Fundamental of Fluid Mechanics for Chemical and Biomedical Engineers
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Lecture 32
Irrotational Flow

Hello. So, in the previous lecture we looked at inviscid flow, where inviscid flow we defined as the flow in which the viscosity or viscous effects can be neglected and that happens generally when the Reynolds number is large. And then we derived the famous Bernoulli's equation by deriving the momentum equations for an incompressible inviscid flow in streamline coordinates and one of the important points with that we derived there was that or we should remember that the Bernoulli's equation is valid for incompressible steady inviscid flow along a streamline.

So, what we are going to discuss today is what is called irrotational flow or potential flow and how this is related to inviscid flow that we will see now. So irrotational flow, if you remember when we talked about fluid kinematics, when we derived the relationship for the translation of the fluid and we developed the relationship for substantial derivative, which is ∂V/∂t + V.∇V. We did that and then we looked at the deformation of the fluid or the angular deformation of the fluid as well as the rotation of the fluid element.

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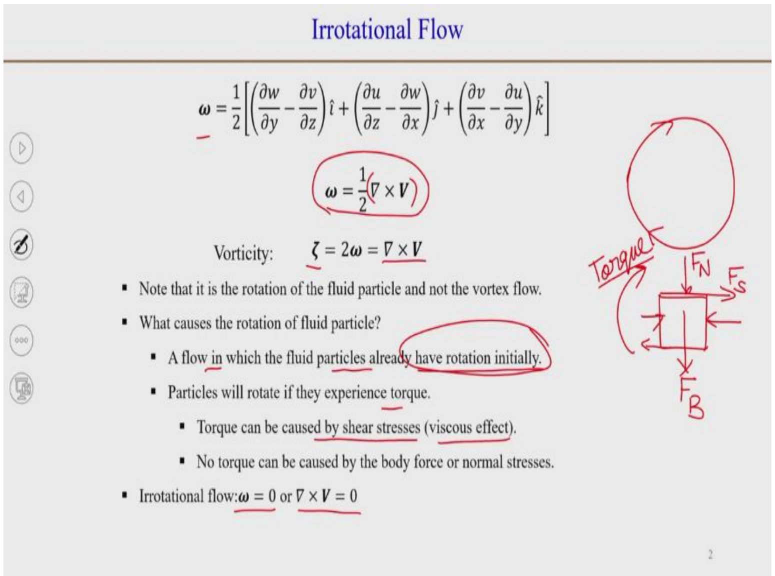

And we derived a relationship which was in the form of ω and we saw that in the simpler terms which is 1/2 of curl of velocity field or ∇×V is what we get as the rotation of the fluid element or we defined vorticity then which is basically ∇× V or curl of the velocity. So, the factor of 2 is taken into account in vorticity. So, when we say that the flow is irrotational that means the rotation in the fluid is 0, there is no rotation in the fluid. So that means ω vorticity ζ is 0.

Now just to remind ourselves, when we talk about rotation, it is not the say if you have a flow field which is a vortex flow field. So, the rotation is not talking about the streamlines which are circular in nature or there is vortex or recirculation in the flow. What, rotation here means the rotation of the fluid particle itself, so when can that happen?

If fluid particle can rotate if, when the flow started the particle has or would have had some rotation by itself. So, if flow in which when the flow started the particles, fluid particles, they had rotation when the flow started that time itself. So it was, there was rotation initially present in the fluid. Or when can the rotation start during the flow, it will be because for the particle or for the fluid particle to rotate you need a torque and torque will be generated when you have a shear stress, because if you take a fluid particle, the body force will be acting at the center of the particle or the normal force will also be passing through the center of the particle.

When you have a shear force, so when you have a shear force that is the thing that will give a torque. So that the fluid particle can rotate. And how are the shear stress generated? Shear stresses are generated by the viscous forces. So generally, we have seen enough that it is the viscous effects that is responsible for the generation of shear stresses and in the previous class we neglected the shear stress term because viscous effects were negligible.

So that means that if we have and inviscid flow then the shear stresses are going to be 0 unless there is some rotation present initially in the fluid. So irrotational flow will mean ω is 0 or we can write $\nabla\times V$ or curl of velocity is 0 that is the definition for inviscid flow. And as we just discussed, not for the inviscid flow, but $\nabla\times V = 0$ for irrotational flow.

As we just discussed that for the fluid particle to rotate, there should be some viscous effect in the fluid or it should be rotating initially. Now there is no viscous effect present in an inviscid flow and if the particles are not rotating when the flow started then inviscid flow will be irrotational.

