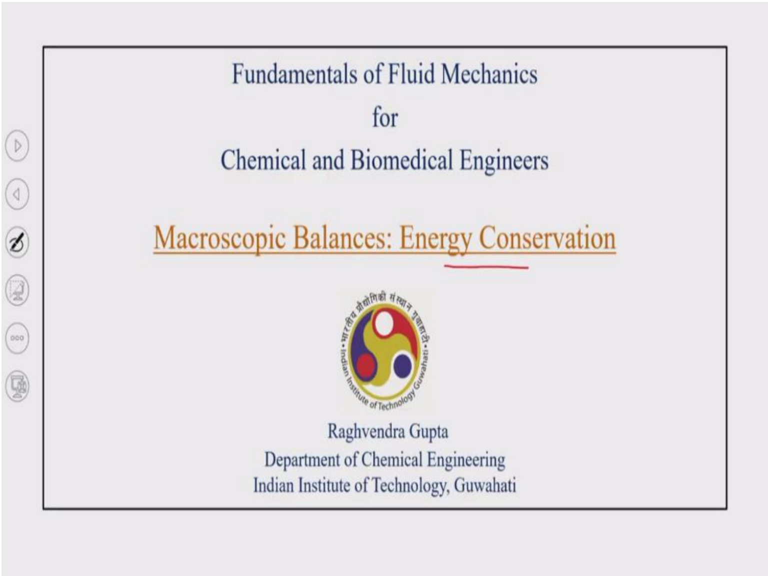
Fundamental of Fluid Mechanics for Chemical and Biomedical Engineers
Professor Raghvendra Gupta
Department of Chemical Engineering
Indian Institute of Technology, Guwahati
Lecture 19
Macroscopic Balances: Energy Conservation

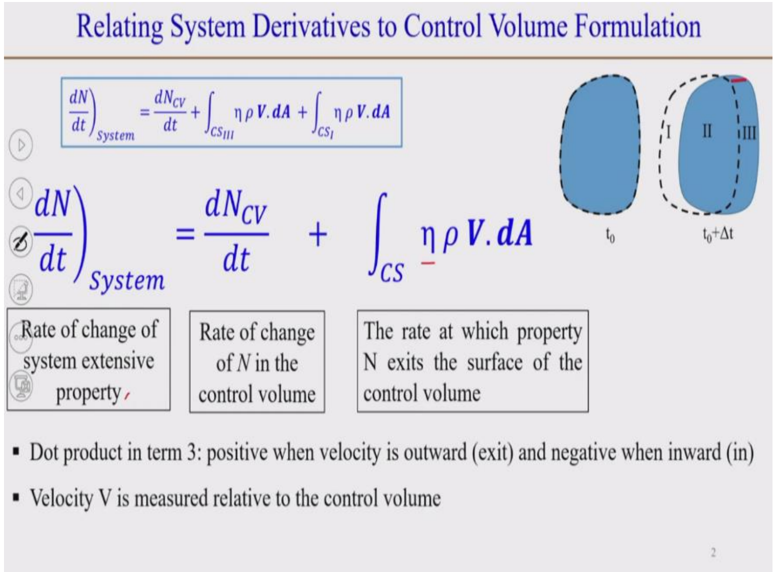
In previous lectures we have applied Reynolds Transport Theorem. First we derived the Reynolds Transport Theorem and then applied it for Macroscopic Balances of Mass and Momentum.

(Refer Slide Time: 00:57)



