Fundamentals of Fluid Mechanics for Chemical and Biomedical Engineers
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Lecture 17
Macroscopic Balances: Momentum Conservation I

So, in the previous lectures we derived the Reynolds Transport Theorem, which relates the system formulation of conservation equations with the control volume formulation for the conservation equations, what it gives us the rate of change of an extensive property which is used in the conservation equation. So, the control volume formulation of rate of change of a extensive property it can be mass, it can be momentum or it can be energy that is written in terms of the rate of change in the control volume formulation.

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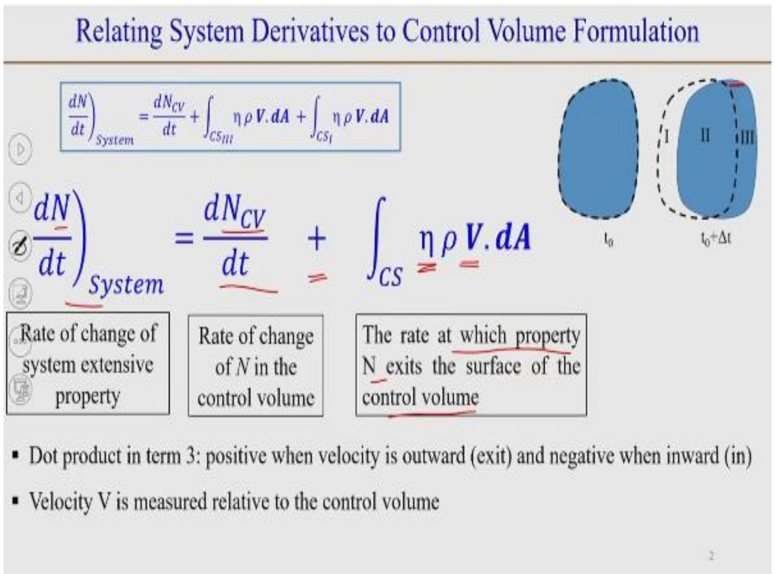

In this lecture, we are going to use this to derive the momentum conservation equation. So, let us remind ourselves the Reynolds Transport Theorem. So, which tells us the rate of change of a system extensive property, which is N here, that is equal to rate of change of N in the control volume. So, N_{CV} basically refers to the property, extensive property N in the control volume plus the rate at which the property N exits the surface of the control volume, η here is the intensive property corresponding to N, so η is basically N divided by mass or N per unit mass. We need to remember here that this dot product is positive and the velocity is outward and negative when it is inward and the velocity V we have here is measured relative to the control volume.

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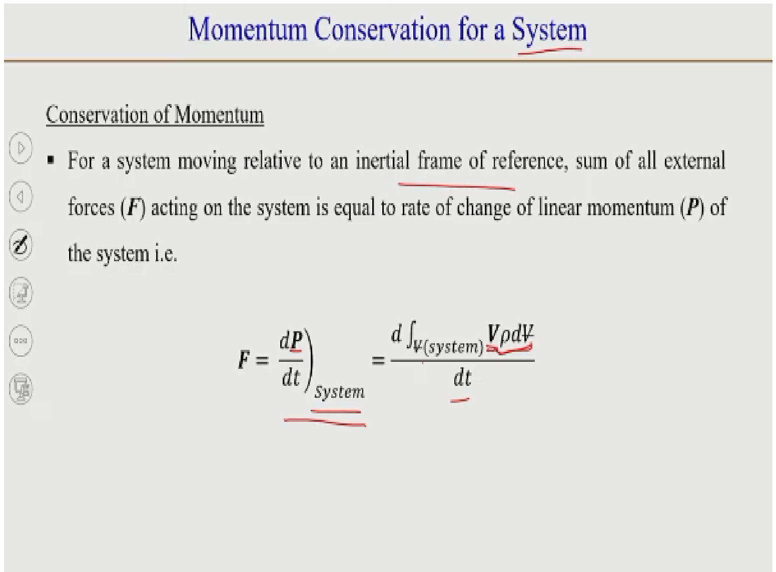

So, remind ourselves what is conservation of mass? For a system, which is basically Newton's second law of motion which states that for a system moving relative to an inertial frame of reference, for such a system the sum of all external forces acting on the system is equal to the rate of change of linear momentum. So, F = d M V/dt or d/dt of the momentum, here the momentum is P. So, that is the system formulation.

