Fundamentals of Fluid Mechanics for Chemical and Biomedical Engineers
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Lecture 26
Navier Stokes Equations: Flow in a Falling Film

In the previous lecture, we derived the Navier-Stokes equation which are momentum conservation equations in the differential form for an incompressible Newtonian and a liquid having constant viscosity or a gas having constant viscosity. So, in this lecture, we will solve a problem which is a falling liquid film due to gravity and we will find out the velocity profile in this liquid film starting with the mass and momentum conservation equations in the differential form.

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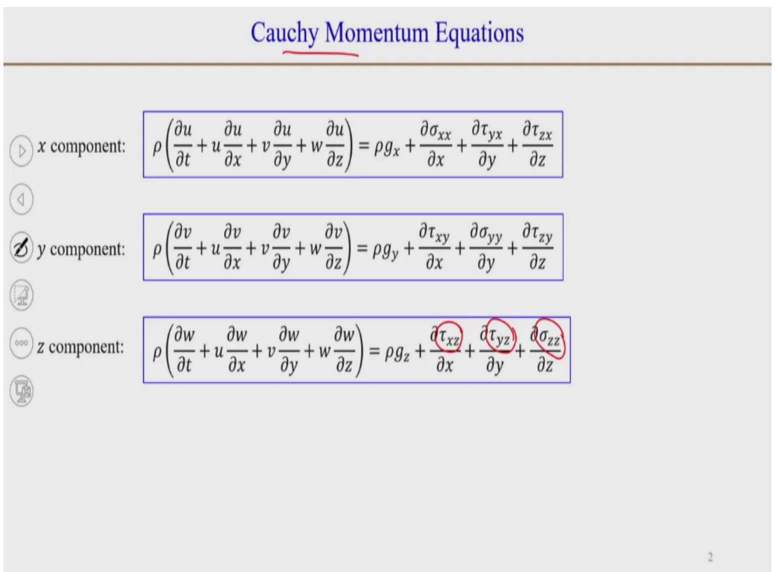

So, to remind ourselves these are the momentum conservation equations in the form of stresses here and these equations are valid for Newtonian as well as non-Newtonian fluids and we call Cauchy momentum equations because we have not still replaced the stresses using the constitutive equation. So, you might remember the constitutive equations are the relationship between stress and rate of strain.

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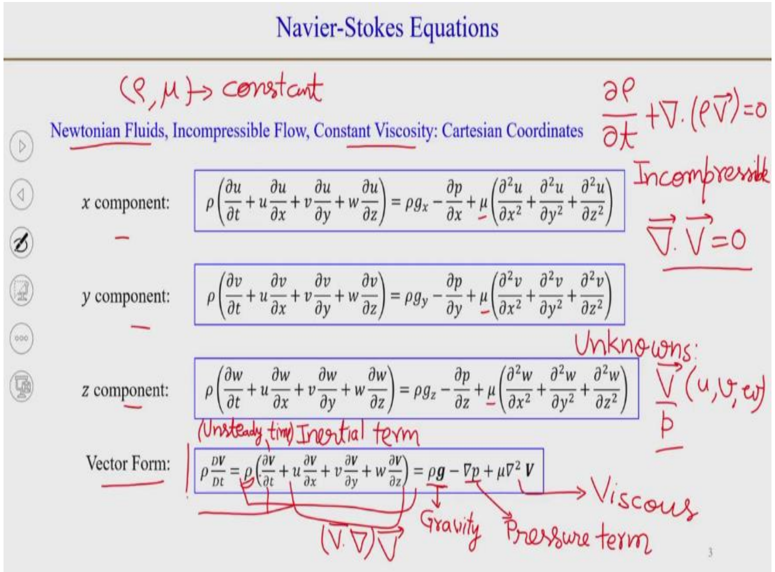

So, once we replace the stresses in terms of rate of strain, what we get is Navier-Stokes equations where we have assumed those constitutive equations for a Newtonian fluid. And we have also assumed that the viscosity is constant so we can take viscosity out of the derivative in these x, y and z component equations.

You might see that in all of these equations this is in the vector form. So, the terms on the left-hand side, they represent the fluid acceleration multiplied by of course density. So, if you look at the unit of each term, on this side, it is ρ into g. So, basically mg divided by volume. So, this is force per unit volume, so the unit of each term will be force per unit volume.

Similarly, on the left-hand side ρ is mass per unit volume, kg/m^3 is the unit and ∂v/∂t is acceleration. So, this is the mass per unit volume multiplied by acceleration. The first term is the local acceleration and these three terms which is basically nothing, but V.∇ operated on V where V is a vector.

So, this is called convective acceleration or the acceleration which is there because of the bulk motion of the fluid. In some cases, or in some places you might also see the name advection. This term is known as advection also. This is also called the inertial term. And the first term is known as unsteady term or transient term because it is time dependent. So, when the flow is steady this term will be 0.

This is the time due to gravity or you can replace g with a body force or ρ g with a body force if there is a body force apart from gravity. ∇p is pressure term or pressure gradient term. So, this is the term due to the pressure forces and the last is viscous term and the term is because of the viscous effect or viscous stresses present in the flow.