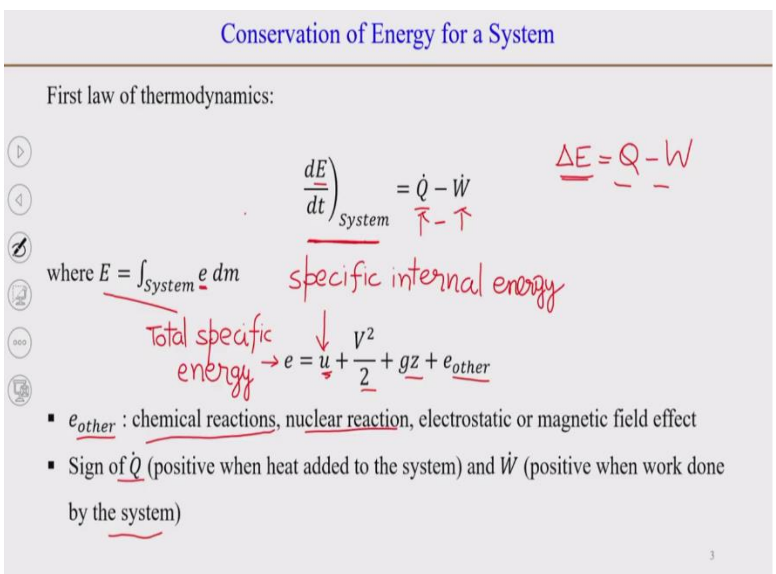
In the lecture today we are going to do it or we are going to apply Reynolds Transport Theorem to derive a equation for energy conservation in Eulerian framework or in the Control Volume Formulation.

(Refer Slide Time: 01:10)



So, the usual Reynolds transport theorem that we have derived, which says the rate of change of system extensive properties. So, when we talk about energy the system extensive property will be the total energy of the system and the corresponding intensive property η in this case will be specific energy or total specific energy, which we will represent with small e.

(Refer Slide Time: 01:49)



So, let us look at the Energy Conservation for a System. So, when we apply first law of thermodynamics for a system, because we are talking about a flow system. So, the change in total energy of the system rate of change of total energy of a system is equal to $\dot{Q}$ which is the energy supplied to the system and $\dot{W}$ which is work done by the system. So, we can see the simple balance that the energy that will be there in the system that will be equal to or the difference will be the change in energy of the system ΔE, the more familiar form of first law of thermodynamics is in terms of ΔE = Q - W.

So, which is ΔE is the change in energy of the system that is equal to the energy that is being supplied to the system minus the work done. So, the energy being spent by the system to do the work, so that is why you see a - sign there. Now, when we talk about a flow system, it is going on continuously. So, in place of having a ΔE we will talk about in terms of rate. So, the rate of change of energy of a system is equal to rate by which the energy is being supplied to the system minus work done by the system. So, rate of work being done by the system.

Now, this capital E the corresponding intensive property as I said is small e, which is called specific energy of the system. So, that will be dE = e dm. So, if you want to integrate it over the systems so E = integral over the system e dm. Now, this specific total energy of the system so, E is total specific energy of the system whereas small u is specific internal energy of the system.

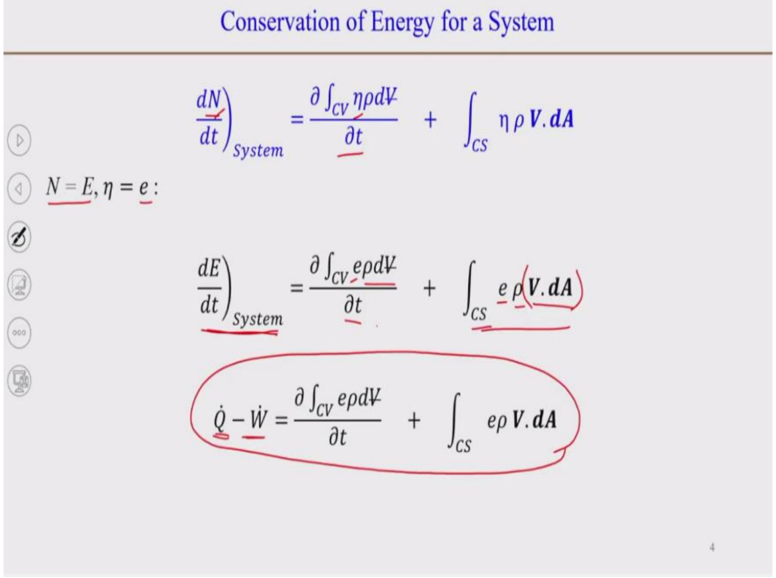
And remember probably this is the only lecture where we are talking about small u being the specific internal energy of the system, everywhere else we have used or we will be using small u as a symbol for x component of velocity. So, please remember that here u is specific internal energy of the system. It can be kinetic energy per unit mass, potential energy or per unit mass plus some other forms of energy that might be there.