Now, P is basically M into V, so we can write this P in terms of integral, so integral over the control, over the system volume and M into V, so V is the velocity vector and M is, if you write a small elemental or for a small elemental volume d~~V~~, so dM can be written ρ d~~V~~. So, that is the momentum conservation equation for a system. And if we need to write in the control volume formulation, then we need to change this for the system dP/dt equal to the forces that is being applied on the control volume. So, this formulation need to be changed in terms of control volume.

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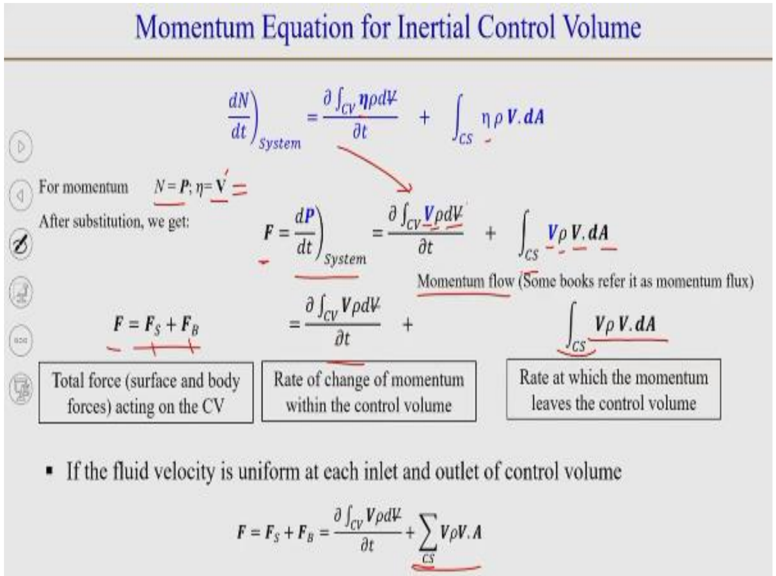

So, in this equation, now what we are going to do is write N = P which is momentum and the corresponding η which is P over V and P is what? M into V, V is velocity here. So, η = V and N = P or momentum. Now, when we substitute it, so when we substitute we get F is equal to which is force dP/dt, this is dP/dt for system, so for the system formulation we replace this with the control volume formulation, so we will get ∂/∂ t, integral over the control volume in place of η we put V, so V ρ dV̶, dV̶ V̶ a horizontal line cut is the volume.

So, integral over the control surface or the area integral, again we replace in place of η we substitute V here, ρ V . dA. So, we get our momentum conservation equation for the control volume, which is basically the force on the control volume that is which forces can be surface forces, which can be body forces and that is equal to the rate of change of momentum within the control volume plus rate at which the momentum leaves the control volume.

Now, it is easier to imagine when we talk about the mass coming in, mass going out, because that we can easily relate with or that we can easily visualize, but it might be a slightly difficult to visualize when we talk about momentum, because momentum is not something that we see as a tangible thing, it is intangible. So, the total force on the control volume that is equal to ∂/∂ t is the momentum that is within the control volume and this term is the momentum which leaves the control volume.

It is also known as so, this term V ρ V . dA it is called momentum flow, in some book it is also called momentum flux. So, as we did for mass flow rate, if the velocity is uniform at the inlet and outlet, then this integral term can be converted to, the integral over the control surface can become the summation for the control surfaces from where the flow is coming in or going out and V . dA becomes V . A, where A is the area of the surface of the inlet or outlet.