Now we have these equations, if you write in the vector form, you have in this equation because we are assuming the flow to be incompressible and of constant viscosity, so ρ and μ are constants, and you know the acceleration due to gravity. So, you if look at, there are unknowns, unknowns in system of equation are vector V or in the component form you u, v and w. And another unknown is pressure p. Now you have one set of equation which is momentum conservation equation in the vector form or you can take three components.

Another equation is mass conservation. Of course, so you can write ∂ ρ/∂ t+∇.(ρV) =0 or if the flow is incompressible, so for incompressible flow you can write this as ∇.V =0. So, you have basically two equations, one coming from the continuity or mass conservation and another from momentum conservation. So, you have two equations and two unknowns and you can solve these equations for the unknowns or velocity and pressure.

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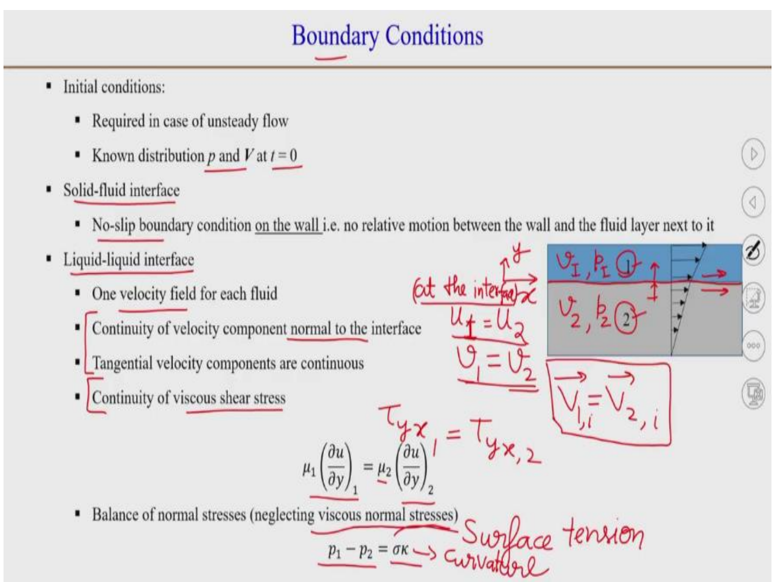

But when you solve this, you also need the boundary conditions and if the flow is unsteady, then you also need the initial conditions. So, if the flow is unsteady then you need to know what is the velocity and pressure distribution because those are your unknowns and these are the things that you need to know at t =0 in case of unsteady flows so you should know the distribution of pressure and velocity in the entire domain at time t =0 or in the entire region of interest.

Now there are boundaries. So, when you have analyse a fluid flow, you have a certain region of interest and in this region you want to analyse or find out the solutions for the velocity and pressure fields. So, you need to know in order to solve these equations, governing equations, mass and momentum conservation equations, you need to find out or you need to know the boundary conditions.

So, the common boundaries that you will have that inlet and outlet boundary of your domain and the walls or if it is open to atmosphere then there might be a free surface or it is in contact with another fluid then it might be fluid-fluid interface. So, for inlet and outlet boundary, when we find out analytical solution, in most of the cases, we assume the inlet and outlet present to be far away at+infinity or - infinity.

So, those problems are we so not need to in general define such boundaries. If we are solving these numerically then of course, your domain need to be finite and then you need to specify the velocity and pressure at the inlet and outlet boundaries or either you might need to define the velocity and pressure or you might need to define their gradients.

So, what we will be looking at here that what are the boundary conditions at the gas-liquid, liquid-liquid or solid-liquid interfaces. So, if you have a solid fluid interface, so it might be a solid-liquid or solid-gas interface, which is basically fluid is flowing on a solid wall or fluid is in contact with a solid wall. So, on this surface we will have no-slip boundary condition, which simply means that there is no relative motion between the solid surface on which the flow is happening and the fluid layer adjacent to it.

So, at the solid wall, the fluid velocity will be equal to the velocity of the wall. Now you will have the tangential component of velocity that will be equal to the velocity of the wall. So, the velocity of the fluid the tangential velocity of the fluid will be equal to the velocity of the wall. If the wall is moving, then the tangential velocity will be equal to the velocity of the wall.

The normal velocity will be also the velocity of the fluid normal to the wall will also be 0 if the wall is impermeable. In case if it is porous wall then you will need to define some flow rate or you need to, those pores will be counted as or will be treated as inlets and outlets as the case may be. So, a for a solid impermeable wall you will have no-slip boundary condition where the normal component of the velocity will be equal to 0 and the velocity which is tangential to the fluid will be equal to the fluid velocity.

Now at a liquid-liquid interface or a fluid-fluid interface in general, what you have is if you have two fluids, two immiscible liquids in contact with each other, fluid 1 and fluid 2, then in order to understand or in order to find the velocity field in such a system, you will need to solve the velocity and pressure field for fluid 1 and velocity and pressure field in fluid 2. So, we need the boundary condition at this interface. This will be, of course, we valid when we talk about liquids which are immiscible because there is no mixing of the two fluids.

So, we will have to have one velocity field and one pressure for each fluid and at the boundary, at the fluid-fluid boundary because of the kinematic boundaries, the kinematic boundary condition will be that we will have the velocity conditions there that what is the velocity of the interface. So, continuity of velocity component normal to the interface.

