Fundamentals of Fluid Mechanics for Chemical and Biomedical Engineer

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Lecture 20

Fluid Kinematics

So, in this module we will talk about Fluid Kinematics. As we know the Kinematics refers to the motion of bodies or motion of particles without actually analyzing or without considering the forces that the body or the particle is subjected to. So, in Fluid Kinematics we study the motion of the body, it can be the position, it can be the velocity, the rate of change of position of the particle or the rate of change of velocity, which is acceleration and if the body or the fluid particle rotates, then the rotation or if it undergoes deformation, then the deformations.

So, in this lecture or in this and subsequent lectures in this module, we will be looking at all these components of motion of the fluid particle, which will be handy for us, when we consider the dynamics of fluid that is we also consider the effect of forces. So, for example, if we want to analyze the motion of fluid because of a force being applied on the fluid, we will use Newton's second law of motion and there we will need F = ma, so we will need acceleration.

So, first we need to find out what is acceleration. Similarly, if we talk about that the fluid is subjected to a shear stress and now, we know that when there is shear stress being applied on the fluid, then there is an angular deformation or shear rate. So, the rate of angular deformation we need to find and that is what we are going to discuss in this module.

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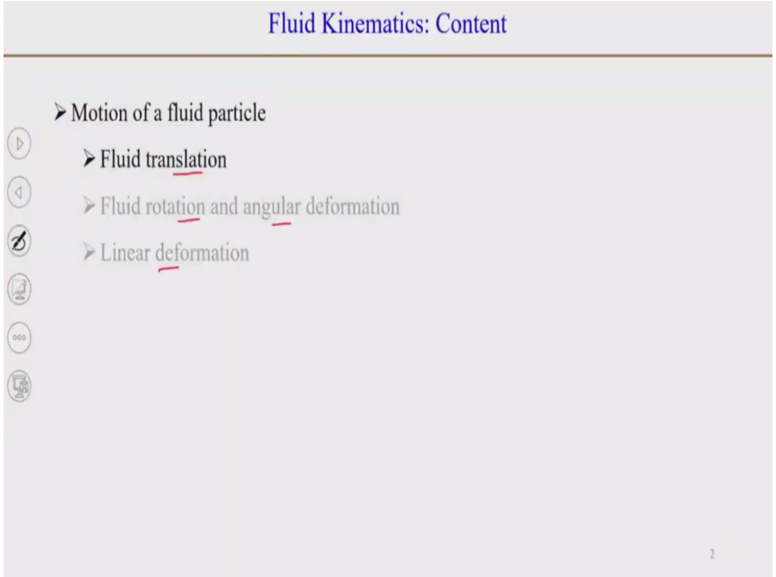

So, here are the contents of this module we will be talking about the translation of the Fluid, we will be talking about the rotation and angular deformation as we will see later on that the rotation and angular deformation they, that mathematical expressions for them will come from one analysis itself. So, we will study them together and then we will talk about linear deformation.

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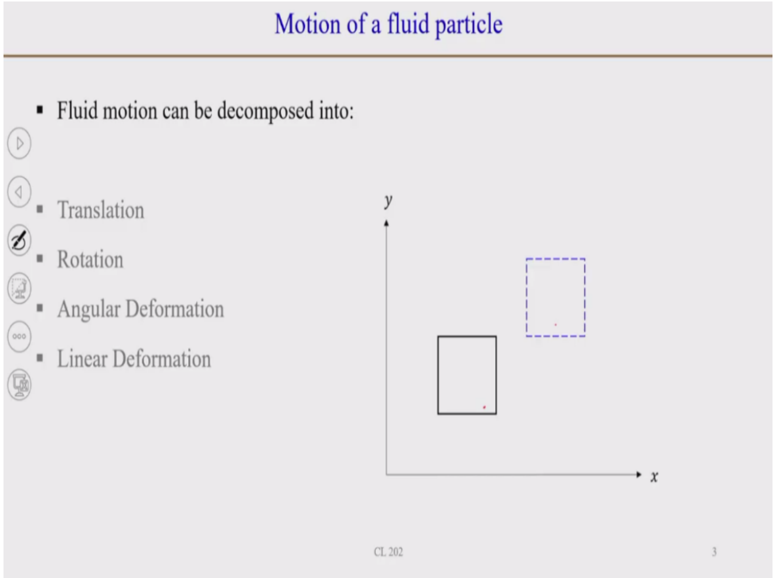

So, let us look at these, all the deformations and all the components of motion for a fluid particle, when a fluid particle if we consider the motion of a fluid particle in two dimensions, let us say in x and y coordinates and if we take infinitesimally small fluid element, now, this infinitesimally fluid element will be a two-dimensional surface. So, if we assume that the sides of this two-dimensional surface are very small, then we can approximate it as a rectangle, the sides can be assumed to be a straight. So, if we take a rectangular fluid element, which is infinitesimally small and consider the motion. So translation refers to that the fluid element has moved from one place to another without actually undergoing rotation or any kind of deformation, so that is translation.

