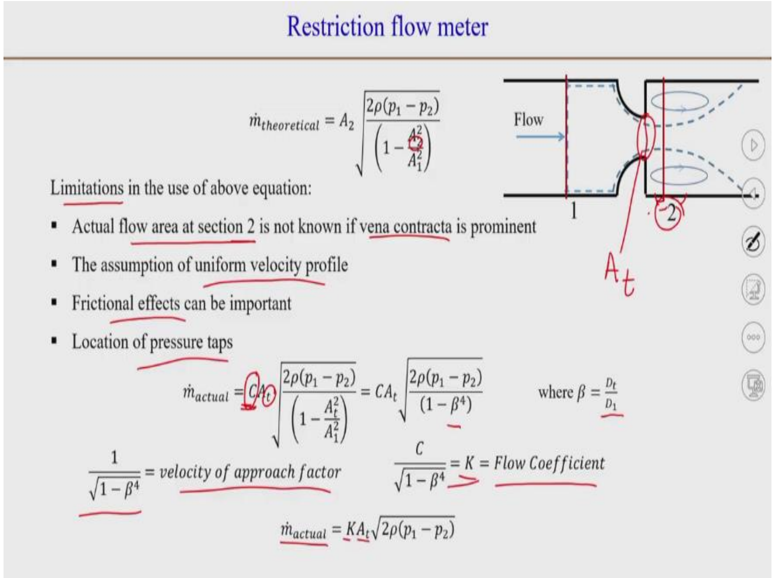

Now let us list down all the limitations that this equation has. This is a theoretical equation and there are certain assumptions we have made. Actual flow area at section 2 is not known, if vena contracta is prominent. So, vena contracta is nothing but the the region where the flow contracted. So, if there is this this contraction is dominant, then you will not know where actually flow area is, it may not happen at the throat region, but downstream of the throat and and it will be a function of Reynolds number, obviously.

So, uncertainty in section 2 or area of section 2, because that is what is going to come into picture in our formula. Then we have assumption of uniform velocity profile. There are, the velocity if the flow is turbulent, then you will have a uniform velocity profile in the bulk of the region except near the walls. So that is another assumption. If the flow is laminar, then you will have parabolic velocity profile.

Now the frictional effects can be important, so we have neglected or we have assumed the flow to be frictionless. So that is another assumption. And the location of pressure taps, so we will put a pressure tap at downstream of this throat region, but depending on the on the flow, the minimum or the section 2, minimum throat area may move at different places. So that is another limitation of it.

So, take into account of all this, a constant is introduced which is C, and in place of A2 we can use A throat, so this is a geometrical parameter. This is the area which is A throat. So, we can assume that A2 =At. So, we know the geometrical parameter and A2 is known and all the non-idealities can be brought into in this constant which is C. So, we can write this in terms of area ratios, Dt/D1 which is β.

And the factor, when you rearrange it, you will get this. So, factor 1 by $\sqrt{1 - \beta^4}$ is termed as velocity of approach factor. And this constant C /1 by, by divided by $\sqrt{1 - \beta^4}$, all these constant terms, they are combined together in one constant, which is called flow coefficient K. So, the actual flow rate formula, or the actual mass flow rate will be equal to constant K times throat area into $\sqrt{2 \rho \Delta p}$. So, we can use this formula.

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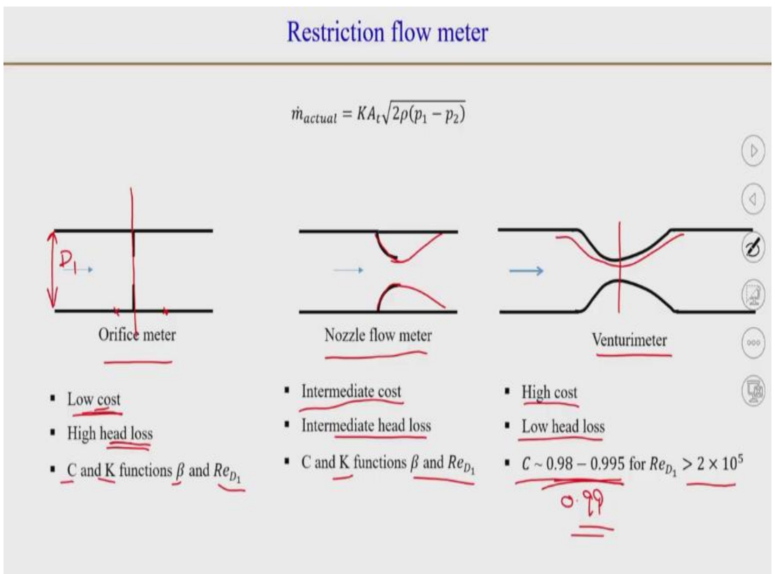

Now you can use different ways or different kind of elements which can be used to restrict the flow. So, the simplest one is that you introduce a plate here. So, when you introduce the plate, this is basically orifice and the flow comes through here and you can measure the flow rate downstream or at different places, and the problem here is that in this case, though the cost will be low, because it does not take manufacturing cost in terms of manufacturing cost, but the pressure loss here will be high because of the sudden change in area.