That means the velocity component normal to the interface will be equal. So, the velocity of fluid 1 normal to the interface, at, in fluid 1 will be equal to velocity normal to the interface in fluid 2, so the normal components of velocity in fluid 1 and in fluid 2 will be equal and they will be equal to the interface velocity. Why? If there is a difference between the two, then that means a certain wide will be created between the interface and the fluid which is not possible or which is not feasible.

This will not be valid when there is some mass transfer or evaporation happening at the interface then you need to take into account the mass transfer or phase change, evaporation or condensation. But for isothermal flows where there are no phase change, no mass transfer, we will have the normal components of velocity at the in the two phases will be equal.

The other component of velocity will be, the velocity component which is tangential to the interface, so the velocity component tangential to the interface will also be equal. So, the tangential velocity will be continuous. That means the velocity if we assume that the coordinate system here x, y, and the interface is aligned along the x direction then we can say that u which is the x component of velocity in fluid 1 is equal to u in fluid 2 and v in fluid 1 from normal component of velocity, this is at the interface.

So, at the interface u1 =u2, v1 =v2, so the normal component of velocity and the tangential component of velocity at the interface will be equal, that means the velocity will be continuous or the velocity vectors at the interface in fluid 1 and fluid 2 will be equal. So, that simply means v1 and v2 at the interface, they are equal.

Now this interface will also be in a mechanical equilibrium, so the forces acting on it should also balance each other, so at the interface because it is a surface, so the forces acting will be the surface forces. So, shear stresses which are tangential to the interface, so viscous shear stresses, they will be continuous.

So, that means τ_{yx} in fluid 1 will be equal to τ_{yx} in fluid 2, or if you write it in terms of the rate of strain, so mu1 du/dy in fluid 1, this will be equal to $\mu 2$ du/dy in fluid 2. So, we can see that the velocity is continuous at the interface and viscous shear stress is continuous at the interface, then we come to normal stresses.

So, most of the time, in most of the cases, the viscous normal stress is negligible, so for the normal stresses because at the fluid-fluid interface, you have surface tension, so the normal stresses, difference between normal stresses will be balanced by the stress generated due to the surface tension.

So, here p1 - p2 $=\sigma\kappa$, which is the stress or normal stress because of surface tension, σ here is surface tension, not the normal stress, so please remember that, σ is surface tension and κ is curvature of the interface. For a sphere we have $\kappa = 2/R$.

So, those will be p1 - p2 which are pressures, we have not considered here viscous normal stresses, but if there are viscous normal stresses then we can say p1 plus the viscous normal stress in fluid 1 - p2 plus viscous normal stress in fluid 2 that will be balanced by force due to surface tension.

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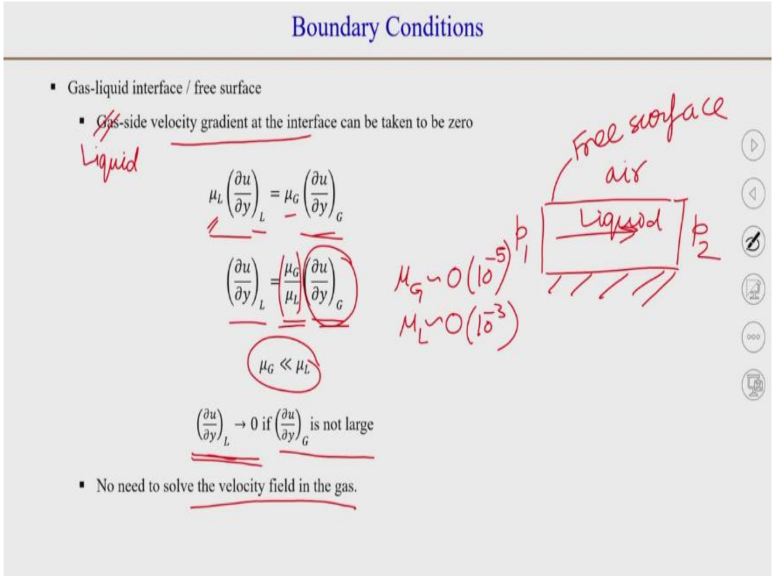

Now a special case when you have a gas-liquid interface, for example a gas-liquid interface is open to atmosphere. You have a liquid film falling over a wall then what happens? At such a gas-liquid interface, we still have, the interface is basically the interface between two fluids.

So, we still have the boundary conditions which we discussed just now to be valid, but we can take or we can simplify that the velocity gradients at the interface can be taken to be 0 or this should have been liquid side. When we are looking at a gas-liquid interface, in such cases, for example, a flow in a channel which is, this is free surface and the flow is driven by pressure here.

Now at this interface there is air on top and the liquid in the channel, so we can write the continuity of viscous here stress, so from that continuity of viscous here is stress, we can write μl ∂u/∂y of liquid, that will be equal to μg into ∂u/∂y of gas. Now we can rearrange this in terms of ∂u/∂y of liquid that will be equal to the ratio of viscosities of gas divided by viscosity of liquid, so this basically ratio of viscosities.

And this is multiplied by the gradient of velocity in the gas side. Now typically at the atmospheric temperature, the gas velocity is of the order of 10^{-5}, whereas the velocity of liquid for example that of water is of the order of 10^{-3}. So, there is two order of magnitude difference in the viscosities.