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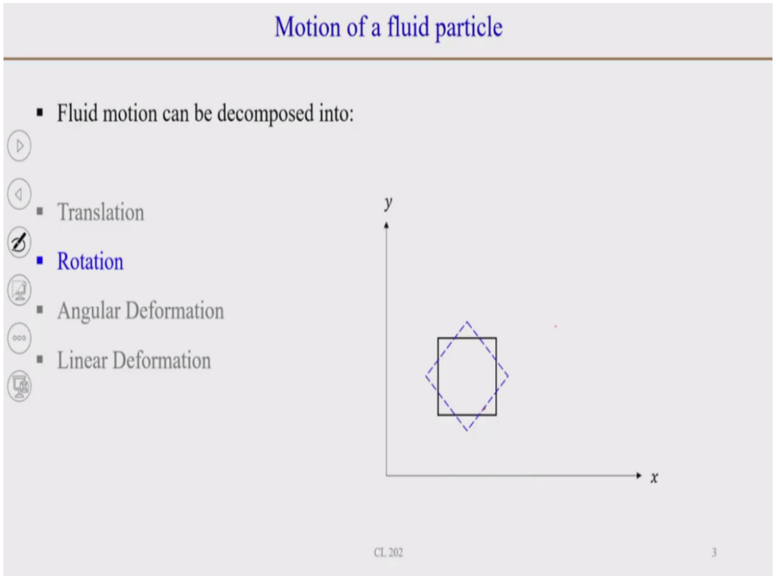

If the fluid particle just rotates about its own axis and it does not translate from one place to another or it does not undergo any deformation then it is rotation.

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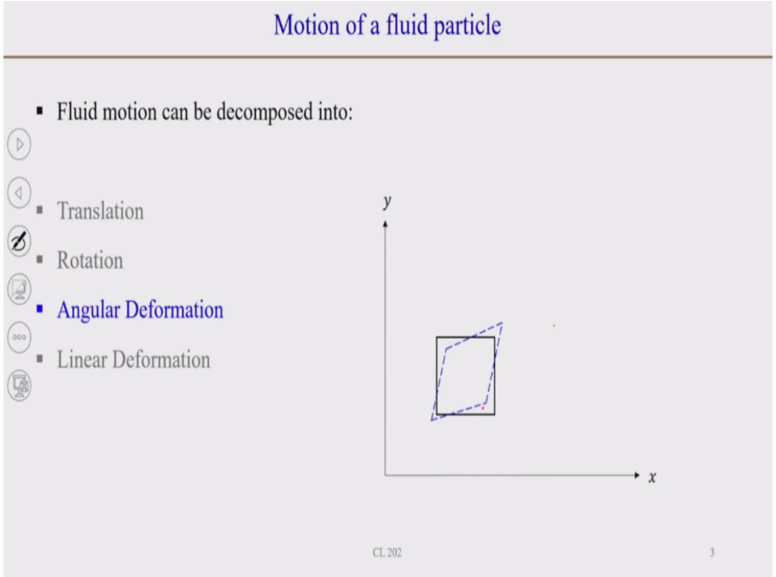

Then angular deformation refers to that the sides of the particle or side of this fluid element, they undergo a angular deformation. So, the shape of the particle changes from a rectangular, it does not remain rectangular anymore.

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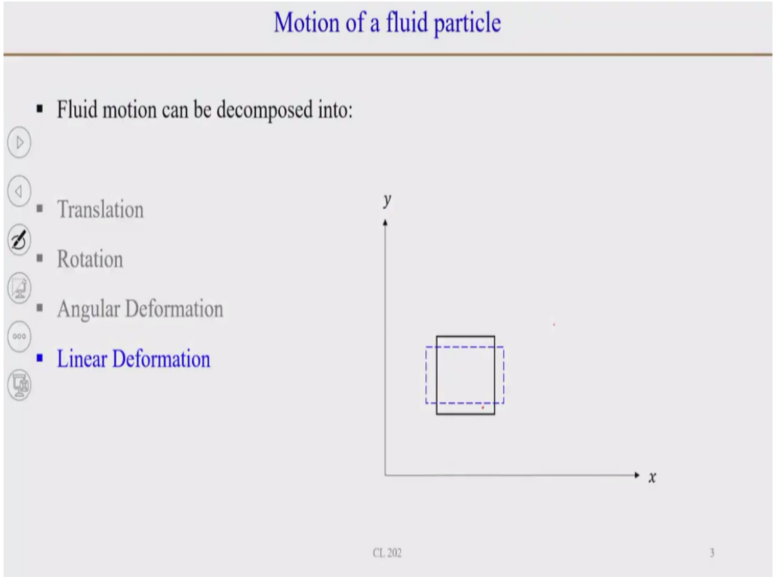

