Fundamentals of Fluid Mechanics for Chemical and Biomedical Engineers
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Lecture 18
Macroscopic Balances: Momentum Conservation II

In this lecture we will look at some examples further for Momentum Conservation Macroscopic Balances.

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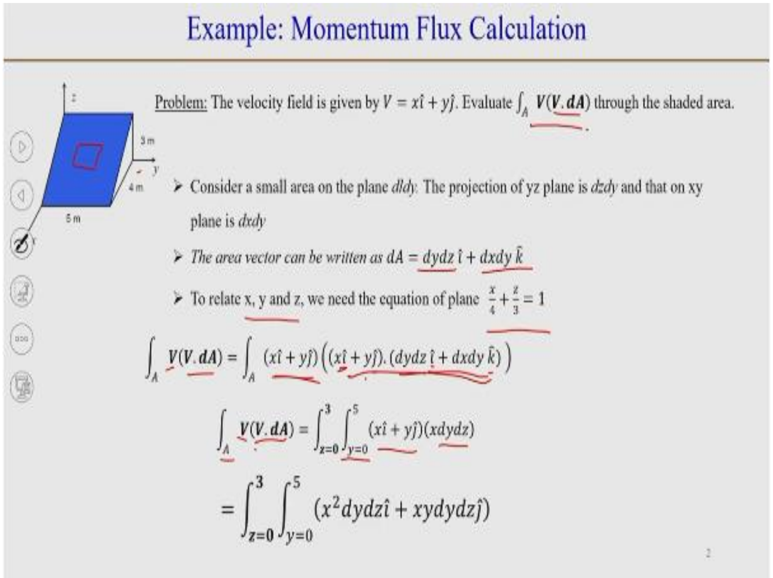

So, the first example we looked at when we were talking about mass conservation, we looked at the problem where we calculated the volumetric flow rate which is V . dA through this plane. Now, we can calculate momentum flow per unit, so if we put ρ into this then that will become a momentum flow. So, we will be calculating the integral, this integral because most of the time what we will be looking at is a, what we will be looking at is an incompressible flow.

So, ρ will be simply multiplied to this integral, so what we want to see is how to calculate this integral or how to calculate momentum flow, we have the plane so, as earlier we will not do it in detail but we will take a small plane on this area dldy and then look at its projection and find the area vector dA, so which we can write dydz $\hat{\imath}$+ dxdy $\hat{k}$, for further details you can go back to the example that we did in the mass conservation lecture.

So, to relate because we have dy, dx, dz we will, x, y, z need to be related and for that we will write the equation of plane which have intercepts a, b, c on x, y and z axis and when we do that we will get the equation of plane. Now, we will substitute these values here, so V is $x\hat{\imath} + y\hat{\jmath}$ and then we have V . dA, dot product so we will take the dot product $(x\hat{\imath} + y\hat{\jmath}).(dydz\ \hat{\imath} + dxdy\ \hat{k})$, when we take the dot product of these two, because $\hat{\imath} . \hat{\imath} = 1$, $\hat{\jmath} . \hat{k}$, $\hat{\jmath}$ and k are normal, so that will become 0 and $\hat{\jmath}$ and $\hat{\imath}$ are normal so j . $\hat{\imath}$ will be 0, $\hat{\jmath}. \hat{k}$ again will be 0. So, we will have only one term which will be $\hat{\imath} . \hat{\imath}$.

So, we will have this part within the bracket it will reduce to xdydz and $\hat{\imath} . \hat{\imath} = 1$ and the first bracket $x\hat{\imath} + y\hat{\jmath}$ remains same. So, remember the difference from mass conservation there it was only V . dA, so it was a scalar quantity but now we have this scalar quantity V . dA or volumetric flow rate multiplied by V, which is a vector, so this is a vector quantity and we have to integrate it over the area of this plane, so if you look at the limits integral y = 0 to 5 and integral z = 0 to 3 as we can see from here. So, we can do the integration, we can take this inside $x^2\ dydz\hat{\imath} + xydydz\hat{\jmath}$.