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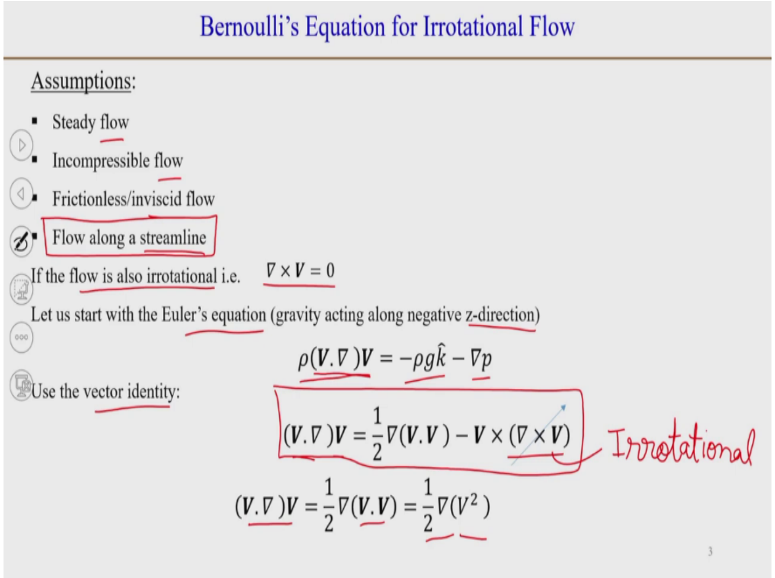

So, we derived Bernoulli's equation already for an inviscid flow. Now we will look at that the derivation of Bernoulli's equation for an irrotational flow. So apart from the normal assumptions or the assumptions that we discussed earlier, we will also have another assumption, which is for irrotational flow that curl of velocity or ∇× V is 0. So, the assumptions were that the flow is steady, flow is incompressible and flow is inviscid or frictionless and it flows along a streamline.

And we will see in a minute that for an irrotational flow it is not necessary that the fluid flows along a streamline. So even though we have listed down this assumption here, we will not use in our analysis and we will consider the motion of a fluid particle in general. So, but we will also consider that the flow is irrotational or ∇× V is 0.

Now, we will start with Euler's equation in the vector form and assuming that the gravity act along the negative z direction, so g will have a - k component there. So, this is the acceleration, the flow is steady. So, a local acceleration ∂/∂t of V is 0 and we have no viscous term and there are gravitational and pressure gradient term present.

Now what we will do is, we will use a vector identity directly, so which is that (V.∇)V. So, when V.∇ is operated on vector V we can show that 1/2 of ∇(V.V)- V × ∇ of, V × (∇× V). So, we will use this vector identity directly without deriving it. So, because the flow is irrotational and ∇×V =0, so this term goes.

So, this term is 0 and what we have is now that V.∇V, which is this term here that will be equal to 1/2 of ∇V.V and V.V, when you do it, it will be basically $Vx^2 + Vy^2 + Vz^2$ or u^2 v^2 w^2, depending on what component do you take so that will be 1/2 of ∇V^2, so that is basically 1/2 of ΔV^2. So, we can use or we can replace this term in the momentum equation by 1/2 of ΔV^2.

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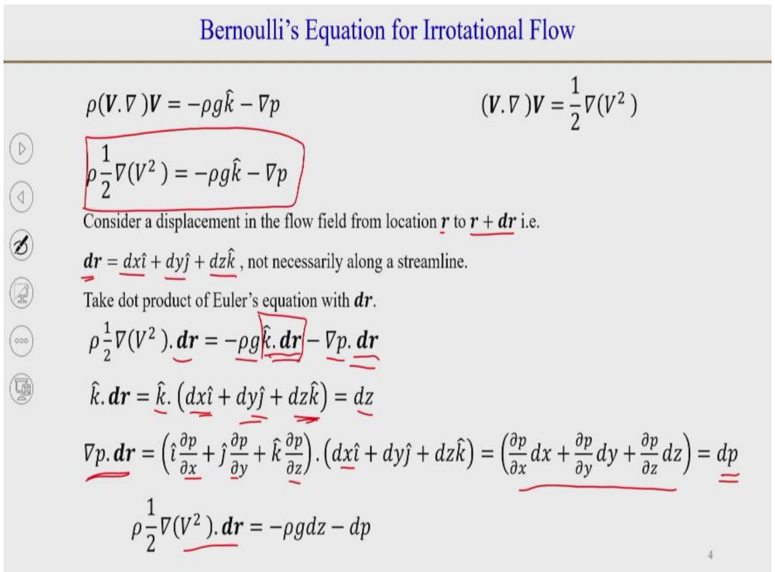

So, we will replace this and our equation becomes in this form. Now what we will do? We will consider, remember when we derived the Bernoulli's equation, we considered that the fluid particle moves along the streamline from point 1 to point 2, and that was a vector ds. Here what we will do? We will consider a displacement but from any point r to r + dr, so from vector r to r + dr and we can say this vector dr =dxi + dyj + dzk. So, the components of dr in x, y, and z directions are dx dy dz.

Now we will take dot product of dr with this equation. So, we can write that ρ into 1/2 Δ (V^2).dr =- ρ gk.dr - Δ p.dr. Now we will try to find each of these term or each of the dot product. So first we can say that because ρ and g, both of them are scalars and for vector product we need to find k.dr. So, k is a unit vector along the z direction and we have dr as dxi + dyj + dzk. So, when you do the dot product, the dot product from the first two terms will be 0 and the only thing you will get non-zero is dz.

So dzk.k and k.k is 1, k.i will be 0 and j.k, k.j =0. So, the first two term will give you 0. Similarly, we can write for Δ p. So, Δ p is basically i ∂/∂x of p + j ∂/∂y of p + k ∂/∂z of p.dr dxi + dyj + dzk.

And when you take the dot product you will get $\partial p/\partial x\ dx + \partial p/\partial y\ dy + \partial p/\partial z\ dz$, because only the corresponding term. So, i and i terms will give you non-zero, similarly the product of j and j terms will give you non-zero value and k.k terms.