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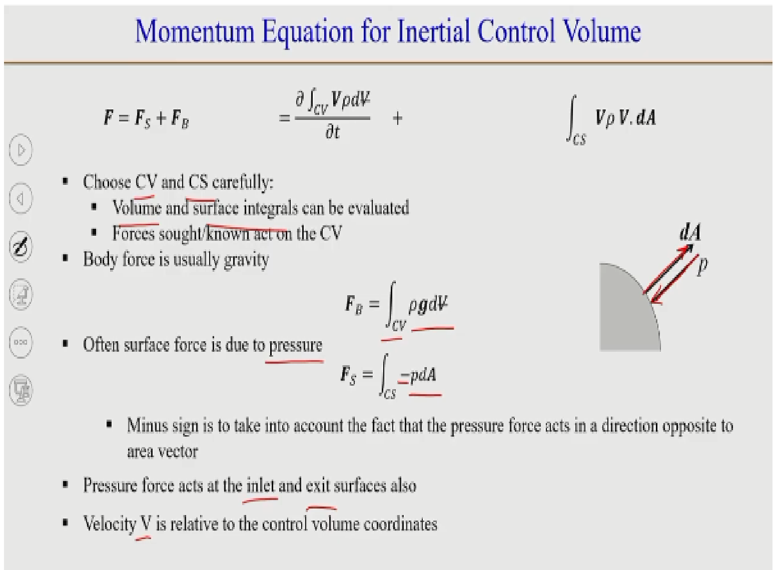

So, few things we need to remember because we need to choose our control volume and control surface carefully. So, that it should be such that, when we choose the control volume and control surface we should be able to integrate because we have integral, surface integrals and volume integrals. So, they should be chosen such that these integrals can be evaluated.

And we also need to find, when we talk about the forces, the forces are being applied on the surface of this control volume. Now, so the forces that we need to find or forces that we already know that we will use in the formula, they should be known. So, we should choose our control volume in such a manner. The body force in this course at least will be gravity, generally when we use it, so body force can be given integral over the control volume ρ g dV.

Now, surface forces can be pressure force or stresses, but most of the time we will be taking our control volume outside the flow. So, just outside the flow, so the stresses will become internal whereas, the force that we have is pressure flows. So, at least for this chapter we will be having a predominantly the pressure force that will be important.

Now, this pressure force will be because the pressure tends to act pointing towards the surface, it is a compressive stress and the area vector act as outward normal. So, to take into account that fact we have a - sign here, so F_s is over the control surface -pdA. So, when we take these forces, we need to remember this that the pressure force will be acting on the surface whereas the area vector will be outward and so, when we take the pressure force, we need to take a -sign because it is opposite to the area vector.

The other fact that we need to take into account that the pressure forces will also be acting on or it will also be applied on an inlet and exit surfaces, so we need to take the pressure at these surfaces also. It is not that the flow is coming in, so the pressure is not going to be or we do not need to take into account the pressure force in our calculations. And the velocity V is related to the control volume coordinates.

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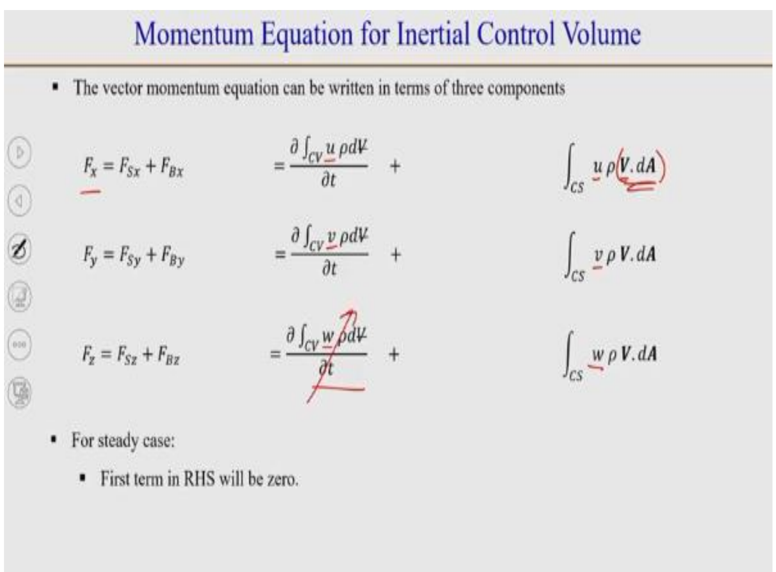