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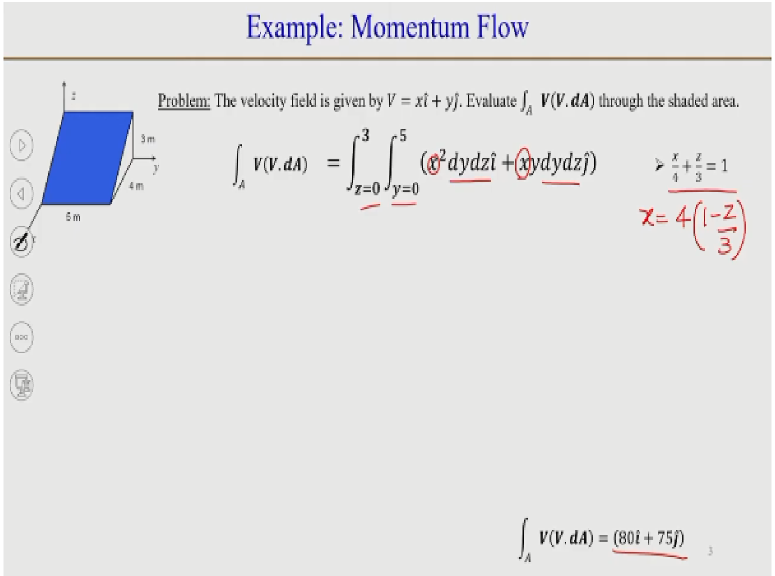

Now, when we substitute the values and because we are doing, we have the limits of y to z and we are doing integration with respect to y and z, so this x needs to be written in both the places in terms of y and z and that is where we can use the equation of plane. So, we can write x =4 into 1 - z by 3 and when we substitute and integrate, we will get the answer in this form, we will get $80\hat{\imath} + 75\hat{\jmath}$. so, I will leave it to you to do the little bit of algebra and calculus and find out the answer.

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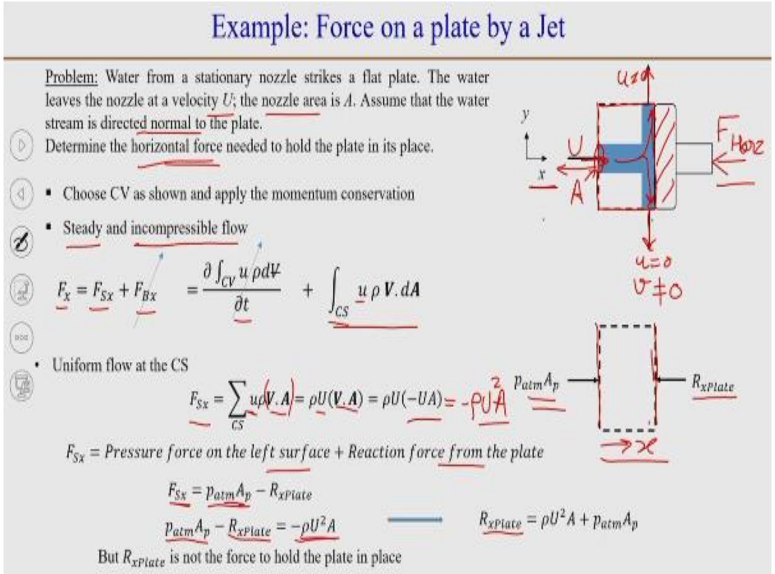

Let us look at the next problem which is force on a plate by a jet. So, if you are holding a plate for example or if a jet hits say from a water hose, the water hits at your palm what is the force experienced by your palm or force experienced by a plate that is what the problem is about.

So, this is an schematic of the problem, that water from a stationary nozzle, it strikes a flat plate, so this is the flat plate and the water jet hits this plate, so when it hits the plate in such a manner that it distributes symmetrically on the two sides, the plate is vertical and we choose the coordinate system such that the jet is along the x direction and the direction normal to it is y direction, so it is a two dimensional problem, we can say or we can assume the problem to be two dimensional.