And the linear deformation refers to that the fluid is being stretched or compressed. So, this linear deformation will not be so important for this course, because we know that if a fluid is compressible then only it will be a we will be able to stretch or compress it and as we are concerned in this course about incompressible fluid then this linear deformation will be 0 as we will see in subsequent lectures.

But unfortunately, it is not so, that a fluid particle will undergo only translation, only rotation or only angular deformation or linear deformation, all these motions are combined together when a fluid particle is subjected to a shear stress are subject to a force or normal stress.

So, what we are going to do we will study all of these motion components translation, rotation, angular deformation and linear deformation one by one and then we can see that what are these different components for a particle, which undergoes all of those together. So, in this particular lecture, we will be talking about translation of a fluid particle.

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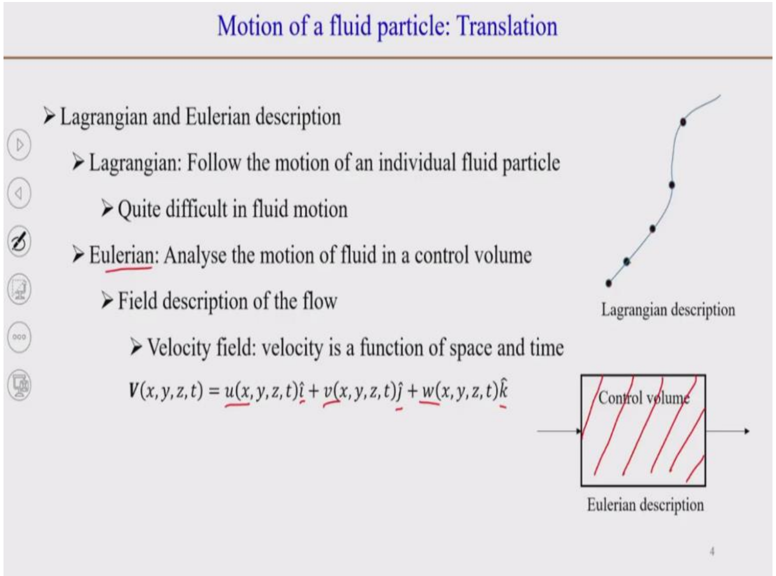

But before we do that, let us remind ourselves what is Lagrangian and Eulerian description of fluid motion. So, as we discussed earlier that Lagrangian motion refers to that if we follow the motion of a particular fluid particle. Now, that is the approach that we use in mechanics or in solid mechanics, where we take a rigid body and find or analyze the motion of this rigid body, but when we talk about fluid mechanics, we are often concerned with fluid flow in a particular domain or in a particular area.

For example, a chemical engineer might be concerned with fluid flow in a reactor, he is not concerned that what happens to or what is happening to a reactor when it comes in and what is happening when it goes out to analyze or to design the reactor he is concerned about the fluid that is in the reactor at a particular time. So, it is important or it is convenient to analyze the fluid motion in an Eulerian description.

So, Lagrangian description defines that one take a fluid particle and follow its motion as with time. So, what we see that the path of a fluid particle, but it is quite difficult to analyze in fluid motion and it is also not required or it is also not the goal. So, generally in fluid flow problems, one uses the Eulerian approach. So, one takes a control volume, it might be a reactor it might be a heat exchanger or it might be a separation vessel or even an imaginary flow boundary.

So, one takes a control volume and analyze the motion of fluid that is present in this control volume at any point of time. So, the description in Eulerian approach is of the field. So, for example, when we talk about velocity it is the velocity field it is not the velocity of a fluid particle, but velocity field is a function of position and time. So, if a fluid particle is at point x, y, z at time t, then its velocity is v.

If the same fluid particle is goes, it goes another point at x_1 y_1 z_1 at time t_2 then its velocity will be v_2 the velocity at that particular space or that particular point in the space. So, the velocity description this is a function of time and if we write in the vector form in terms of its component, so, x component u, v component or the y component v and z component w. So, the velocity as well its component can be, they can be function of time x, y, z and t where x, y, z are the position vector and t is time.