So, the running cost will be high in terms of capital cost or in terms of the cost of this equipment, it is low, but the running cost, because the energy that is lost in terms of pressure drop or head loss, it is it is high. So that will also cost something. So, one need to think about which one to use. And the constant C or the flow coefficient K, both of them will be the function of area ratios or the diameter ratios β and the Reynolds number, D1 indicates that the Reynolds number is based on the diameter of the pipe, D1.

Now another restriction one can use is a nozzle flow meter. So, in place of a sudden change in area one uses a gradual change in area. So, this might be a quarter of an ellipse and so the flow accelerates slowly and then it will decelerate. So, it is better in terms of performance and at the same time its cost is higher, higher than orifice meter, but the head loss will be lower than that in orifice meter. Here also the flow coefficient word will be a function of β and ReD.

Then another one is Venturimeter, so it is generally designed in such a manner that the flow is attached here. So, in terms of construction is difficult and that is why it is, when you go to buy it, its initial cost is high, but the running cost or the head loss is lower. So, you may have to invest more, but but when you put it into use it the head loss is lower.

The value of constant C here comes out to be depending on β and for higher Reynolds number, Reynolds number more than 2×10^5, the value is about 0.98 to 0.995 and you can take typical value of about 0.99. So, you can see that it is very close to what we get. It is almost used or close to the ideal equation because C is very close to 1.

So Venturimeter, in terms of Venturimeter because the flow will always almost must remain attached. You will, you will actually have the throat, you can find out the throat here and so many assumptions are you can relax. So, this is a quite approximation, quite a good approximation of the ideal case which we derived or all the assumptions that we made.

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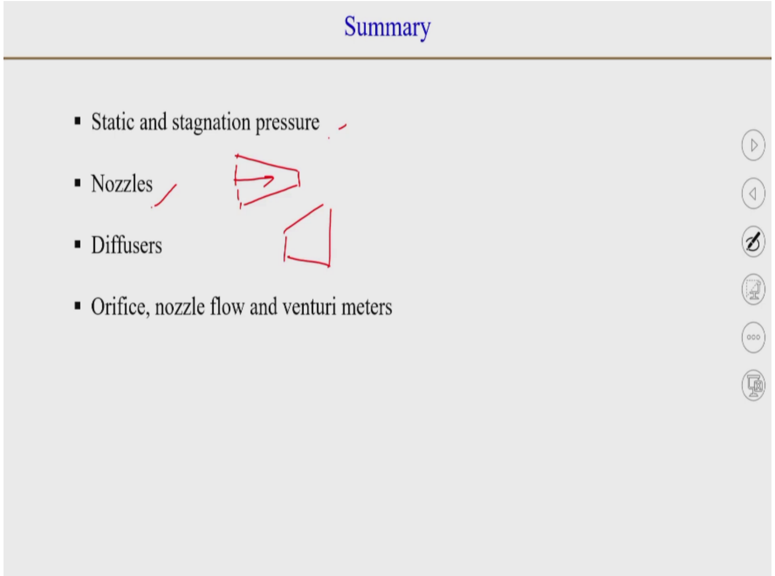

Now so what we have discussed today is, we first started with the Bernoulli's theorem and the conditions that we should use and the and the different forms in terms of units. It might be p in terms of p or in terms p/ρ or terms of p/ρ g, we should remember that unit should be consistent.

We talked about what is a static pressure, stagnation pressure. So, a stagnation pressure is the pressure when you bring the flow to a stationary, you make the flow stationary in a frictionless manner. So p0 stagnation pressure =static pressure + 1/2 ρ V^2, you also call 1/2 ρ V^2 as dynamic pressure. You can use it, the Bernoulli's theorem in supersonic nozzles, so nozzle are the devices which you used to accelerate the flow, for example. So, they are basically the area reducing for subsonic flows and the flow accelerates in the nozzle.

On the other hand, diffusers, the area increases as the flow goes downstream, so flow decelerates in, or the subsonic decelerates in diffusers. Now you have nozzles and diffusers, so we should remember that wherever there is flow separation, it may be a diffuser, it may be sharp corners or sudden bends on all those places we should use Bernoulli's theorem judiciously.

As we just saw restriction flow meters that to take into account flow separation and other non-idealities that may exist in the system, we put into a coefficient C there. So that takes into account the non-idealities in the system. And then we discussed the restriction flow meters and its different variants, orifice meter, nozzle flow meter and Venturimeters. So we will stop here. Thank you.