If this term, if the velocity gradient in the gas side is not too large, then because μ_g/μ_l is a small number so we can neglect the gradient in the liquid side of, and then gradient at the interface in the liquid side and this helps us that to solve the velocity field in the liquid we do not need to find the velocity field in the gas.

So, in such cases where we are not interested, for example, that flow in a river or flow in a canal or flow in a falling liquid film, we are not interested in finding the velocity in the gas and the gas velocity, there is no significant velocity in the gas, then only we will have this condition to be valid that the gradient in the velocity is not large. So, then we can assume that the velocity gradient at the liquid side will be at the interface will be equal to 0.

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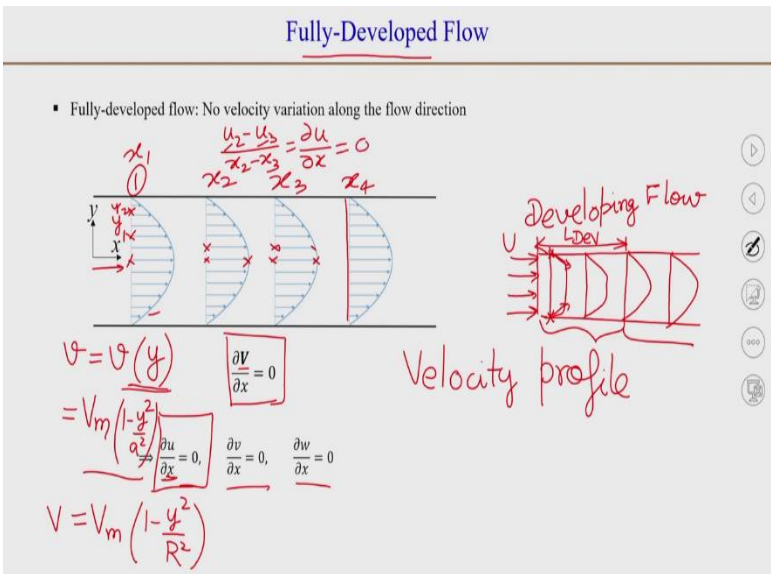

So, let us look at an example, but before that let us talk about what is fully developed flow. So, when you have the flow entering in a channel or in a pipe or in any closed conduit, so let us say that the flow enters in this channel from a, this might be a pipe that is in a free stream of liquid, so for example, pipe in a river far apart from the boundaries, so the velocity in the river is uniform and it comes in with a velocity U.

But because of the no-slip boundary, the velocity here will become 0, so velocity at the walls because of the no-slip boundary condition, the velocity at the walls will become 0 and the fluid will need to rearrange itself, so some of the fluid will start moving towards in the transverse direction to rearrange itself and eventually this will keep happening until you achieve a velocity profile which remains unchanged.

And when that happens, then such flow is called fully developed flow. So, fully developed flow refers to the flow in which there are no gradients in the velocity in the flow direction. So, ∂by ∂x of V in this case where the velocity or the flow is along the x direction, so $\partial V/\partial x = 0$, if it is a pipe flow then x might be the axial direction.

If it is flow between two parallel plates, then x might be the direction parallel to the plates. We can write this in the component form, so $\partial u/\partial x = 0$, $\partial v/\partial x = 0$ and $\partial w/\partial x = 0$. In of the most of the cases, you will see that for example when we talk about Cartesian coordinates then $\partial w/\partial x$ will be 0 because there will be no flow in the z direction, so w will become 0 and as we will see that v is equal to also be 0.

So, most of the time when we use the fully developed flow, the relationship that we need to use to solve equations will be $\partial u/\partial x = 0$ where u is the flow along the x direction. So, that is one thing. Now we will also come across a term which is called velocity profile and that we will encounter again and again now onwards.

So, velocity profile simply refers to the variation of velocity along the transverse direction. So, for example, if this is a flow between two parallel plates or flow in a rectangular channel, the velocity profile means that the velocity as a function of y coordinate. As you can see here that the flow is fully developed, so the velocity profile you take x1 at location 1 or location x2, location x3 or x4, on all the locations the velocity profile is same.

But if you look at say, location y1 or y2, the velocity is varying, so there is variation in velocity along the transverse direction. And this variation is what we call velocity profile. So, for example, when we say the velocity profile is parabolic, that means this equation, the variation of velocity with respect to y, this equation is parabolic.

When we say this is linear, then this equation is linear, when we have a flow between two parallel plates, then the relationship of this velocity profile is Vm into $1 - y^2/a^2$ where a is the distance between parallel plates or when you have a flow in flow in a pipe then you have a velocity V =V maximum into $1 - y^2/R^2$ where R is the pipe radius. The flow that occurs before the flow become fully developed is known as developing flow.

And the length that is required for the flow to become fully developed is known as development length or the entrance length. So, you can see from here that if you take two corresponding points at locations, any location, say x2 and x3, then if you write u2 - u3 divided by x2 - x3 as an approximation of ∂u/∂x, the you can see because u2 and u3, they are equal, so this will become 0, so that is the definition of fully developed flow.