So, these other forms of energy can include the chemical energy caused by chemical reactions exothermic or endothermic reactions energy due to a nuclear reaction or electrostatic or magnetic field effects causing energy. So, most of the cases, almost all the cases that we consider in this course, e_{other} or this contribution to total specific energy will be 0. Now, as we said that, the sign convention that we follow here $\dot{Q}$ is the heat provided to the system.

So, this will be positive if heat is being given to the system. So, $\dot{Q}$ will be positive if heat is being given to the system and $\dot{W}$ which is work done by the system is positive when work is being done by the system. So, if work is being done on the system, then $\dot{W}$ will be negative, if heat is being taken away from the system or system provides heat to its surroundings, then $\dot{Q}$ will be negative.

In some places, you might find that this sign between $\dot{Q}$ and $\dot{W}$ is positive. So, in that case $\dot{W}$ will be work being done on the system. So, now we have refreshed our memory in terms of the first law of thermodynamics or energy conservation for a system. So, let us apply Reynolds Transport Theorem to change this formulation to a control volume formulation. So, what we are going to do we are going to write the rate dE/dt for a system. We will right or convert this using Reynolds Transport Theorem, the formulation for a control volume and at a particular time when the system and control volume coincide, then $\dot{Q}$ and $\dot{W}$ are same.

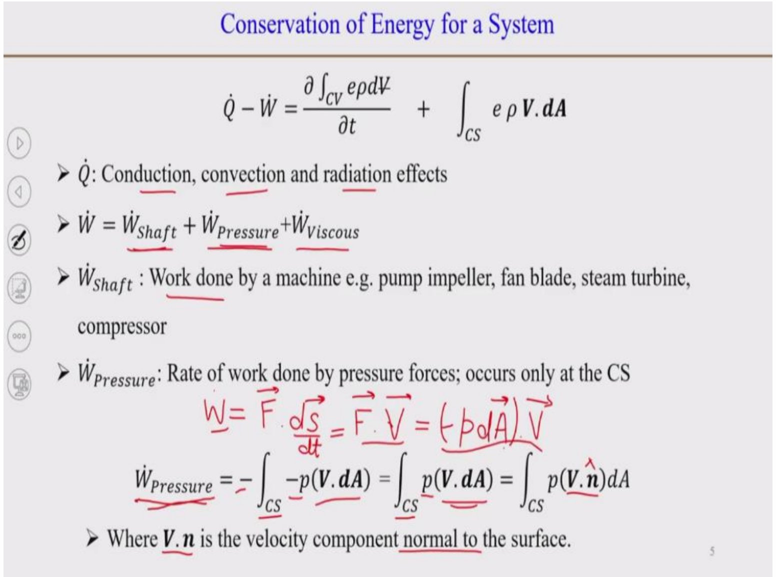
(Refer Slide Time: 07:56)



So, we write the Reynolds Transport Theorem in terms of general variable N, and substitute N is equal to E which is total energy of the system and intensive property η is equal to small e which is total specific energy of the system. So, we replace N/E So, dE/dt of system = integral $\frac{\partial}{\partial t}$ integral over the control volume and η is replaced by e. So, e ρ dV that is rate of change of total energy of the system with time plus the energy that is coming in or going out through the control surface.

So, area integral over the control surface e ρ V.dA. So, we use this in the first law of thermodynamics. So, dE/dt = $\dot{Q}$ - $\dot{W}$ so we replace dE/dt of system = $\dot{Q}$ - $\dot{W}$ and we have the energy conservation in the form of or for a control volume or in control volume formulation. Now, we will look some of the variations or some details about $\dot{Q}$ and $\dot{W}$.

(Refer Slide Time: 09:40)



So Q $\dot{Q}$ is basically, it is heat being supplied or taken away from the system or it is being given to the system. So, it is heat transfer and heat transfer can be caused by conduction, convection or radiation effects. Because in this course, most of the times what we will be looking at is isothermal systems. So, we will not go in further details of it. Now the work so, $\dot{W}$ can have the shaft work or work being done by a machine.

So, that is the shaft work which can be work being done by a pump impeller or work being done by a fan blade or steam turbine or compressors. So, depending on the machine the shaft work can be positive or negative work being done on the fluid or work being done by the fluid. So, that will have, that will be the shaft work component.