So you will get the value and from the definition of this derivative you will see that this =dp or the total derivative of p. So, we can write in place of $\Delta p.dr = dp$. Now we have solved the two terms or we did the dot product for the two terms on the right-hand side of this equation. And now we need to do the same for the equation on the left-hand side.

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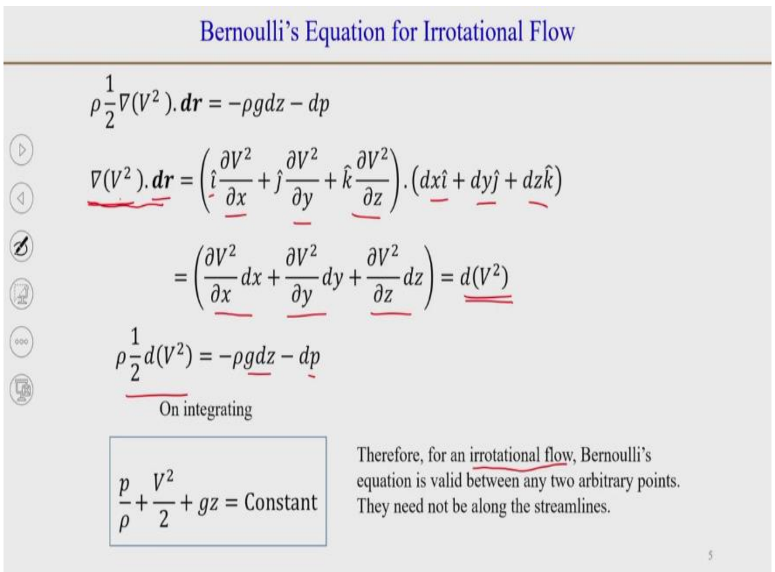

So, as you would guess what we will get is by doing this for d of V^2. So, what we can do is when we write Δ of V^2.dr, it will be like what we did for p. So, i $\partial/\partial x$ of V^2 + j $\partial/\partial y$ of V^2 + k $\partial/\partial z$ of V^2, dot product with dxi + dyj + dzk. And when you take thedot product you will get $\partial V^2/\partial x\ dx + \partial V^2/\partial y\ dy + \partial V^2/\partial z\ dz$ and that will be equal to d V^2.

So, we can write this as when we substitute this value of $\Delta V^2.dr = d V^2$ we can substitute in this equation. So, the equation becomes $\rho/2\ dV^2 =- \rho g\ dz - dp$, and we can integrate this term and bring everything on one side. So, what we will get is $p/\rho\ V^2/2 + gz$ =constant. So, we have again reached the Bernoulli's equation, but without using the assumption that the flow happens along the streamline.

So, what we did is we took the or we made an assumption that the flow is irrotational. So, if the flow is irrotational then Bernoulli's equation is valid for any two arbitrary points, and these points need not be along a streamline. So similarly, you could use the same approach to derive Bernoulli's equation for an inviscid flow. What you could have done in place of dr we could have taken a displacement along ds and from that we could have derived the Bernoulli's equation, but we are not going to do that here. You can see that for yourself.

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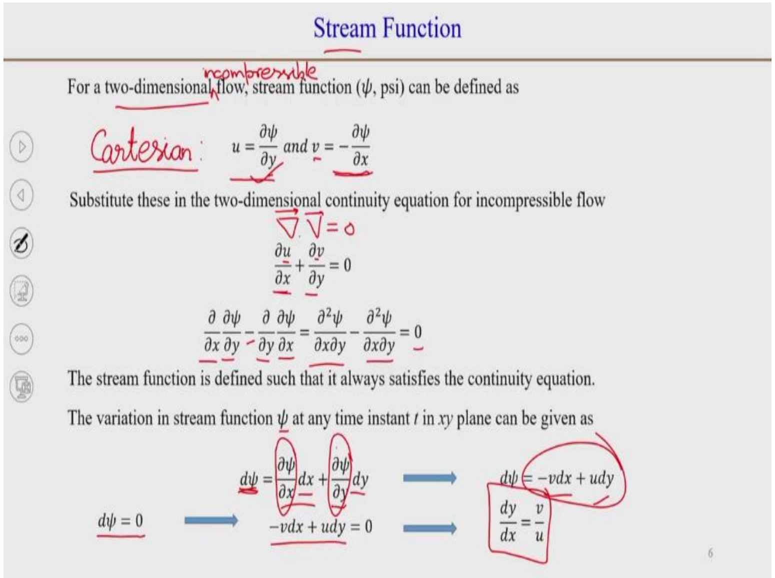

So, the other thing that we are going to do is define a potential function. But before we define potential function, we will look at what is called stream function. And as you would guess that stream function is related to streamlines. So, the stream function can be defined as for if we consider a two-dimensional flow, the concept is more useful for two-dimensional flow and the stream function can be defined as the x component of velocity if we talk about Cartesian coordinates.

So let us just write here explicitly that in Cartesian coordinate the stream function is defined as such that that $u = \partial/\partial y$ of stream function ψ and v or y component of velocity $= -\partial/\partial x$ of stream function ψ. So that is the definition of stream function.