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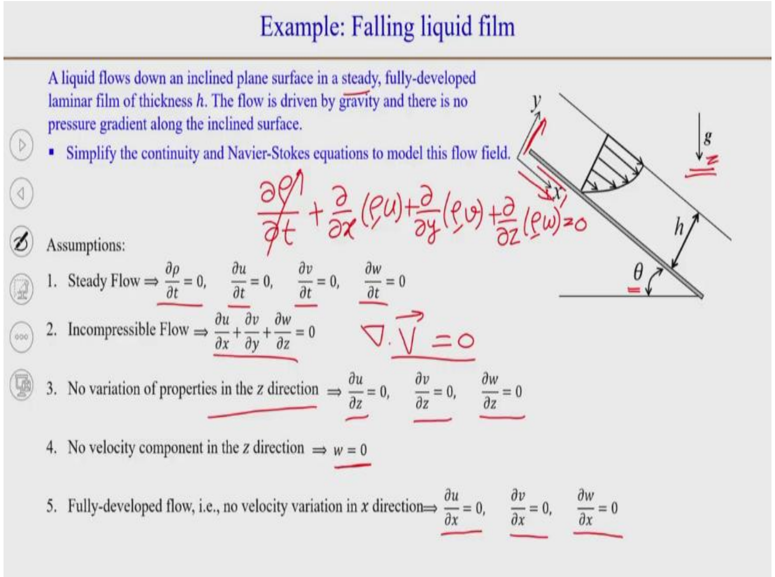

Now we will take an example of flow in a falling liquid film which is a common example. We can see it in many places, for example in a cooling tower, you can see a liquid film falling, so in this case what we take, a liquid film flows down through an incline plane surface in a steady, fully developed laminar film which is of thickness as, so the flow happens over this wall and it drains only because of gravity, it says that the flow is driven by gravity and there is no pressure gradient along the inclined surface.

So, we can see that in most of the cases that we deal with in this course, the flow will be driven by one of the three mechanisms, one is the pressure driven flow which is the most common occurrence, that the pressure at one point is high and pressure at other point is low and because of the pressure difference there will be flow happening.

The other driver for the flow will be gravity as here that from higher elevation the flow or the liquid goes from one place to another. The third driver is shear driven flow. So, for example, if we have some liquid on our hand, and if we blow over it then than that liquid will be flowing, there will be some flow in the liquid and that is because of the shearing notion that we provided by the air that we blew.

So, that will be shear driven flow or one of the problems that we solve is which is called Couette flow, so in the Couette flow, there is flow between two parallel plates, but there is no pressure gradient that drives the flow, but the flow is because the upper plate is moved by a certain force and it has a velocity. So, because of the velocity of that force, the fluid next to that or the fluid layer next to it, if the upper plate is moving with the velocity U, the fluid next to it will also move with velocity U here.

So, coming back to the falling liquid film, we need to write down the mass and momentum conservation equations continuity and Navier-Stokes equations and simplify it based on the assumptions that we have been given that the flow is steady, fully developed, laminar and incompressible. In this course, until and unless we have been specified or it has been specified, specifically that the flow is compressible, we will treat the flow to be incompressible.

And once we have simplified the equations, we will need to find the velocity profile in the liquid, the variation of velocity in this liquid film, the shear stress distribution in the liquid film and the volume flow rate in the liquid film, and once we find the volume flow rate we need to find the average velocity. So, let us write down first all the assumptions, so it will help us in simplifying the equations.

So, the first assumption is that the flow is steady, so that means that ∂by ∂t term for the all the variables will be 0, so in the continuity equation we will have $\partial\rho/\partial t = 0$. Similarly, in the Navier-Stokes equation, $\partial u/\partial t$, $\partial v/\partial t$ and $\partial w/\partial t$ in the three x, y and z component, they will be 0.

Now the flow is incompressible. So, when the floor is incompressible our continuity equation which is ∂ρ/∂t+∂by ∂x of ρ u+∂by ∂y of ρ v+∂by ∂z of ρ w =0. So, because the density is constant as the flow is incompressible then this ρ can be taken out and the first term is 0, because the flow is steady as well as the flow is incompressible.

So, we will have this ρ removed and the equation will remain, the equation will come out to be ∂dot V equal to 0 or in the expanded form ∂dot dou, $\nabla . U = 0$ or ∇.Velocity equal to 0 which in the expanded form ∂by ∂x of u+∂by ∂y of v+∂by ∂z of w =0. The next assumption is that there is no variation in fluid properties in the direction normal to the screen.

So, the flow happens along this plate and we can assume that the coordinate along the plate is x and the coordinate normal to it is y and the coordinate which is normal to this screen is z. So, there is no flow normal to the screen or there is no flow velocity normal to the screen and there are no gradients along those directions.

So, we can have there, when there are no gradients, so ∂by ∂z of u =0, ∂by ∂z of v =0 and ∂by ∂z of w is equal 0, the gravity acts along this direction, so it will have x and y components. And there is no velocity component in the z direction, so w =0 because there is no flow in the z direction.

And we also assume that the flow is fully developed as has also been given in the problem, so there is no velocity variation along the x direction, that means $\partial u/\partial x = 0$ and we already have w =0, so of course, ∂by ∂x of w =0 and ∂by ∂x of v is 0. So, with all these assumptions and the understanding of coordinates that x direction is along the plate, y direction is normal to it, and the z direction is normal to the screen. The plate is at an angle θ from the horizontal and the gravity acts in this direction.