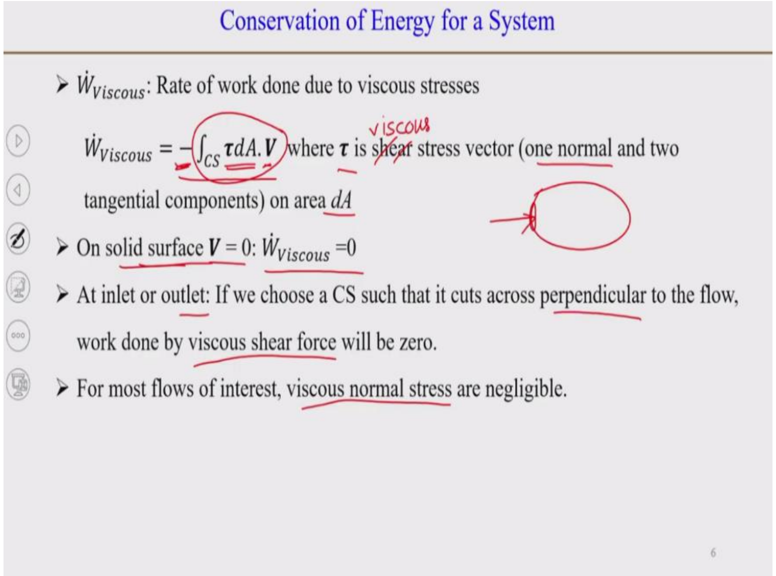
Now, the flow will be coming in or going out. So, there will be work due to pressure and work due to viscous forces. So, the pressure forces and viscous forces they will also contribute to the work. Now, these viscous forces can be normal viscous stress or viscous stress can have normal components as well as shear component or tangential components. In general, we will see that the normal viscous stresses are negligible. So, we will neglect them and most of the cases what we will consider is viscous shear stresses.

So, if we come to work done due to pressure, so, or rate of work done due to pressure then we know that work is force dot product of displacement. So, if we write displacement as D and rate of work done will be equal to F.dS over dt. So, that will result in we can write this F.V. So, dS/dt is nothing but velocity. So, rate of work done is F dot, dot product of force and velocity. Now, this is the work done by the force and this force when we talk about pressure force this will be - pdA.

If we talk about the small force, force on a small area dA, so, that will be equal to dot V. Now, this - sign is to take into account the fact that the pressure force will be acting opposite to the normal or outward normal to the surface. So, this is the work done by the pressure force whereas in our formulation we have work done by the system. So, the pressure work or rate of work due to pressure force that will be equal to negative or minus one times integral over the control surface this force. So, - p V.dA we can write dA.V = V.dA.

So, that will be minus, minus will cancel out so we will have integral p V.dA that is the rate of work done by the pressure force. We can also write this in terms of V.n into the dA, which is the area of the surface, so where V.n is basically the velocity component which is normal to the surface. So, we can say the unit vector n is the unit vector normal to the surface. So, we can write in this form or this form. So, this is the rate of work done due to the pressure force.

(Refer Slide Time: 14:45)



Similarly, we can write rate of work done due to viscous stresses. Now, the rate of work done due to viscous normally stresses will be same as the rate of work done by pressure force except that viscous normal stresses will not have a - sign when we write force, force will be τ dA. Now, viscous stress we can write minus times integral over the control surface τ dA, which is the force dot velocity vector, this term is because of the work done due to the viscous stresses on the control volume.