Now we can see that that this definition of stream function satisfies the continuity equation directly. So, if we write continuity equation, which is for an incompressible fluid, so the concept comes for a two-dimensional and incompressible flow. So, if you have an incompressible flow, then $\nabla . V = 0$ or in two dimensions and you can write $\partial u/\partial x + \partial v/\partial y = 0$.

Now to check if the stream function satisfies it, we can replace u and v by the definition in terms of stream function and see if we get the right-hand side, which is 0. So, we can write this as $\partial/\partial x$ and u is, we can write in place of u $\partial/\partial y$ of ψ -, so this - comes because we will now replace for v, $\partial/\partial y$ of - $\partial\psi$ of ∂x, so this - sign comes here and $\partial/\partial y$ of $\partial\psi/\partial x$.

So that becomes equal to $\partial 2\psi/\partial x\ \partial y$ - $\partial 2\psi\ \partial x\ \partial y$. So, these two terms as you can see, they are equal and what you will get 0. So that means the/definition stream function satisfies the continuity equation for an incompressible flow.

Now we can define or the variation in stream function, which is basically d ψ. This is a ψ. So, d ψ is defined as, we can write from from the definition of derivative that =$\partial/\partial x$ of ψ into dx + $\partial/\partial y$ of ψ ∂y. That d ψ =$\partial/\partial x$ of ψ dx + $\partial/\partial y$ of ψ dy.

Now we can replace $\partial/\partial x$ of ψ and $\partial/\partial y$ of ψ with u and v here. So, we will get d of ψ =- v $\partial/\partial x$ of ψ is - v. So that will be - vdx + udy, and if this =0 that means if there is no change in the stream function, then we will have d of ψ =0. So that means we can write down this =0.

So, if we do that then what we will get - vdx + udy =0 or we can get dy/dx =v/u. And if you remember how, we defined a streamline for a two-dimensional flow streamline is the slope of velocity vectors and we saw that dy/dx =v/u, where v is the y component of velocity, u is the x component of velocity. So, what we see here is the stream function, this equation d ψ =0 basically represents the equation of a streamline for a two-dimensional incompressible flow.

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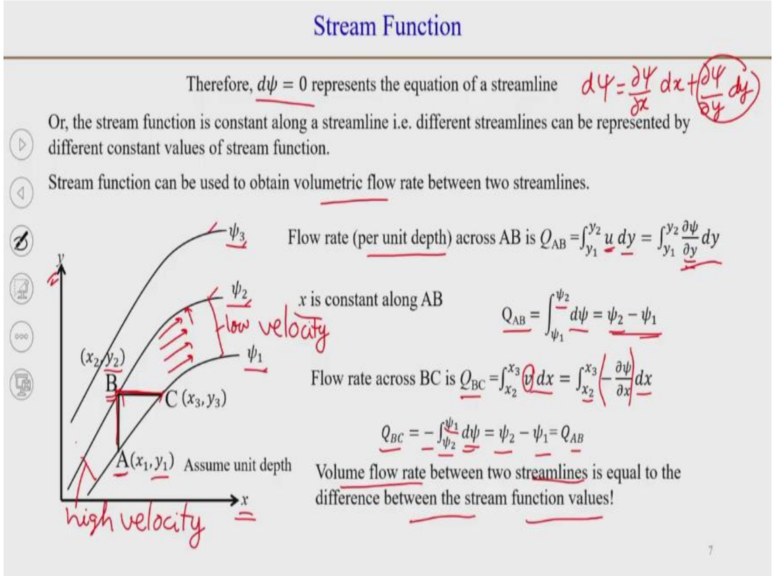

So, we now know a bit more about streamlines in terms of that we can represent streamlines as the lines of constant value of stream function, because that is what d ψ =0 means that a ψ =constant. So, if streamlines are basically lines of constant stream function or iso stream function values.

Now, there is another good, another a useful relationship, we can say we can find here that if you have an x, y plane which is in which the flow is happening. You can have a different streamline and these streamlines; each streamline will have a stream function value corresponding to it. So let us say these three streamlines shown here. They have the stream function values of ψ1, ψ2 and ψ3.

And we can try to see that what is the flow rate between the two streamlines? And we will see that the flow rate between two streamlines is constant. We know that the flow cannot cross a streamline. So, it cannot pass through this direction. So, flow will be between the streamlines only.

Now we will take one line AB and another line BC and try to calculate the flow rate at through coss-section AB and through section BC and try to find are they same. So, if we take the that the dimension normal to this screen is 1, so that is what we say that flow rate per unit depth. So, the depth which is normal to the screen is 1, and we can find the flow rate across this Q AB.

So, the flow rate through this line will be integral from point A to B and flow normal to it, which is u dy we can write this as v.dn and from that you will get u dy into of course 1, which is the depth. And we can integrate it from y1 to y2. So, we will get the flow rate. Now from the definition of stream function ψ, we can replace u with d/dy of ψ, you can just write the definition of ψ here. So, d/d ψ =dx into ∂/∂x of ψ + ∂ψ/∂y dy.

Now u =∂ψ/∂y into dy, but as you can see along the line AB x is constant. So, the dx along the line AB is 0. So, at, when we look at along line AB, we can write that d ψ =or ∂/∂y of ψ into dy. So, we can replace this with d ψ itself and d ψ and we can change the limits. So, this will be from y1 at y1, the value of ψ is ψ1 and at y2 the value of ψ is ψ2 or at A, the value is ψ1 at B the value is ψ2. So, we can write this when you integrate it you will get ψ2 - ψ1 after substituting the limits. So, the flow through this section AB =ψ2 - ψ1.