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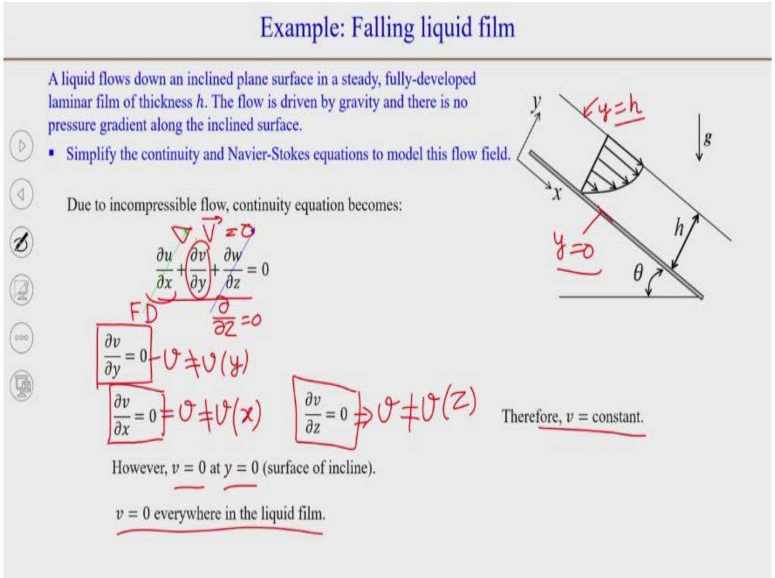

So, let us write down the equations. First, we write down the continuity equation, which is already simplified to this form, ∇.V. Now we can start looking at each term one by one. So, the first term, ∂u/∂x =0, because the flow is fully developed, ∂by ∂z for all the variables is 0, or w =0, so the last term is 0, so we will have only this term that ∂v/∂y =0.

So, our continuity equation is simplified now. Now we already know, because the flow is fully developed, so ∂by ∂x of v =0., and because there are no gradients in the z direction, so ∂by ∂z of v =0. Now that means that v is not a function of either of x or y or z. So, v is simply a constant. So, this tells us that v is a constant.

Now we know that along the y direction at y =0 which is the wall where y will be equal to 0 and at the other end y will be equal to h which is open to air or open to atmosphere. So, at y =0 there is no-slip boundary condition, so v and u will be 0 there, so because v =0 at y =0 and because v is equal to constant in the entire fluid, so v is going to be 0 everywhere in the liquid film.

So, now we know that the flow is one dimensional because w is 0, v is 0, so there is only one component of velocity u, which is along the x direction which is non 0, so the problem is one dimensional now.

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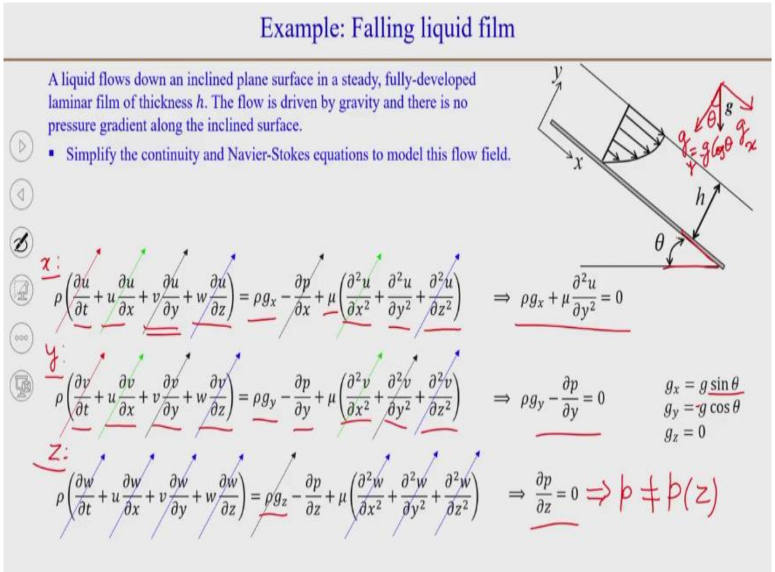

Now let us write down the momentum conservation equations. So, this is the momentum conservation equation along the x direction. So, when we write down the momentum conservation equation along the x direction we can try to simplify this equation by seeing or by finding out what terms can be neglected or can be canceled. So, the first term which is time dependent term, ∂u/∂t =0.

Then the next term u ∂u/∂x and this term on the right hand side, second derivative of u with respect to x, both of them are 0 because they are fully developed flow, so ∂u/∂x is 0. Next w is 0 or ∂by ∂z is 0, so this term is 0 here. And this term is also 0, so all the derivative with respect to z are 0. Now we know that v =0, so this term is 0 and we can simplify this term, so we are left with one term which is ρ gx and another term which is μ multiplied by $\partial^2 u/\partial y^2$.

So, our equation simplifies the x momentum equation simplifies having only two terms. The next we look at the momentum conservation equation or Navier-Stokes equation in the along the y direction. So, again, in this the first time will be 0, because the flow is steady. Next term is 0 because v =0 or ∂v/∂x =0 and this term will also be 0 because this is v is 0, w is 0 or ∂by ∂z is 0, so these terms which are derivative with respect to z, they will be 0.