So, we can write, it will be generally convenient to decompose the equation in vector form, in its component form. So, often we will be doing in rectangular or Cartesian coordinate system. So, we can write down the force balance and the rate of change of momentum in terms of the components, so F_x, F_y and F_z and in this equation we have u, v and w replaced to the velocity vector, velocity components.

Again in the second term we will have the first v replaced by the respective components, but note that this term V . dA is a scalar, so when we are even writing in the component form this will be V . dA and you will need to write the entire expression for the velocity vector, so V . dA, this is an important thing that we need to remember. If, the case is steady, if the flow is at steady state or if this term is steady, then this term, if the flow is steady then this term can go, so this term will be 0 if in the case when the fluid is steady.

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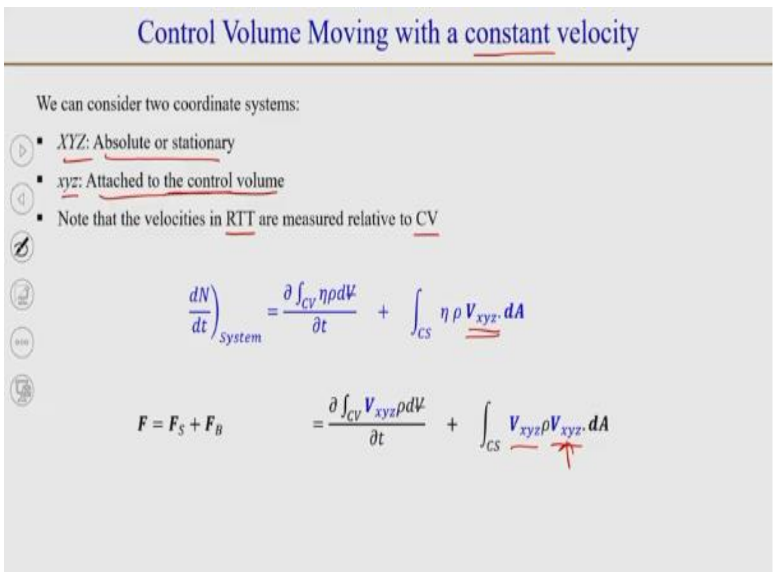

Now, if the control volume moves with a constant velocity, then the equation does not change, but we need to re emphasize this that let us say if in the absolute frame of reference the X, Y, Z, capital X, Y, Z is the coordinate system and small x, y, z is the coordinate frame of reference that is attached with respect to our, with the control volume, then the velocities that we have in the Reynolds Transport theorem, they are measured relative to control volume, so RTT is nothing but Reynolds Transport Theorem.

So, in this, in the general transport theorem, this V is V_{xyz}, so when you write the momentum conservation equation, the V here is V_{xyz}, V_{xyz} which is the velocity in a frame of reference or in a coordinate system which is attached to the controlled volume. So, that we need to remember, especially when the control volume is not fixed and it is moving with a constant velocity.