Now we will do the same exercise to find flow through this section BC again unit depth. So, flow rate across BC is Q BC and we will have v.dA and that will give us v dx from we will integrate it, from x2 to x3, and that will give us, we can replace v by the definition of v in terms of stream function, we can replace it with - ∂/∂x of ψ into dx.

Now again, we can look at BC y =0, so this term will be 0 here and we can replace ∂/∂x of ψ into dx with d ψ. So, we can write this =Q BC =- the value of ψ at x2. So, the value of ψ at x2 is ψ2 and value of ψ at x1 is ψ1 of d ψ. So, when we put the limit, we will get Q BC or flow rate through BC is ψ2 - ψ1 that is again equal to Q AB.

So that means you take any two points between the streamlines the flow rate between them or the volumetric flow rate between the two streamlines is same. Now you see here that the distance between the streamlines is small here. And this distance between the streamlines is large here. So that means the cross-sectional area between the streamlines is changing. So, if the distance between the streamlines is small because the flow rate is same, if the distance between the streamlines is small than the velocity is high here and the velocity is low here.

So, we saw that volume flow rate between two streamlines =the difference between the stream function values. And it is a constant. So, if you have two streamlines, and if we know the stream function values ψ1 and ψ2 then we can directly find the flow rate from here. It's in 2D plane, or we can say flow rate per unit depth as we see here.

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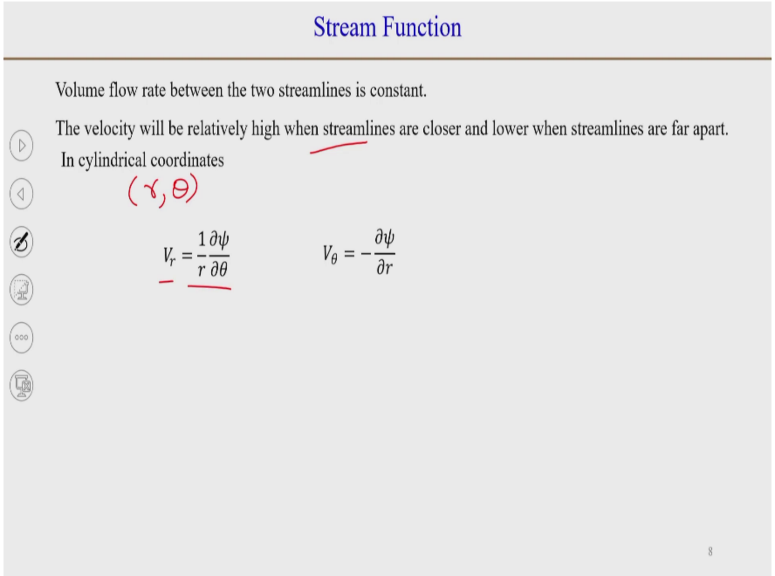

So, we already saw that if the streamlines are closer than the velocity is high and if the streamlines are far apart, then you will see that the velocity is low. So generally, when you see streamlines plot or the contour plots of streamlines analyzing any CFD results, you might see that when they say or explaining the figure, they say that when the streamlines are placed closer to each other then the velocity is supposed to be high there and when the streamlines are far away from each other then the velocity is relatively lower there.

We can also do a similar exercise for cylindrical coordinates and can define the stream function ψ in cylindrical coordinates again in 2D. So, it will be in terms of r and θ. So Vr =1/r ∂/∂θ of ψ and Vθ will be - ∂/∂r of ψ. So that will be the definition of stream function in the cylindrical coordinates in r θ plane.

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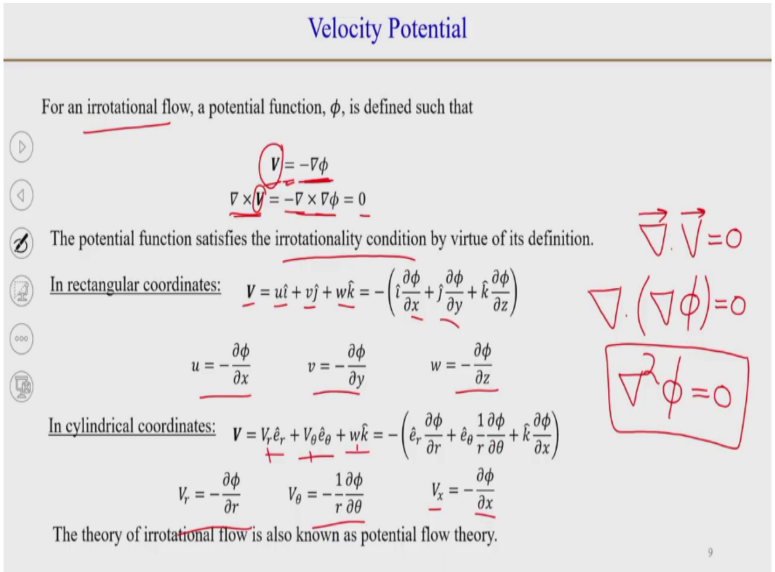

Now we will define another function which is potential function and as we will show that the potential function is relevant for irrotational flow only and because it is defined for an irrotational flow. So, the is irrotational flow is also known as potential flow. So, it is defined as V = -Δϕ, so ϕ is the potential function here and it is defined in terms of that velocity vector that = -Δ ϕ.