Now, we have been given the velocity, so the jet comes with a velocity U and the area of this nozzle from which the jet comes out, so the jet area is A of course, the intrinsic assumption here is that the nozzle area and jet area both are equal and we assume that the water stream is directed normal to the plate, so it is normal to the plate. Now, what we need to find out is the horizontal force that is required to hold the plate in place.

So, the force that is required to hold the place or hold the plate in its place, now we are going to solve this problem using two different control volumes, the objective of using two different control volumes is to demonstrate that the choice of control volume is important, if you choose a correct control volume or if you choose control volume judiciously, then that effort becomes less.

So, the first control volume that we have chosen is as shown by the dotted line here, so this is our control volume, now we can write down the momentum conservation equation. We need to find the horizontal force, so we need to write the momentum conservation equation only along the x direction, the liquid is being distributed in the vertical direction but because the horizontal force is a part in the momentum conservation equation along the x direction, so we will solve the equation along the x direction only.

So, we have chosen the control volume and now we will apply the momentum conservation equation, of course, we will assume the flow to be steady and it is flow of water, so it is an incompressible flow. The force that can have a surface forces and body forces but there is no body force along the x direction, so this body force will eventually be 0, the flow is steady so this term will be 0. Now, when we do that both these terms are 0 and the flow is uniform, so we can replace this integral with a summation and the problem simplifies a bit.

So, we write F_{sx} because there is no body force, so the surface force along the x direction that is equal to on replacing the integral with summation you have u ρ V . A, so V . A is a scalar quantity and u is the x component of velocity. Now, if we simplify this then u is x component of velocity and x component of velocity, the flow is along x direction and in this control volume the flow comes in with a velocity U, there is only one surface on this control surface at the x plane where you have the non zero velocity in the u direction.

At two places at these two places the x component of velocity is 0, x component of velocity is 0, of course, there will be a v component which is not 0 in both these places, but we are not considering v component of velocity, so we do not need to find this out. Now, if we replace it in place of u we write down capital U, so that is x component of velocity we need to find V . A and the area vector at this point will be pointing outward, so we will have a U into the jet area, so that will be - U A. So, V . A becomes - U A.

So, $F_{sx} = - \rho U^2 A$. Now, this is the total force along the x direction and this force will be equal to if we take the control volume and think about the forces that act on this control volume, so we have a diagram showing the forces along the x direction on this, on this surface the x plane, the pressure will be acting on the left surface that will be equal to the atmospheric pressure multiplied by the area of the plate because this area is equal to the plate area.

Now, when the water jet hits the plate here a reaction force will also be applied on the water, so in this control volume the force on this side will be the reaction force from the plate. So, there are two forces, the pressure force on the left surface and the reaction force from the plate. So, we can write down F_{sx} is equal to the force due to pressure, so $p_{atmospheric}$ into area of plate and which acts in the positive x direction and the reaction force which is acting in the negative x direction, remember that this is the x direction, positive x direction that we have chosen.

So, from this we can find or we can rearrange write down $F_{sx} = - \rho U^2 A$, so $p_{atmospheric}$ multiplied by A_p which is pressure force on the left surface - reaction force by the by the plate on the control volume that will be equal to F_{sx} which $=- \rho U^2 A$.

So, now if we rearrange and find what is $R_{x\ plate}$, the reaction force on the plate that will be equal to $\rho U^2 + p_{atmospheric} A_p$ but this is the reaction force that the plate applies on the control volume and remember what we need to find is the horizontal force that is needed to hold the plate in its place. So, we have not found the horizontal force yet.