And because v is 0, so these two terms will also be 0. And finally, we have two terms left in this, that ρg_y - ∂/∂y (p) =0. The third or equation which is the momentum conservation equation along the z direction. All the term which have w in it will be 0, because w =0 and there is no component of gravity along the z direction, so this term is also equal to 0, so this gives us the $\partial p/\partial z = 0$ and that means that p is not a function of z or p is constant with respect to z.

So, with this we have simplified the x, y and z momentum equations and let us try to simplify these equations further or integrate it, so we will be able to find out the velocity profile. Now we can sort it out here that g will have component along x and y direction. So, this will be the component of g along y direction and this will be along x direction.

So, along the x direction, we will have $g_x = g \sin\theta$, we know that the angle between the horizontal and the plate is equal to θ, so the angle between the normal to this plate, which is this and the vertical direction that will θ. So, this angle is θ, so gy will be equal to g cos θ but - sin because y is on the other direction and $g_x = g \cos\theta$, so gy will be - g cos θ.

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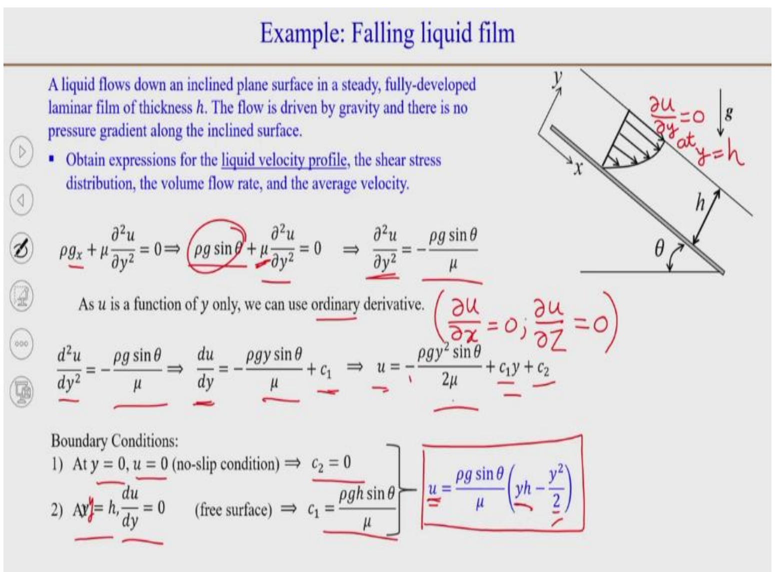

Now we can simplify this equation and integrate it, so we can substitute what is gx, we just saw that gx is g sin θ, so this term become ρ g sin θ plus you have $\mu\, \partial^2 u/\partial y^2$ and we can integrate it. So, before integration, we can take this term on the other side and μ also on the other side, so this becomes $\partial^2 u/\partial y^2 = -\rho g \sin\theta/\mu$ and integrate it.

Note that u is a function of y only, we know that $\partial u/\partial x = 0$, so it is not a function of x and $\partial u/\partial z = 0$, so it is not a function of z. That means u is a function of y only, so we can change this partial derivative to an ordinary derivative, so we can write the second derivative of u that will be equal to - ρ g sin θ/μ and we can integrate it.

So, when you integrate it, you will get du/dy =- ρ g sin θ into y but divided by μ plus a integration constant, let us say this constant is c1. And when we integrate it further, we will get u =- ρ g y^2d/2 into sin θ/μ+c1 into y plus another constant of integration c2. So, now we have two unknowns, c1 and c2, and we can use two boundary conditions, one at the wall and another at the free surface of the film.

So, at the wall, we already know because this wall is stationary, so at y =0, u =0, that is because of the no-slip boundary condition. So, when you do that at y is equal 0, u is 0, so this term is 0, this will also be 0 and this term will also be 0, so you will get c2 =0. Now at the free surface at y =h, we have du/dy =0, because we just saw when we discussed the boundary conditions on the free surface at the gas-liquid interface.

The velocity gradient in the liquid film can be approximated to be 0, so we can say that at this point $\partial u/\partial y$ =0 which at this point is at y =h. So, we can use that and we already know what is du/dy and when we write that at this is 0 at y =h, so we will simply get c1 =ρ gh sin θ/μ.

So, when we substitute that, the values of c1 and c2, we will get u =ρ gy sin θ/μ of - ρ gy^2 sin $\theta/2\mu$+c1, which is ρ g h sin θ/μ into y and we can take ρ g sin θ/μ out from it, so from here we will get h into y, so yh - y^2d/2. So, that is our velocity profile in the liquid film which gives us the velocity variation or variation in u with respect to y.

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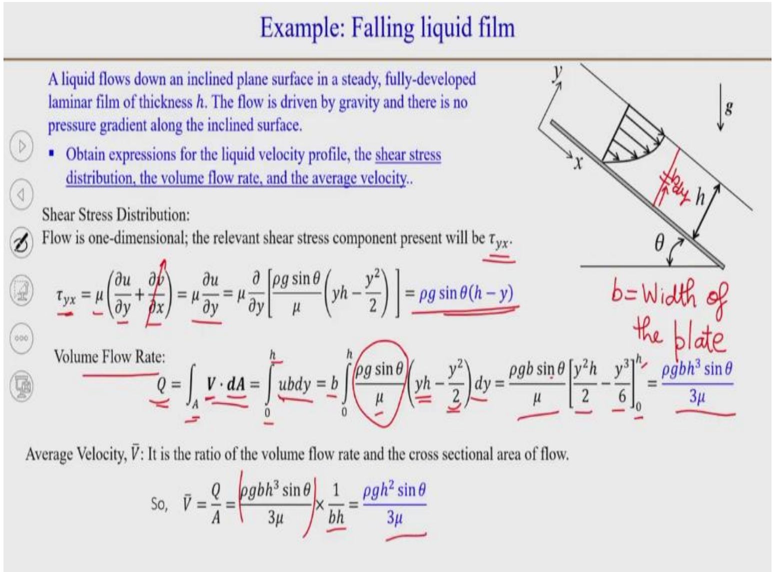