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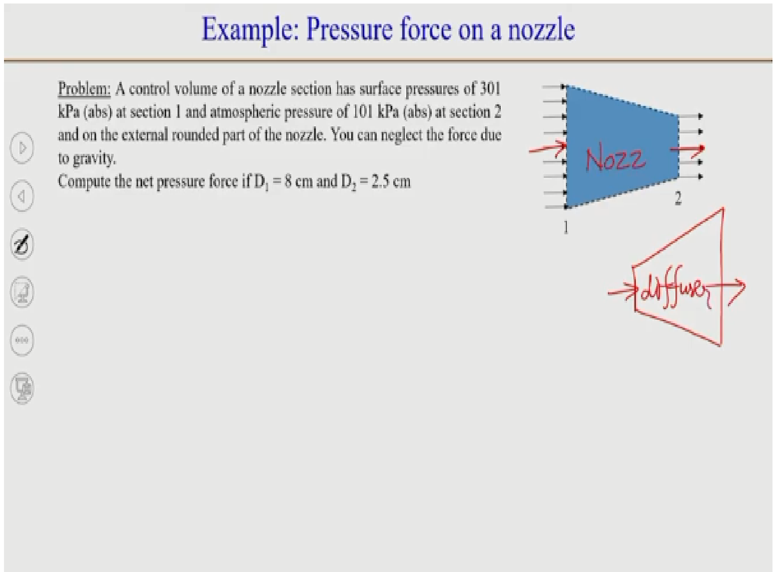

So, let us look at some examples. First example is about pressure force on nozzle. So, we are just looking at the forces on nozzle, so nozzle as you might know that nozzle is a device in which the flow is accelerated in at least in subsonic flow. So, the flow area as the flow comes in, the area of this device decreasing, so flow comes in and it passes through, so if the flow is subsonic, incompressible then the flow will be accelerated. So, the device is used quite frequently and the opposite device, opposite to it is called diffuser in which the area of the flow increases. So, this is called diffuser and this is nozzle.

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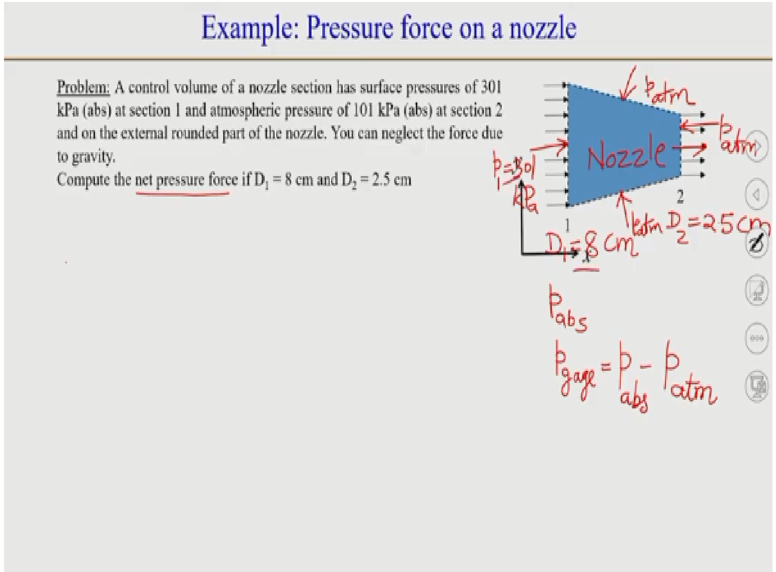

So, let us get back to the problem, what we need to find is the force on the nozzle, compute the net pressure force, we have been given the dimension at section 1. So, diameter at section 1, D_1 = 8 centimeter and the diameter at section 2 D_2 = 2.5 centimeters. Now, it says that has a surface pressure of 301 kilo Pascal absolute at section 1, so the pressure that is acting at section 1, p_1 = 301 kilo Pascal and that is absolute pressure. And atmospheric pressure of 101 kilo Pascal at section 2 and on the external rounded part of the nozzle.

So, on all other surfaces, at this surface the pressure = $p_{atmospheric}$, same on these round curved surfaces on the walls of the nozzle, the pressure is atmospheric. So, before we discuss further, let me just talk about absolute pressure and gauge presser. So, in many cases you talk about pressure, pressure measurement with respect to the atmospheric pressure. So, this is called gauge pressure which is pressure minus atmospheric pressure. Now, if we look at this, so this is $p_{absolute}$, we look at this nozzle the pressure forces acting, there will be pressure forces acting on along the x direction and along the y directions.