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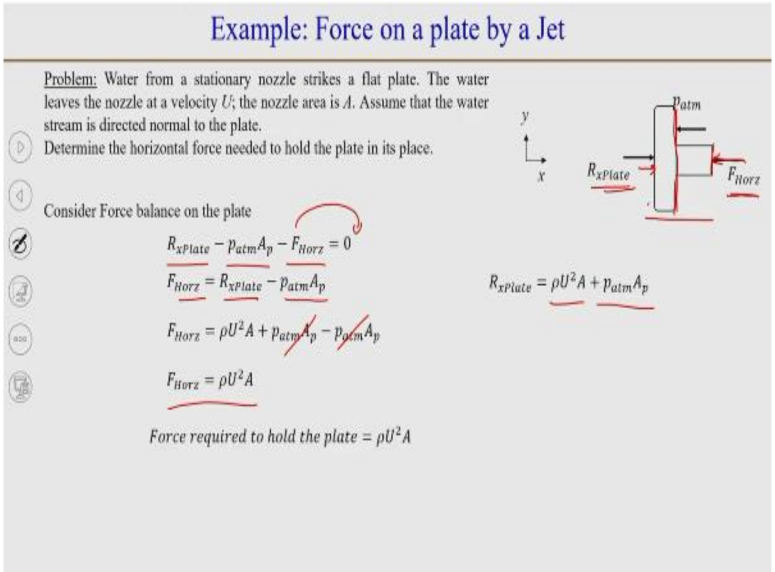

So, now what we need to do is we can do the force balance on this plate itself and find out the horizontal force. So because the plate has to be held stationary, so the forces, to all the forces will balance so that the plate is in equilibrium. So, we will apply the force balance on this plate. Now, if we look at the plate there will be a reaction, equal and opposite reaction force on the plate in the positive x direction, then there will be a force due to atmospheric pressure in the entire plate surface area, in the entire, so you can consider this area and this area ultimately, this will be the plate area.

So, $p_{atmospheric}$ into A_p will be the force because in the negative x direction because of the atmospheric pressure of the surrounding air and the horizontal force that you will need to put additionally to hold the plate stationary. So, when we write down this balance we can write the $R_{x\ plate}$, the reaction force that is being applied on the plate in the positive x direction - pressure force $p_{atmospheric}$ A_p in the negative x direction and the horizontal force which is in the negative x direction.

So, from this we can write an expression rearranging and find the expression for horizontal force, so horizontal force goes on this side and we find that F horizontal =$R_{x\ plate}$ - $p_{atmosphere}$ into A_p. Now, we can substitute the value of $R_{x\ plate}$ which we find on the, which we found on the previous slide. So, $R_{x\ plate}$ is equal to ρ U^2 A + $p_{atmospheric}$ A_p and we can substitute this here, so $p_{atmospheric}$ A_p that will cancel out and we get F horizontal = ρ^2 A. So, we have been able to find, we are able to find the horizontal force that is required to hold the plate stationary.

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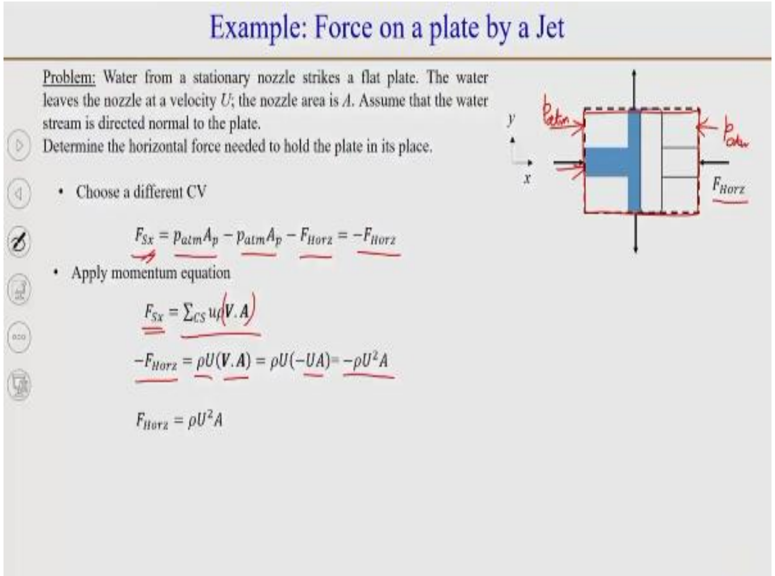

Now, we can choose a different control volume and try to solve problem again. So, this time we choose a control volume which surrounds the jet as well as the plate. Now, if we see here that in this control volume the horizontal force acts on the control volume itself and the flow is coming in inside the control volume. So, if we do this, if we apply the momentum conservation in this control volume and write down the equations.