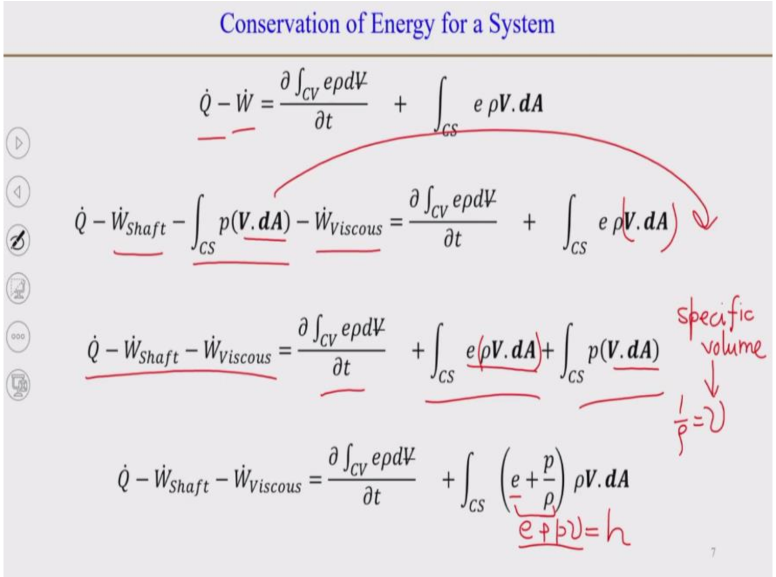
So, if we want to write the work done by the control volume, then it will be with a negative sign. So, τ is where τ is the shear stress vector. So, remember that we are talking about a particular surface dA, where we will have three components of stresses. So, at that particular surface, we will have a stress vector and on the surface you will have three components of stresses. So, it can be a viscous stress vector actually, because it will have a normal component and two tangential or shear components on an area dA.

Now, when you have a control surface, which is coinciding with the solid surface or solid wall, so, on a solid wall we know because of no slip boundary condition velocity is 0. So, when the velocity is 0, then the rate of work done because of the viscous stresses will be 0, because V is 0 in this.

At inlet or outlet, so on any control volume where the flow is coming in or going out. So, let us say this is your control volume and this is. Now if we choose a control volume in such a manner that the flow enters because most of the cases when we are solving a problem, we have the freedom to choose the control volume. So, if we choose our control volume in such a manner that the control surface that is the boundary of the control volume is normal to the inlet and outlet ports or normal to the inlet and outlets.

So, in that case the normal vector to the area will be, the shear stress will be acting normal to the velocity. So, they will be perpendicular and the dot product will result in 0 viscous shear force. As I said earlier that the viscous normal stresses are often negligible for most of the flows that we are going to look into. So, we can neglect viscous normal stresses and for these two cases where the control surface is a solid surface or the flow enters normal to the area the velocity is normal to the control surface then on that surface the rate of work done by viscous stresses is 0.

(Refer Slide Time: 18:22)



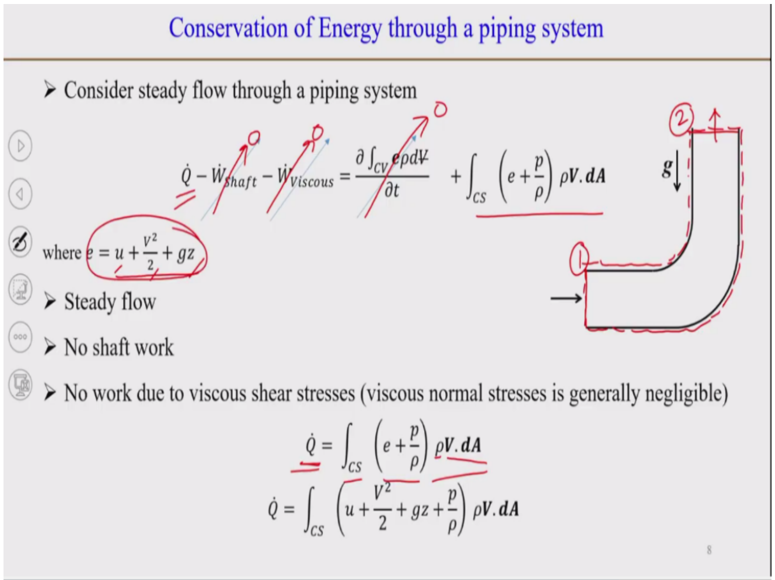
So, with these three different components of viscous stresses, we can write down in the general formulation of energy conservation for a control volume. So, we can write $\dot{W}$ is equal to the work done due to shaft, the work contribution due to the pressure force and the work contribution due to the viscous stresses or viscous forces. Now, we can take this term because we have a V.dA here and V.dA on the right side. So, we can try to combine this.

So, we can rearrange it on the left side we will be left with $\dot{Q}$ - $\dot{W}$ shaft - $\dot{W}$ viscous is equal to the unsteady term plus integral over the control surface e ρ V.dA plus the work done due to the pressure force. So, this terms is brought on this side. Now, if we combine these two terms, so, take ρ and V.dA outside the bracket then we will have e + p over ρ.