So that means the flow happens from from higher of ϕ value to lower ϕ value and that's why we have a - sign there. So, if we substitute this definition of ϕ which is the potential function in the irrotationality condition, so if we substitute that ∇× V in place of V if we write - Δ ϕ then we will get - ∇× ∇ϕ and this will give us a 0.

So, by definition itself the potential function satisfies the irrotationality condition. So, the velocity potential is defined for an irrotational flow. So, like stream function we saw that the stream function by virtue of its definition it satisfied the continuity equation. Similarly, the potential function by virtue of its definition it satisfies the irrotationality condition.

Now you can write this in terms of component and you can do it in for 2D or for 3D. So, in rectangular coordinate when you expand velocity vector you can write ui + vj + wk in terms of its three-component u, v, and w along x, y and z directions and you can write Δ ϕ, so - i ∂/∂x of ϕ + j ∂ϕ/∂y + k ∂ϕ/∂z. So, if you equate the component respectively, then you will get u = - ∂ϕ of ∂x, v =- ∂ ϕ /∂y and w =- ∂ϕ/∂z.

You can also do the same thing in cylindrical coordinates and there you will get Vr er + V_θ e_θ + wk, so the velocity components along radial, angular and axial directions, that will be equal to - er ∂/∂r of ϕ + e θ/r ∂/∂θ of ϕ + k ∂/∂x of ϕ. And we can again equate the components, respective components.

So Vr will be - ∂ϕ/∂r. Vθ will be - 1/r ∂θ of ∂ϕ. And Vx =- ∂ ϕ /∂x. So, we have the definition of a potential function in cylindrical as well as rectangular coordinates. And if we substitute this in the continuity equation as we will see now.

So, if we take a two dimensional incompressible, irrotational flow. So, if we, if we write down the continuity equation, which is ∇.V =0, ∇.∇ ϕ =0. So, we can write this as ∇^2 ϕ =0, which is a Laplace equation in mathematics and it is a linear differential equation.

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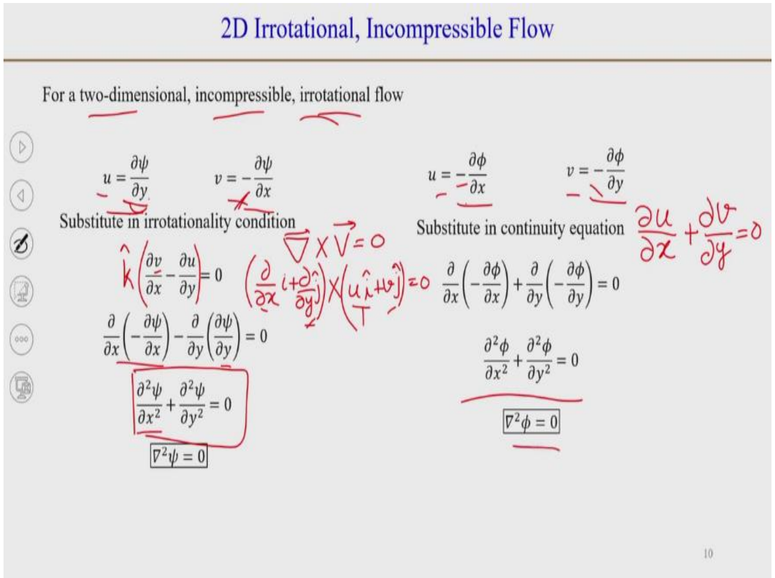

So, you can find the solution of this equation easily, analytical solution. And that is the reason there has been lot of interest in solving inviscid or inviscid and irrotational flow because the solution for such flows is easily possible because the flow can be represented by the Laplace equation. So now if we consider a two dimensional incompressible and irrotational flow and we can write down both.

So, we can write down the stream function and the velocity potential. So here we write the definition of u in terms of stream functions. So, u =∂ ψ /∂y and v is equal - ∂ ψ /∂x. By their definition itself the u and v they satisfy continuity equation for when we tried those in terms of stream function. And we let us, because the flow is irrotational as well.

So, we can substitute these definitions in the irrotationality condition, which is basically $\nabla \times V = 0$ and for a 2D we can write $\partial/\partial x$ i, + $\partial/\partial y$ j cross product u i + v j =0. And when you do that, you will get $\partial/\partial x$ of u. So, when you relate the first term in the two brackets, you will get 0 and when you write take $\partial/\partial x$ of v. So, first term you will get i × j, which is k the direction of this of course will be k.

And you will get $\partial v/\partial x$ and then when you relate the second term in the first bracket with the first term in second bracket, so you will get $\partial u/\partial y$ j × i. So, it will give - k. So, you will get - $\partial u/\partial y$ and again, when you do a cross product of the second term on both the bracket you will get 0. So, this is your irrotationality condition.

Now, you can substitute the values of u and v there that is basically the k component of ω that we derived. So, we can substitute in place of v - $\partial/\partial x$ of ψ, so $\partial/\partial x$ within bracket - $\partial/\partial x$ of ψ - $\partial/\partial y$ of u, and u is $\partial/\partial y$ of ψ. So again, what we get from here is - $\partial^2 \psi/\partial x^2$ - $\partial^2 \psi/\partial y^2$ or we can multiply by - or - 1.