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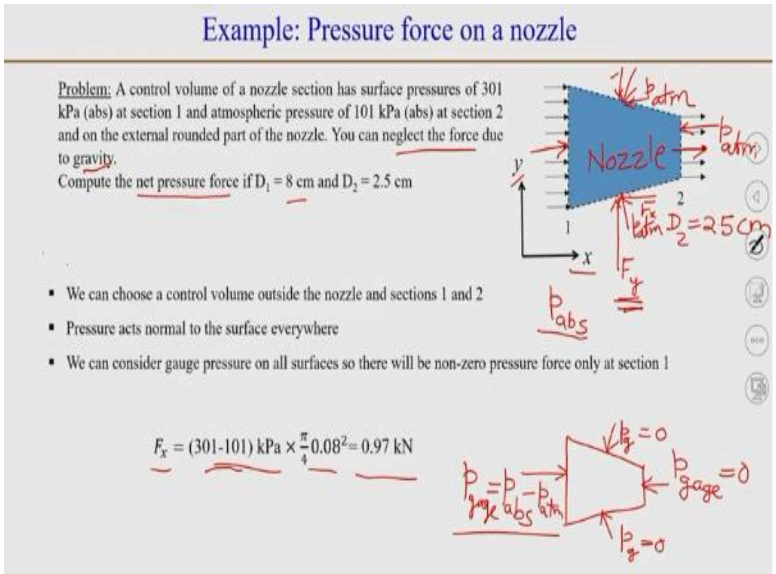

So, if we consider our coordinate system such that this is the x direction and this is y direction, then we can write down the momentum conservation equation if we want to solve this problem, if we would have been asked to use the momentum equation, here we just need to find the forces. So, if we look at the forces in the y direction, the pressure force along the y direction, then this pressure we can divide into two components, the force due to pressure along y direction and along x direction.

Now, if you look at the other side, you can decompose, so the pressure due to the y direction they will cancel out, F_y and $-F_y$, so there will be no net force in the y direction unless we consider gravity and we have assumed that the force due to gravity can be neglected. So, the only thing we need to do is look at the forces which are acting along the x direction.

So, if we look at this nozzle and the force acting on all the surfaces, then this is p, let us say $p_{absolute}$ and on the other surfaces the pressure is atmospheric, so the forces will we be p into area. Now, because the pressure is atmospheric, so if we assume or if we give the pressure in terms of gauge pressure, then the pressure will be p_{gauge} will be 0 Pascal on all the surfaces and the problem becomes simple, because we just need to find that pressure on this face.

So, we can use in place of absolute pressure to find out the force we can use the gauge pressure formulation. Now, the control volume it has been already drawn, the dotted line, so, which you just encompasses the nozzle, just outside the nozzle and section 1 and 2 has been given, now pressure it acts normal to the surface that we have already discussed and we can consider gauge pressure on the all the surfaces. So, there will be nonzero pressure force only at section. So, we have considered gauge pressure on all surfaces, so that there is nonzero force only at section 1.

Now, if you do that, then the force in the x direction because of pressure will be gauged pressure multiplied by the area. So, gauge pressure is absolute pressure 301 kilo Pascal minus atmospheric pressure which is given as 101 kilo Pascal, so 301 - 101 kilo Pascal into the area π/4 the diameter of this section is 8 centimeters, so 0.08^2 and when you solve it you will likely get 0.97 kilo Pascal, I suggest that you verify the numbers.

So, what we have learned in this problem is, we learnt about that if we work with gauge pressures, then the problem can be simplified, so we should consider this on going along when we solve the problems, we now know what is nozzle, what is a diffuser and when we take the forces we might see that in number of cases the forces on the other side of the walls they might get cancelled. So, the y component of force we can so, we should try to simplify the problem as much as possible, so the calculations become simpler.