So, we can generally write this in this form or 1 over ρ if you remember from thermodynamics, which is often present and it is called specific volume. So, 1 over density is called a specific volume is commonly used in thermodynamics. So, this is e + pV if you look at this, this term is nothing but e + pV where V is specific volume and what is this, this is called enthalpy h.

So, in thermodynamics you might see this term being written in terms of enthalpy in general and pV is generally known as flow work. So, we can use this general formula to solve problems for energy conservation in a control volume.

(Refer Slide Time: 21:08)

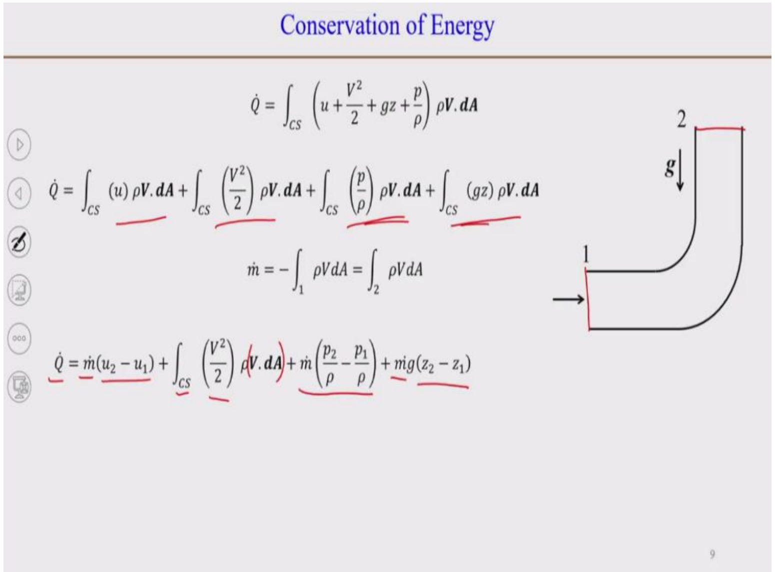


Now, if we choose a small system which is a simple piping system, so, let us say you have a elbow where the flow is coming in at section 1 and it goes out at section 2, the gravity acts vertically downward on this. So, we can write down the conservation equation in this, now we can further expand or we can write that e = u + V^2. So, specific internal energy plus kinetic energy and potential energy.

If the flow is steady, which is generally the case for most of the problems, then this term will be 0 and in this there is no machines. So, in this system there are no machines. So, the shaft work is also 0. Now, we can choose our control volume in such a manner. So, we can choose control volume coinciding with the wall and normal to the flow at the inlet and outlet ports. So, the shear stresses on the wall will be 0 and shear stresses or the work done because of the shear stress, not the shear stresses on the wall will be 0, but the work done because of the viscous shear stresses will be 0 on the walls and the work done because of the viscous shear stresses will be 0 on inlet and outlet ports because the control surface is normal to the flow entering.

So, viscous work done due to the viscous stresses is also 0. So, we end up with 2 terms $\dot{Q}$ and the net flux of, net flow of energy. So, that $\dot{Q}$ will be equal to integral over the control surface e + p by ρ into ρ V.dA and if you notice V.dA is the volumetric flow rate into ρ which becomes mass flow rate. Now, you can substitute e = u + V^2 + gz.

(Refer Slide Time: 24:01)