Of course, $\hat{\imath}$, $\hat{\jmath}$ and $\hat{k}$ are the unit vectors along x, y and z direction respectively. Now, with this knowledge, the goal here will be that how we can because we are going to analyze the principles of fluid motion in an Eulerian description. So, we need to knowing the velocity field in the Eulerian description, the goal will be to find out the acceleration of a fluid particle in an Eulerian description and it will not be simply just dv/dt or $\partial v/\partial t$ because it is a function of position.

So, when a fluid particle goes even in a, from one place to another the velocity of the fluid particle will change even if the flow is steady and the particle will have some acceleration. So, we need to derive an expression for acceleration in terms of velocity field. So, let us look at this.

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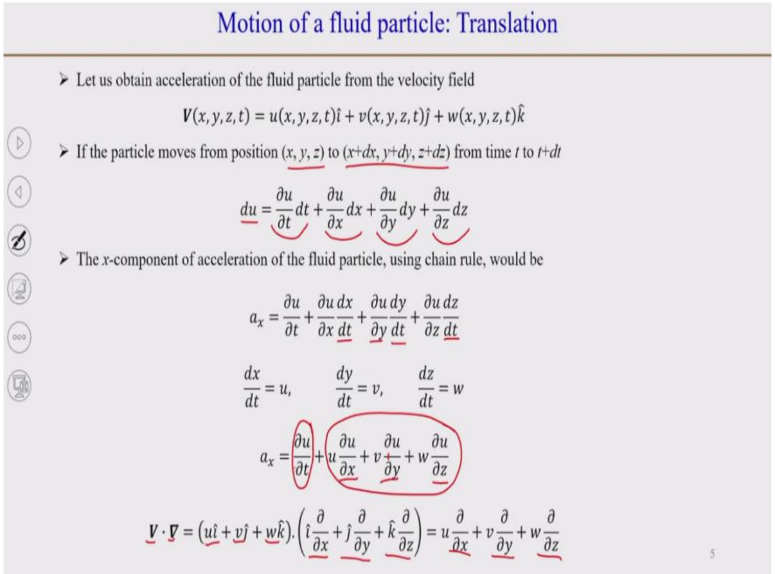

If we write down the velocity in terms of u, v and w vector form and consider that a point the particle moves from position a point, which has coordinates x, y and z to another point is small displacement. So, the displacement along the x direction is dx, along the y direction is dy and along the z direction is dz. So, the particle moves from point x, y, z to x + dx y + dy z + dz in time t to t + dt.

So, in a dt time it moves distance dx, dy, dz along x, y and z directions. So, the change in x component of velocity du in this case will be the change in fluid velocity because of time so, ∂u/∂t multiplied by dt that is because the velocity of fluid particle changes with time, or the velocity field changes with time or is a function of time. So, that is, but then particle also moves from point x to x + dx.

So, in moving from this distance the gradient or the change in u along the x direction so, ∂u/∂x multiply by dx that is because the particle has moved a distance dx along x direction similarly, ∂u/∂y is the change in the velocity because the particle has moved distance dy in the y direction and the last component because the particle has moved distance dz along the z direction.

And if we divide this by time, we will get the x component of acceleration which is du/dt or a_x = ∂u/∂t + ∂u/∂x dx/dt + ∂u/∂y dy/dt ∂u/∂z dz/dt and dx/dt dy/dt dz/dt they are u, v and w. So, we get that a_x = ∂u/∂t + u ∂u/∂x + v ∂u/∂y + w ∂u/∂z. So, that is x component of acceleration of fluid particle.

So, even though we see here we can clearly see even though $\partial u/\partial t$ is 0 the acceleration of the fluid particle will be nonzero depending on if the velocity is a function of u, v and w. We can slightly see that what how this function or how this expression look like. So, if we take V.∇ where V is the velocity field in the vector form and ∇ operator. So, in the vector form V is $u\hat{\imath} + v\hat{\jmath} + w\hat{k}$ in the Cartesian coordinate system and ∇ operator is $\hat{\imath}\partial$by ∂x + $\hat{\jmath}$ $\partial$by ∂y + $\hat{k}$ $\partial$by ∂z.