So, first let us look all the surface forces because the body force again will be negligible so the surface forces on the two sides, so from the left side there will be atmospheric pressure, $p_{atmospheric}$ multiplied by plate area and from the other side also you will have $p_{atmospheric}$ and this will be multiplied by area.

So, you have p_{atm} A_p pressure force on left side, pressure force on the right side, they will cancel out and the horizontal force, the reaction force does not come in picture here, so you have the net force in the x direction = - F horizontal, you can substitute in the momentum conservation equation. So, if you remember when we wrote down momentum conservation equation the body force term was 0 and the unsteady term ∂ by ∂t term was 0.

So, this is summation over u ρ V . A. So, we replace F_{sx} = - F horizontal ρ U and this will be V . A again - U A, so that - F horizontal = - ρ U^2 and F horizontal = ρ U^2. So, we can see by the choice of two different control volume, when we chose a control volume on which the force required was acting, we were able to solve the problem easily, we just needed to do or apply the momentum conservation equation only once.

However, if we would have been asked that what is the reaction force on the plate or by the plate, then the first choice of control volume was appropriate, so it depends on what is it that we are looking for, what is the force, what are the flow rates that will determine the choice of control volume.

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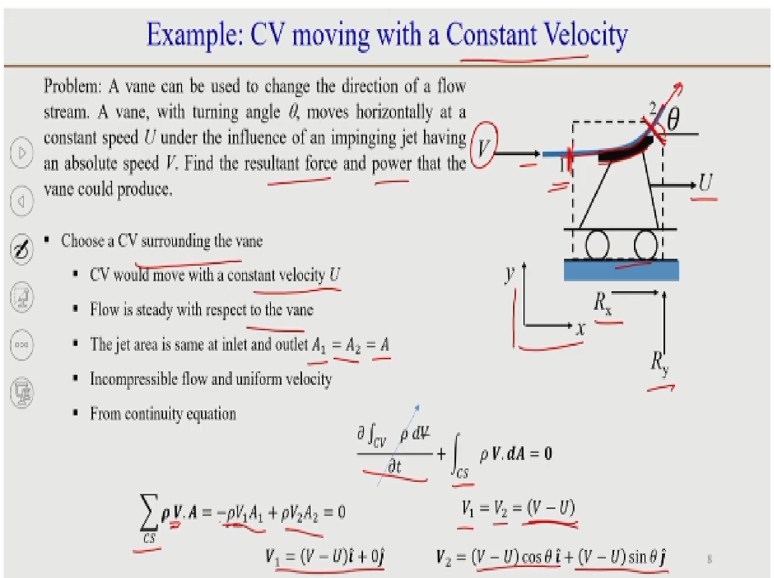

So, let us look at the next problem. Now, in this problem we will consider a case where the control volume moves with a constant velocity. So, if we have a vane, vane is generally a device which can turn the flow, so this is the vane and a jet hits this vane and the flow is the direction of flow is turned from up, so it comes along x direction but then it turns at an angle θ from the horizontal edge, we can see from this figure and this vane is connected with the trolley.

So when the jet hits there is momentum transfer and because of the jet momentum, that momentum transfer the trolley moves with the velocity V, the jet comes with a speed V and it is aligned with the x direction when it approaches the vane. So, it moves under the influence of an impinging jet having an absolute speed V, what we need to find is the resultant force and the power that vane could produce.

Now, we assume that the vane moves along the x direction and the direction normal to it is y direction, when this trolley moves with a velocity U so there will be a reaction force and we assume that the components of this reaction force are R_x and R_y along the x and y directions respectively. So, we choose a first thing we need to choose is, choose a control volume, so we choose the control volume in such a manner that the control surface at point 1 is normal to the jet and control surface at point 2 is normal to the jet and that may make our life a bit easier.

And we also assume that the control volume move with a constant velocity U, because in this case the vane moves with the velocity U, so if we consider that our control volume also moves with the velocity U, then it will be easier for us to apply the momentum conservation equation. And when we are sitting on vane moving with a velocity U, so the flow becomes steady with respect to the vane.