So, you will get this equation $\partial^2 \psi/\partial x^2 + \partial^2 \psi/\partial y^2 = 0$, which is a Laplace equation in two dimensions or you can write in vector form ∂^2 or ∂^2 of ψ =0. Now we can do a similar exercise by assuming the definition in terms of velocity potential. So, we can write u and v in terms of velocity potential. So, u =- $\partial/\partial x$ of ϕ and v =- $\partial/\partial y$ of ϕ.

You might notice the differences between the two definitions here. So, you can see that u is defined as $\partial/\partial y$. So, when it is x component of velocity, it is differentiated with respect to y for stream function for velocity potential it is differentiated with respect to x. So, in velocity potential you will have - sign for both, whereas in stream function it is - sign only for v component or y component of velocity.

So, when we substitute this definition of velocity potential it satisfies rotationality or irrotationality condition by the definition itself, and we will substitute in the continuity equation. So, we will substitute these values in the continuity equation. So, when we do that continuity equation is basically ∂ u, or $\partial u/\partial x + \partial v/\partial y = 0$ for a 2D incompressible flow.

So, we can substitute the value of u - $\partial\phi/\partial x$ + y and v we can substitute by - $\partial/\partial y$ of ϕ. Again, we will get a Laplace equation in the term $\partial 2\ \phi/\partial x2 + \partial 2\ \phi/\partial y2 = 0$. So, for a two-dimensional incompressible irrotational flow both stream function as well as velocity potential, they satisfy the Laplace equation.

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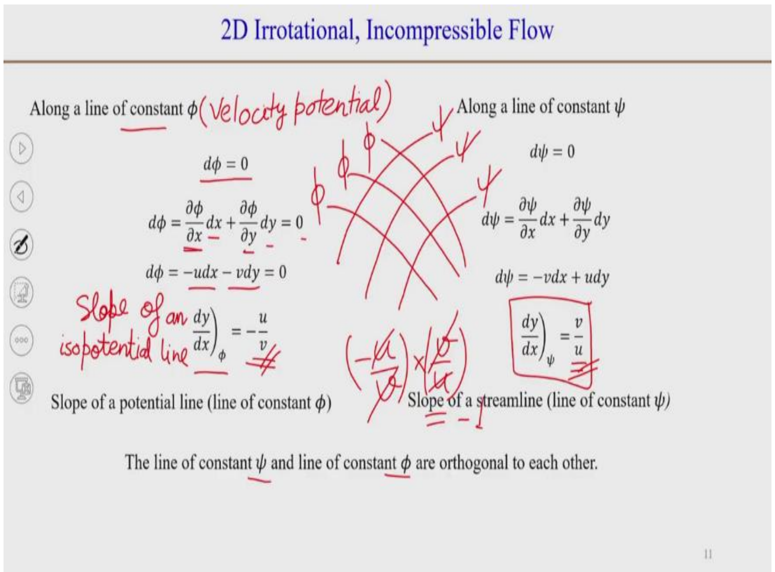

Now if we consider a line of constant ϕ, which is a iso potential where ϕ is the velocity potential and we can find out its slope. So, for an iso potential line, d ϕ =0. And this we can write in terms of $\partial\phi/\partial x$, dx + $\partial\phi/\partial y$ dy =0 and we can replace the value. So, $\partial\phi/\partial x$ will be - u, so you can write - u dx. Similarly, this will be - v dy. And from that we will get the slope dy/dx of iso potential line. So that is slope of an iso potential line that will be equal to - u/v.

Now we can also find that what is the slope of a line, which has stream function constant of slope of streamline, which we already found, which we already saw that this =v/u. And if you look at these two, the slope m1 and m2 of these two lines, they are when you multiply - u/v into v/u, what you get is v and v cancel out u and u cancel out and what you get =1. So m1 into m2 =- 1 and those are orthogonal to each other. So, line of constant stream function and constant velocity potential, they are orthogonal to each other.

So, if you plot of plot lines of constant stream function and of constant velocity potential, they will be normal to each other. And by their properties, we can, if we because we know both of them in terms of velocities. So, if we know one, if we know stream function, we can find out velocity potential. Of course, all this we are talking about for a flow which is incompressible as well as irrotational because the definition of velocity potential is valid only for an irrotational flow.

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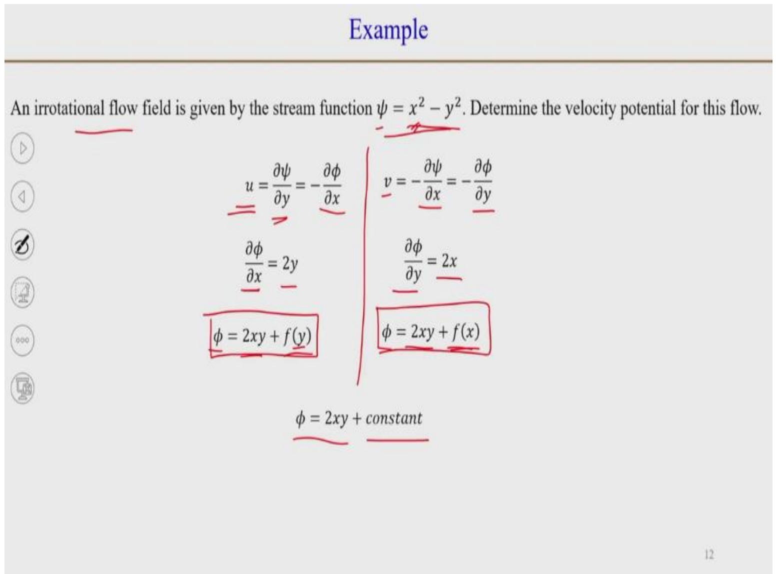