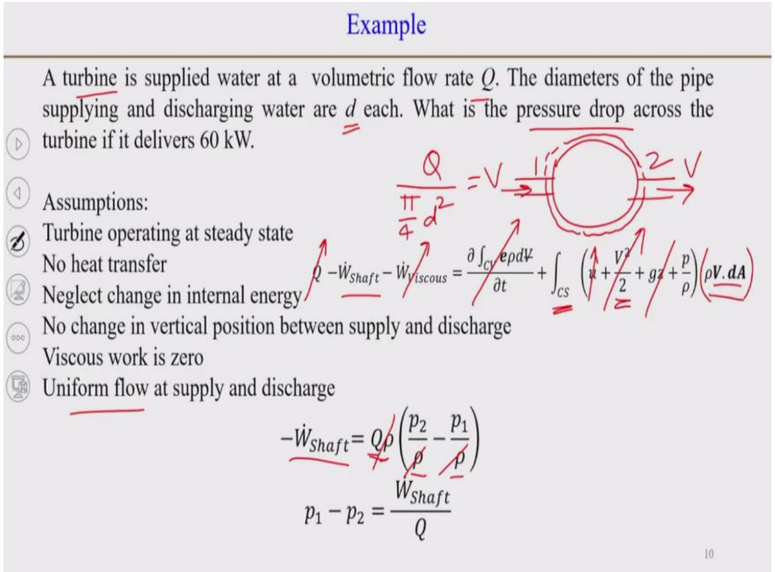


And then we can take each take up each term for the separately and expand or evaluate these integrals. So, the V.dA will be because at the inlet it will be pointing, the area vector will be pointing outward whereas velocity will be pointing inward. So, it will be - $\dot{m}$ the integral, whereas at the outlet it will be $\dot{m}$ where $\dot{m}$ is flow rate ρ into V into area of inlet 1 or outlet 2.

So, we can simplify this $\dot{Q}$ = $\dot{m}$ u_2 - u_1 plus, because of this term, we will get $\dot{m}$ p_2/ρ - p_1/ρ and because of the gravity term $\dot{m}$ g z_2 - z_1. This integral will have integral over the control surface V^2 ρ v.dA. So, it will depend if the velocity at the sections are uniform, then we can replace $\dot{m}$ into V^2, V_1^2 - plus $\dot{m}$ into V_2^2, but we will keep it in this form.

So, when we solve later on in the course, when we look at the flow in pipes and solve the problems of flow in pipes or pipe networks, then we will use this equation and we will again come back to this equation.

(Refer Slide Time: 26:06)



Now, let us take a simple example. So, there is a turbine which is supplied water at a volumetric flow rate Q and the diameters of the pipe that supply and discharge water through this turbine, let us assume they are same. So, they both have these are d and the question is what is the pressure drop across the turbine if it delivers the 60 kilowatt of shaft work? So, let us draw that if you have a turbine and the flow is coming in, flow goes out from here.

So, we can take our control volume, now when we write to the general equation for energy conservation for this control volume and try to see that what will be each term. So, first list down the assumptions, we assume that the turbine operates at steady state. So, the unsteady term will become 0, then we also assume that there is no heat transfer in the system so, $\dot{Q} = 0$. We can neglect the change in internal energy of the fluid. So, there will be no change in u.

So, Δu will be 0. So, that term can go and there is no change in vertical position between supply and discharge. So, z there will be no change in z at section 1 and 2. So, this term will also go and we integrate this term and the viscous work is 0. So, we can make viscous work 0 or by the choice of our control volume that the flow enters normal to the inlet and outlet. So, the viscous work is 0, so, this term is also 0.

Now, we are left with 2 terms V^2 into p/ρ. So, we can also assume let us say that the flow is uniform at the supply and discharge pipes. So, because the flow is uniform and the diameters are same, so, from mass conservation the velocity at the supply and discharge side will be equal that will be V which will be Q over pi/4 d^2. So, that will be equal to V. Now, when we integrate or we do not need to integrate because the flow is uniform.

So, this becomes ρ V.dA is mass flow rate and mass flow rate it is same at section 1 and 2. So, this term will also cancel out V^2 because V_1^2 and V_2^2 are same and this will be multiplied by mass flow rate. So, this term will also cancel out and what we will end up with that minus shaft work that will be equal to V.dA is Q into ρ and because there will be a - sign at the inlet. So, V.dA will be - Q at the inlet so, you have - p_1/ρ and at the outlet p_2/ρ. So, you have minus shaft work = Q ρ p_2/ρ - p_1/ρ. Now, ρ and ρ will cancel out.

So, you will end up with $p_1 - p_2$ = shaft work divided by Q. So, this is a simple example now. We will use the conservation of energy as I said for solving the pipe flow problems and one can also derive the Bernoulli's equation from Energy Conservation. So, we will do Bernoulli equation when we talk about inviscid flows, but it can be derived from the energy conservation principle also.

Let us stop here. Thank you.