So, let us look at the problem and the jet area, it is also assumed the jet area at section 1 and section 2, so section 1 where it is coming in the control volume section 2 at both the places the jet area remains same and the flow is incompressible and the velocity is uniform. So, first thing because before finding out the force we need to find the velocities, we know the velocity at 1 but we do not know the velocity at 2, we know the speed because the area is going to remain same, so V . A will be same but let us see.

So, first apply the mass conservation principle on this vane, the flow is steady, so the first term becomes 0, now when we apply this the flow is uniform, so the integral sign can be replaced by the summation sign and we will have ρ V . A, that will be equal to at section 1 - because the flow is coming in V and A will be in the opposite direction. So, $-\rho V_1 A_1 + \rho V_2 A_2$ at the exit.

Now, we need to substitute the values of V_1 and V_2 and A_1 and A_2, A_1 and A_2 are same, so they will be equal to A_1 and A_2. So, $V_1 = V_2$ which will be equal to V - U. Remember, that this V is the velocity with respect to the velocity, that is with respect to the control volume. So, V_1 = V - U, so V_1 and V_2 they are equal to V - U and $A_1 = A_2 = A$ which is area of the jet.

So, from this we can find the velocity vector at section 1, which is V_1 = V - U $\hat{\imath}$ + there is no y component, so 0 $\hat{\jmath}$. At V_2 the flow is directed at an angle, the magnitude of this or the speed is V - U, so its x component will be V - U cosθ $\hat{\imath}$ and the y component will be V - U sin θ and so when we write it in vector form V - U cosθ $\hat{\imath}$ + V - U sin θ $\hat{\jmath}$.

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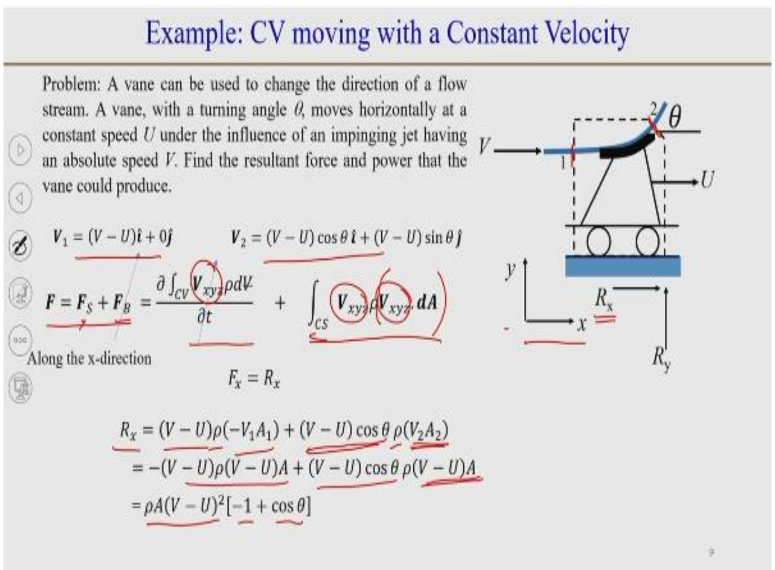

So, we have been able to find the vectors V_1 and V_2 which are the velocity vectors with respect to, at section 1 and section 2 these are the velocity vectors or the velocity of the jet with respect to this control volume which is moving with vane with the velocity or with the constant velocity U. Now, we can apply the momentum conservation equation to find out the velocity components or the force on the vane.

So, let us write down the momentum conservation equation, $F = F_S + F_B$ the forces or surface and body forces the transient term + integral over the control surface, remember that the velocity is here Vxyz they are velocities with respect to a frame of reference which is moving with the control volume. So, even though what we have shown here this x, y here but this is moving with a control volume attached with this control volume, the coordinate frame. Along, the x direction there is no body force, so we can neglect the body force when we consider the force balance along the x direction.

So, F_x will be equal to the force on this control volume will be along the x direction is Rx, the pressure forces from all side, from the left and right because they are all open to atmosphere, so atmospheric pressure and the pressure forces will cancel out and that is the reason we have not considered the pressure forces here. So, F_x we substitute R_x now, this term we will have summation and we need to write down Vx component of velocity into ρ V vector xyz . dA or . A.