So let us look at an example, where we have a stream function. So, an irrotational flow field the stream function is given by $\psi = x^2 - y^2$. And what we need to find is the velocity potential for this flow. So, we can write down the value of u for this. So, u =∂/∂y of ψ and that is also equal to in terms of velocity potential - ∂/∂x of ϕ.

And we already know what is ψ, so we can find that ∂/∂y of ψ will be equal to, when you differentiate this with respect to y first term differentiation will be 0 because x^2 is not a function of y and when you differentiate - y^2, what you will get is - 2y. So, you will get - ∂/∂x of ϕ =- 2y.

So, you get ∂/∂x of ϕ =2y. When you integrate it, then you will get, you have to integrate it with respect to x. So, you will get ϕ =2xy + a function which is independent of x, but can be a function of y. So, you will get ϕ =2xy + a function of y.

Now you can write the v component of velocity. So, v =- ∂/∂x of ψ and that will be equal to -, in terms of velocity potential, - ∂/∂y of ϕ. So now when we differentiate ∂/∂x of ψ, or when you differentiate with respect to x what you will get is - 2x and - 2x =- ∂/∂y of ϕ. So, ∂ϕ/∂y =2x.

And when you integrate it with respect to y, you will get $\phi = 2xy$ + a function which is independent of y, but it can be a function of x. So, if you look at two values of ϕ that we have got is $2xy$ + a function which may be a function of y, $2xy$ + a function which may be a function of x and if both needs to be true then fy and fx they will be equal and constant. So, what we get is $\phi = 2xy$ + a constant. So that is the value of velocity potential given stream function and we could find the value of velocity potential.

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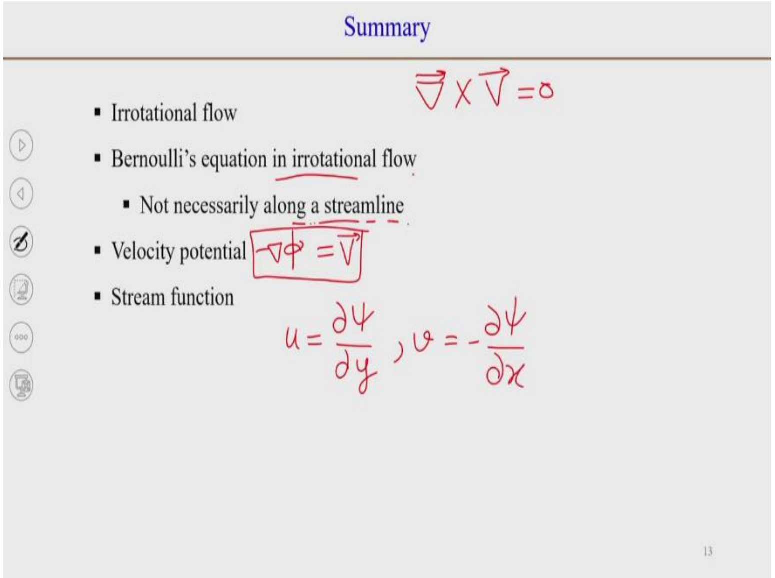

Now, so that is all what we have discussed today. Let us summarize what we discussed. First, we defined that what is an irrotational flow and irrotational flow is where the rotation of fluid particles is 0, and mathematically we can write that the curl of velocity or $\nabla \times V$ should be 0 that is the irrotationality condition.

And from that irrotationality condition we also derived Bernoulli's equation for an irrotational flow and a good improvement there or an advantage in this is that in an irrotational flow it is not necessary that we need to apply the Bernoulli's equation along a streamline, you can apply the Bernoulli's equation between any two arbitrary points for an irrotational flow.

Now then we defined velocity potential for which is defined for an irrotational flow and velocity potential is defined as ϕ =or V = - Δ ϕ. And we also defined stream function for a two-dimensional incompressible flow where we looked at that $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$.

And we saw that both for a two-dimensional incompressible irrotational flow, both the stream function ψ and velocity potential function ϕ, they satisfy the Laplace equation. You can see that there are the streamlines or the lines of constant value of stream function and the iso potential lines that is the lines of constant value of velocity potential. They are orthogonal or normal to each other.

So, when you want to analyze such problems, you may want to analyze in these two coordinates. Now remember from here what we also got is that the inviscid and irrotational flow it can be represented by a linear equation, which is Laplace equation. We saw that when we got this value, or when we got Laplace equation $\partial^2 \psi = 0$ that satisfies the continuity equation in itself.

But apart from that it is not being shown here, but it can be shown that it also satisfies the Euler's equation. So that is the governing equation for an inviscid irrotational flow. The Laplace equation $\partial^2\psi$ is the governing equation for inviscid flow. That is the reason that lot of work has been done on the theory of inviscid flow. We will stop here. Thank you.