When we take their dot product, so, $\hat{\imath} \cdot \hat{\imath}$ will be 1, $\hat{\jmath}.\hat{\jmath}$ will be 1, $\hat{k}.\hat{k}$ will be 1, but the multiplication of two non similar vectors or non similar unit vectors $\hat{\imath}.\hat{\jmath}$ will be 0. Similarly, $\hat{\jmath}.\hat{k}$ will be 0 and so on because they are normal to each other. So, what we will end up with that u $\partial$by ∂x + v $\partial$by ∂y + w $\partial$by ∂z. And if we look at this term this is basically u $\partial$by ∂x + v ∂y ∂y + w $\partial$by ∂z of u.

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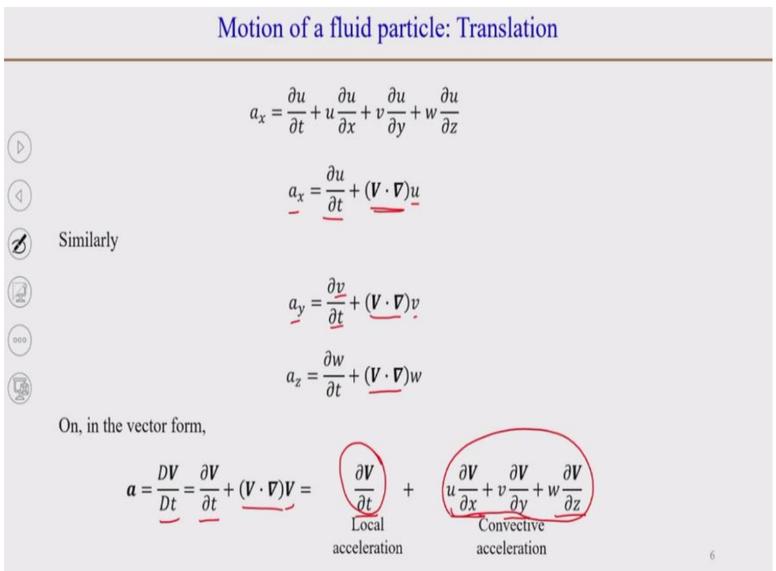

So, this can be referred as ∇.V or V.∇. So, we can write this in the vector form that $a_x = \partial u/\partial t$ + V.∇ of u. So, we can use this operator V.∇ operator. Now, in a similar manner, we can find out the y and z components of the acceleration of fluid. So, a_y will be $\partial v/\partial t$ + V.∇ y component of velocity v, similarly, z component will be $\partial w/\partial t$ + V.∇ w.

So, we need to see here or we need to remember that this term is V.∇ it is it becomes because this is a scalar product So, this becomes a scalar or we can write this acceleration in the vector form, which will be a_x $\hat{\imath}$ + a_y z + a_z $\hat{k}$ or a = Dv/Dt that will be equal to partial derivative of velocity vector + V.∇ operated on vector V. Or if you expand it, then you will have ∂V/∂t + u ∂by ∂x of V + v ∂by ∂y of V + w ∂by ∂z of V and this is a very important result.

The derivative of the velocity vector with respect to time is what is called local acceleration. So, time derivative is local acceleration and this term V.∇ v or u ∂by ∂x + v ∂by ∂y + w ∂by ∂z of vector V is called convective acceleration. So, this term comes because the particle is moving or the fluid is moving from one place to another and because of the change in velocity because of that, because of the change in velocity in space this comes into picture.

You can see that if ∂by ∂x ∂by ∂y and ∂by ∂z if they are 0 then this component will be 0.

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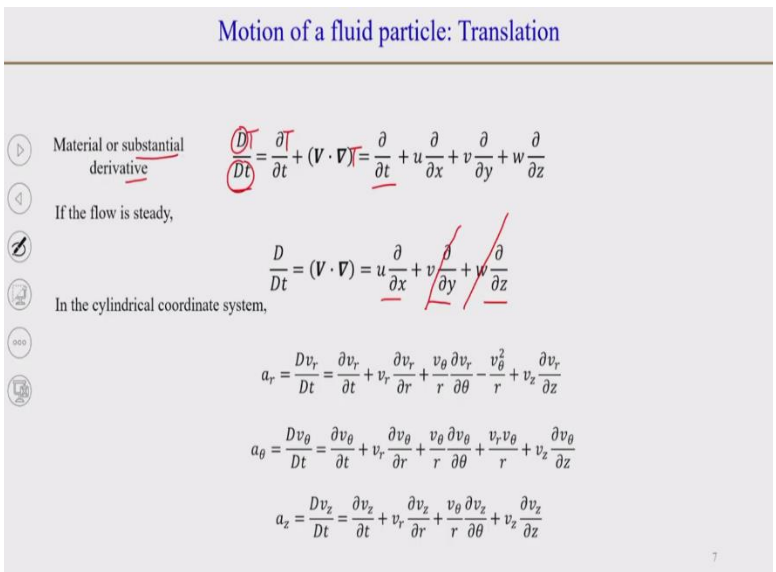

Though we have derived it for a velocity component or for a velocity vector, but this result that D/Dt = ∂by ∂t + V.∇ is a powerful result and it can be used to find out the variation of any scalar field in a control volume. So, for example, if we have a temperature then that capital DT/Dt = ∂ T/∂t + V.∇ of T. So, that will represent the variation of temperature in a particular velocity field and this derivative the capital D here refers to material or substantial derivative.