So, at section 1, the x component of velocity is V - U into ρ and V . A is - V_1 A_1, at section 2 the x component of velocity is V - U cosθ into ρ and multiplied by V_2 A_2. So, we substitute the values - we can bring out the first place - V - U into ρ V - U into A. Now, this will be V - U cosθ ρ and V_2 . A_2 will be again V - U A. So, we need to see here that the difference that comes, V - U cosθ is the x component of velocity and this will be V_2 A_2, so that will be V - U A.

So, we will have that giving us ρ V - U^2 A, which will be common in both this so we will take this outside the bracket ρ V - U^2 within a bracket, - 1 from the first term + this will also have ρ V - U^2 A, so, the only thing left is cosθ, so R_x will be equal to ρ A V - U^2 into within bracket - 1 + cosθ, that is the reaction force.

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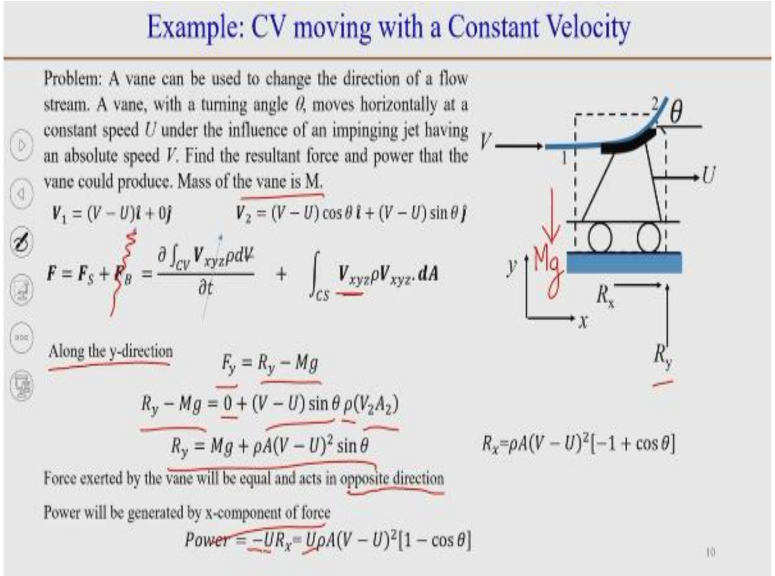

Now, we have been able to find the reaction force in x direction, what we need to find the reaction force in the y direction. Now, we do not neglect, let us do not neglect the mass of the vane which is M. So, we do not neglect this M here, we do consider F_B the flow is steady so along y direction if we write we have $F_y = R_y - Mg$, so R_y is the force, the reaction force and because of the mass there will be a force Mg acting in the downward direction.

So, R_y - Mg which is the force along the y direction. Now, there is no velocity at section 1 along the y direction, so V component of velocity along the y direction is 0, so the first term at section 1 is 0, first for the momentum flow, the second term will give the V component of velocity at section 2 is V - U sin θ, flow is going out so we will have V_2 A_2 which will be V - U into A and ρ. So, we will have R_y - Mg or = ρ A V - U^2 sin θ and we can take Mg on the other side, so we will have an expression for R_y.

Now, we know R_x and R_y and that will give us the force that will be exerted by the vane, so that is the reaction force and the force that the vane will exert will be equal to this, but it will be in the opposite direction. Now, we need to also find the power that will be produced by vane. So, power will be generated because the displacement is along the x direction only, so power will be generated only because of the x component of force.

So, that will be equal to the x component of force into U and this - sign represents that it will be equal to - R_x because the force is acting in the opposite direction. So, we substitute the value of R_x which is ρ A U, ρ A into V - U^2 into 1 - cosθ, when you take - sign inside so this - 1 + cosθ becomes 1 - cosθ and this is multiplied by U here, so that is the power.

So, we have looked at quite a few examples in the moment conservation equation and it is important to solve different problems so that you get a feel of the problems, you get to see the choice of control volume or the problems that one encounters. So, I suggest that you should solve as many problems as possible from different sources. Thank you.