Now we will find out the shear stress distribution in this liquid film, because the flow is one dimensional as we saw that the flow happens only along the x direction. So, the only shear stress component which will be non-0 will be τyx or τxy, so we can write τyx that will be equal to μ into ∂u/∂y+∂v/∂x and because v =0 or ∂by ∂x =0, so this term will be 0 and you will τyx =μ ∂u/∂y and we already know what is ∂u/∂y.

So, we will have μ multiplied by ρ g sin θ h - y. So, that is the general expression for the shear stress in the liquid film and you can see that this equation is linear. So, you will have the linear variation of shear stress, whereas the velocity profile as you saw in here, the velocity profile this equation is parabolic. So, you have a parabolic variation of velocity profile here.

And then we need to find the volumetric flow rate which will be, the volumetric flow rate will be integral V.dA over a cross-sectional area because the flow is fully developed, so you take any section at any x and integrate the velocity over the cross-sectional area because the velocity varies along the y direction, so we will need to integrate it.

We cannot take it to be uniform flow, and the area will be, if you take any cross-section at a distance y and this will be dy, let us take that the depth of the plate of the width of the plate normal to the screen is b, so this area will be b into dy, where b is width of the plate, which is the dimension of the plate normal to screen.

So, dA will be bdy and you will have ubdy integral from 0 to h, so b is a constant, you can take it out and you can replace u with the expression for velocity with is ρ g sin θ/μ into yh - y^2d/2 into dy, and we can take all this also out of the integral sign, because this is constant with respect to y, so we will have a ρ gb sin θ/μ into when we integrate yh, so h is a constant into y^2d/2, first term.

From the second term we will get $y^3/3$ into 1/2, so that is $y^3/6$. When you put the integral limits from 0 to h, so the first term will give you $h^3/2$ - $h^3/6$ and you will get from here $h^3/3$, so you will get ρ gb sin θ μ into $h^3/3$ that is your flow rate in the liquid film. Now finding out the average velocity which is nothing but the flow rate divided by the cross-sectional area and cross-sectional area is b into h. So, you divide the flow rate expression by bh and you get the average velocity.

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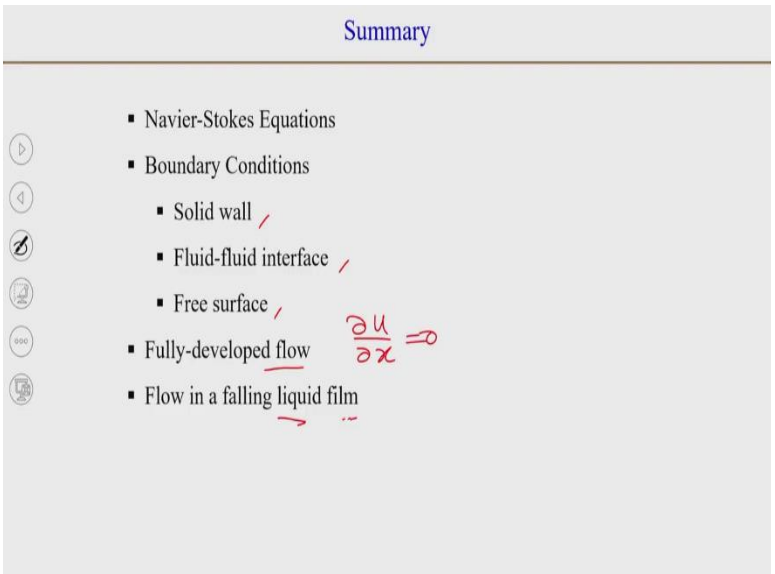

So, in summary, in today's lecture we have looked at the Navier-stokes equations and how we can use the assumptions that are inherent in the problem or that have been explicitly given in the problem, how we can use those assumptions and apply the boundary conditions so that the equations can be simplified and we can integrate the equation and find a solution of the flow.

So, looked at the boundary condition at the solid wall will be no-slip boundary condition, at the fluid interface we will have continuity of velocity and continuity of shear stress and at the free surface we can neglect the shear caused by the gas, which is above the liquid film, so at the interface at a free surface, we can assume that the velocity gradient is 0.

And we also looked at that the flow is fully developed which is that ∂by ∂x where x is the direction of flow, ∂by ∂x of V is 0, where V is the velocity vector or we can write it in component form and the most important part will be that $\partial u/\partial x = 0$ where x is the direction of flow. And we applied all of these concepts to solve the flow in a falling liquid film which has a number of applications in chemical as well as biomedical engineering for example, the flow in the film which we have in our eyes which lubricates our eyes.

So, we will stop here. Thank you.