So, if a moving observer is measuring the temperature of it is measuring the density, the rate of change of that particular quantity that is being measured that will be obtained by this expression. So, this is not only valid for the acceleration of a fluid particle, but also for the time derivative of other quantities in Eulerian description. Now, if the flow is steady then the first term will become 0 and we will have u ∂by ∂x + v ∂by ∂y + w ∂by ∂z.

If the flow is 2 dimensional then this term will become 0 because the third component of velocity will be 0 if the flow is 1 dimensional then the other component of velocity will also be 0. So, you will have just u ∂by ∂x for 1 dimensional flow. Now, what we have done is we have analyzed the motion in a Cartesian coordinate system, but often in chemical as well as in biomedical engineering we are concerned with flow in an axisymmetric cylindrical pipe or in a Cylindrical channel. So, there it is convenient to use cylindrical coordinate system.

So, we can without going into the detail of derivation of this we can write the acceleration in terms of components a_r, a_θ and a_z in terms of v_r, v_θ and v_z and these are the expression. You see here that the expressions are not as simple as from the Cartesian coordinate system and this complexity comes because the derivative of unit vectors e_r, e_θ they are not 0 because as the, if the direction θ changes, then along with θ the e_r vector, the direction of e_r vector will change even though its magnitude is 1, but its direction change similarly, the e_θ vector direction will change with θ.

So, one need to take that into account but we will not derive this in this lecture. However, you can use these expressions to find the acceleration in a cylindrical coordinate system.

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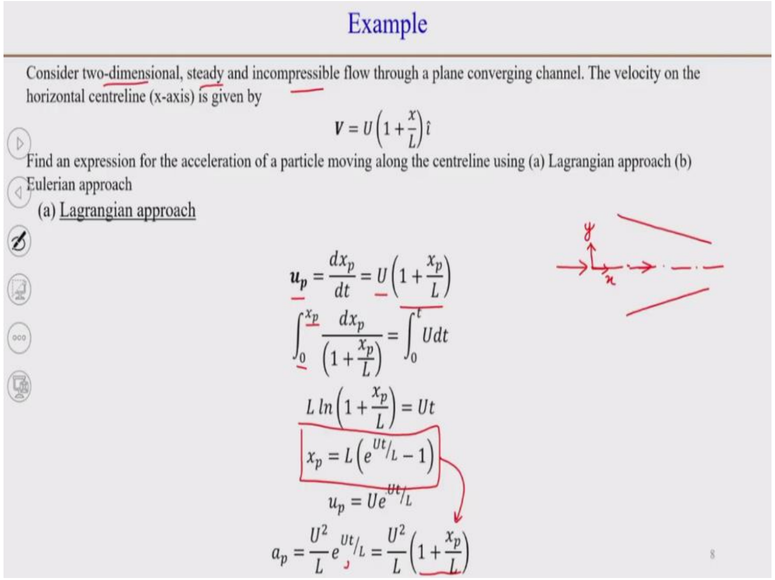

Now, let us look two examples to So, first in this example, what we are going to look at that verify for ourselves that if a particle is defined the acceleration of a fluid particle in using a Lagrangian approach and if we find the acceleration of a fluid particle or if we find the acceleration of fluid using Eulerian approach will they be same. So, here is that a flow is two dimensional, it is steady, so ∂by ∂t will be 0.

The flow age incompressible and throws, it flow through a plane converging channel. So, the converging means the diameter of the channel is decreasing as you go along the downstream direction of the flow, flow is happening in this direction, the velocity on the horizontal center line, so, x axis. So, we can say that this is the x direction and normal to it will be y direction. Now, the velocity we have been given at the central line which is velocity vector V = U (1 + x/L) $\hat{\imath}$.

So, there is only x component of velocity at the central line, which is obvious because of symmetry the flow will be happening only in, along the x direction. So, on this is 1 dimensional flow other components of velocity the y component of velocity small v and z component of velocity w will be 0. The goal is to find the expression for acceleration using Lagrangian approach and Eulerian approach.

So, first we will consider the Lagrangian approach. So, using Lagrangian approach the velocity of the fluid particle will be equal to U(1 + x_p/L), where x_p is the instantaneous location at time t what is the location of the fluid particle is x_p or the position vector of the fluid particle, because this is the most only along the axis. So, the position will be along the x axis. So, the position of the fluid particle is x_p at time t and u_p gives its velocity.

Now, if we integrate it, we will get the position of fluid particle at time t. So, we substitute $dx_p/(1 + x_p/L)$. So, we bring x_p on one side and time on other sides. So, integral 0 to t Udt and if we integrate it, we will get l natural log of $1 + x_p/L = Ut$ the limits of integration are from 0 to x_p if the initial position at time t = 0 if the particle is 0 and then at any time t the position of the particle is x_p.

So, from, after rearranging this you will get L goes on the other side so, it will become Ut/L and 1 get $1 + x_p/L$ = exponential Ut/L when we write it in terms of x_p then x_p or the position of the particle is L $e^{Ut/L}$ -1. So, that is the position of the fluid particle as a function of time. Now, from this position of fluid particle we can obtain U as well as V.

So, up with respect to time will be $Ue^{Ut/L}$ when we derive or when we differentiate this with respect to time we will get $u_p = Ue^{Ut/L}$ or we can directly get it if we substitute the value of x_p here then we get up in terms of time and the acceleration of the fluid particle will be the differentiation of up with time $U^2/Le^{Ut/L}$ that = $U^2/L(1 + x_p/L)$.

Because this I have written because when we obtain the expression from the Eulerian we will get the similar expression. So, this comes because if we substitute the value of $e^{Ut/L}$ from this expression, then we will get the acceleration in terms of x_p. So, this is the acceleration from the Lagrangian description. Note that x_p here is the position of the fluid particle.

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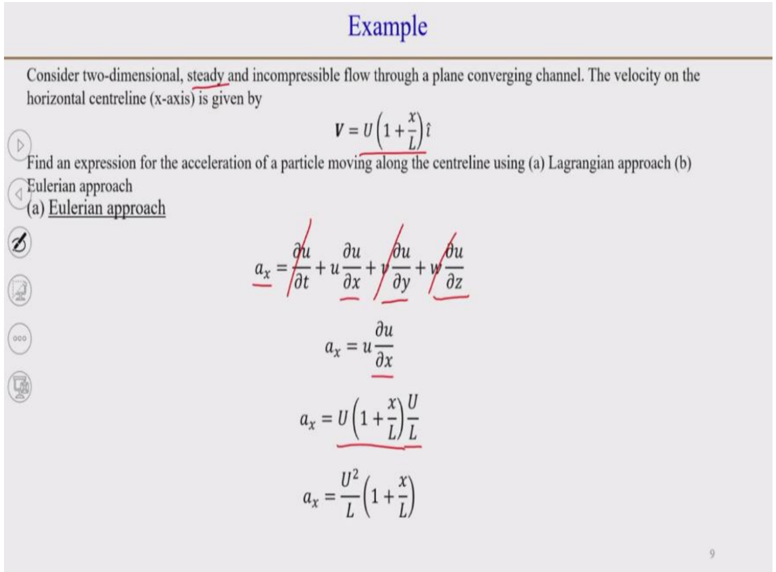

Now, we will use the Eulerian approach to find out the velocity of the particle. So, we know because it is only motion along the x component or motion along the x direction only. So, we consider the acceleration along the x direction a_x = ∂u/∂t + u ∂u/∂x + v ∂u/∂y + w ∂u/∂z. Now, because v is 0, so, this term will be 0 and w is 0. So, this term will also be 0 and we will have a_x = u ∂u/∂x because the flow is steady. So, ∂u/∂t will also be 0.

So, we have ax = u ∂u/∂x and we can substitute the value of u there, which is capital U 1 + x/L. So, U is 1 + x/L and ∂u/∂x will be U/L. So, we substitute the value of U/L and a_x = U^2/L 1 + x/L and it was the same where x was the instantaneous position of the particle. So, that was a problem to finding out acceleration using the different approaches and verifying that what we get the same acceleration from the two approaches.

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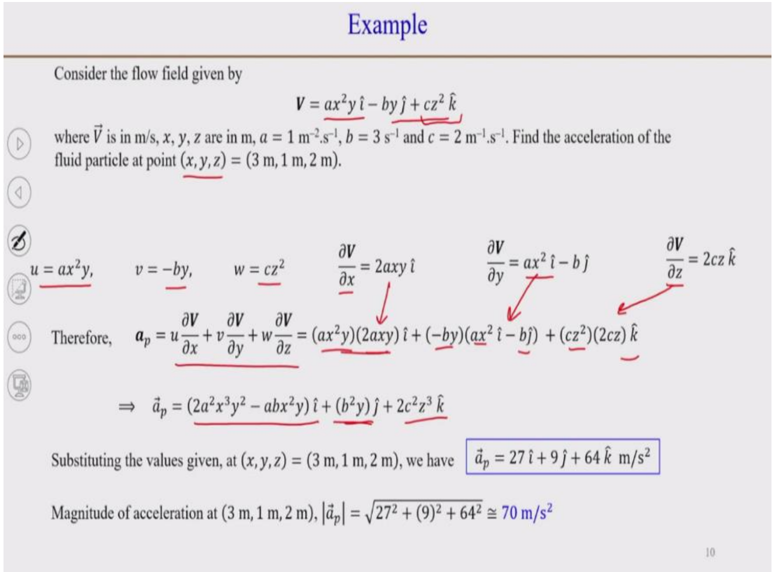

Now, let us look at another example, where we have been given that all the three components of velocity x, y and z component of velocity they are nonzero and then we need to find the acceleration of the fluid particle at a particular point, which coordinates are 3, 1 and 2. So, because small u, small v, small w that is the velocity components along x, y and z direction they are all nonzero.

So, this velocity field will have acceleration or acceleration components along x, y and z directions. So, we will need to find out the derivatives of different components. So, let us write this down first the u component or the x component of velocity is ax^2y, the y component of velocity is -by and z component of velocity is cz^2. Now, we will find because we need to find ∂y ∂x.

So, we will find the derivatives of these components or derivatives of even better because we can find, we will need the derivative of the velocity vector if we find or if we calculate in terms of vectors. So, derivative of velocity vector with respect to x. So, the first component from the first term we will get 2axy the other two terms are not function of x. So, the derivative of those terms with respect to x will be 0. So, $\partial$by ∂x of vector V is 2axy $\hat{\imath}$.

Now, if we find the derivative with respect to y you have the vector then the first term will give us $ax^2\hat{\jmath}$, from the second term we will get -b$\hat{\jmath}$ and the third term is not a function of y so, that the derivative with respect to third term will be 0. Now, derivative of the velocity vector with respect to z, so, that will be 2cz because the first two terms they are not a function of z. So, the only nonzero term will be the derivative of third component of the w component of velocity, so, 2cz $\hat{k}$.

So, having found that let us write down the expression for acceleration, because the flow is not a function of time. So, ∂V/∂t or the time derivative of velocity vector v is 0. So, we can write down the expression in terms of u ∂by ∂x + v ∂by ∂y + w ∂by ∂z of vector V and substitute the values. So, u is ax^2 y and ∂v/∂x is 2axy. Similarly, v is -by and ∂V/∂y is ax^2 $\hat{\imath}$-b$\hat{\jmath}$ and the z component of velocity is cz^2 multiplied by derivative of V with respect to z so, 2cz.

And if we simplify this, then we get multiplying this $2a^2$ x^3y^2 that is one component along the x direction, the other component along the x direction will be this term. So, $-abx^2$ y that will be the acceleration along the x direction + along the z direction will be b^2 y and along the $\hat{\jmath}$ direction sorry along the y direction that will be the velocity component and then along the z direction, the velocity component will be $2c^2$ z cube $\hat{k}$. So, that is the velocity field in general expression.

Now, we can substitute the values of a, b, c and x, y, z and find the value of vector a_p, which comes out to be 27 $\hat{\imath}$+ 9 $\hat{\jmath}$ + 64 $\hat{k}$. I request you to check the calculations once in case there is a calculation mistake. And then the magnitude of this acceleration at this point will be root 27^2 + 9^2 + 64^2 that will be about 70 meter per $second^2$. So, that is all about Fluid Acceleration and this in this lecture, what we have learned is that, how to represent the or how to find the acceleration of fluid given a velocity field.

And we also need to remember or we also should remember that the substantial derivative can be used to find the derivative of any scalar or vector quantity in an Eulerian description. So, that is a very useful result in fluid mechanics in particular, and in transport phenomena in general when we are as a Chemical Engineer or even in Biomedical Engineering when we are looking at transportation of heat or transport of different species, then we need to look find the derivative of c or concentration field or the derivative of temperature field we can use substantial derivative. So we will stop here. Thank you.