For your understanding it will be better if you solve this problem assuming the pressure as absolute pressure on all the walls and then again find the answer that you get the same answer. So, in place of gauge pressure you can solve the problem using absolute pressure and should be able to get the same numbers.

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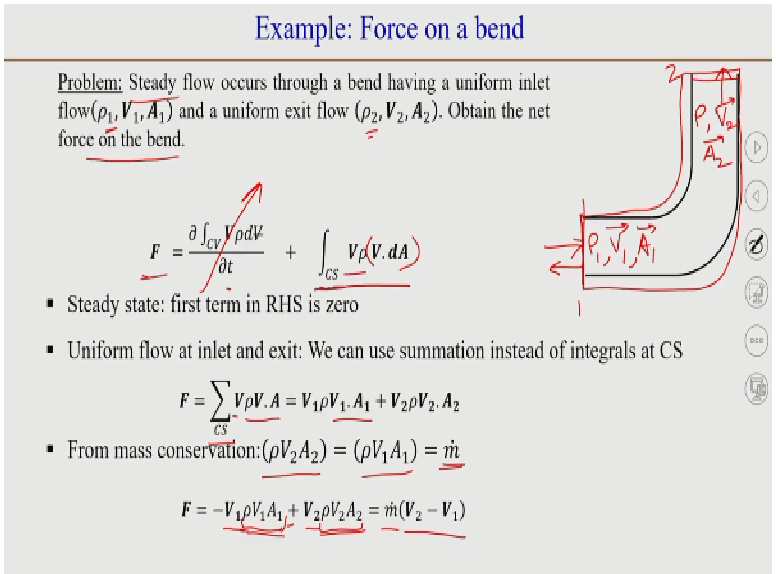

So, let us look at another problem which is you have a bend or in many places when you have a piping system or in the pipelines in our household, we use elbows and these elbows turn the flow from one direction to other direction. So, as a result, because of the change in momentum, because V_1 to V_2 at least the velocity vector will change, so there will be a net force on this. So, this problem asks us to ask us to find the force on the bend whereas ρ_1, V_1, A_1 are the density, velocity vector and area vector. So, area vector of course, it will be pointing outward and it displays ρ, V_2 vector and A_2 these are the, the flow is considered uniform at the inlet and exit of this and we need to find the net force on this.

So, we can write down the momentum conservation equation in the vector form F = ∂/∂ t over the control volume into integral V ρ dV, or integral over the volume of V ρ + surface integral of V ρ and V . dA. So, because we can consider the flow to be steady, so the first term will be 0. Now, we need to find the net force on the bend, so we do not need to do anything on the left hand side and we just need to expand the control surfaces or we need to obtain this integral over the control surface.

So, we can take a control volume which is encompassing this or you can take different control volume. So, the flow is uniform at the inlet and exit, so this integral can be converted or can be written in terms of summations and you have only two surfaces, the surface 1 let us say and surface 2 on this control surface from where the flow is coming in or going out. So, we can write

F is equal to summation over the control surface V ρ V . dA. Now, we write in the vector form, so we can just expand this summation V_1 vector ρ V_1 . A_1 + V_2 ρ V_2 . A_2.

Now, we consider the directions, so ρ when we take this integral the area vector will be pointing outward whereas the flow is coming in, so - ρ V_1 vector V_1 A_1 + V_2 vector and ρ V_2 A_2. Now, we know from the mass conservation that this ρ A_2 V_2 = ρ A_1 V_1, what I have considered here is I have taken this ρ_1 and ρ_2 to be equal. So, that will be equal to $\dot{m}$, and so force that will be equal to we can write ρ V_1 A_1 and ρ V_2 A_2 both are equal to $\dot{m}$.

So, we can write them as $\dot{m}$, so $\dot{m}$ V_2 vector - V_1 vector is the net force on this bend. So, this is how you can apply the conservation, momentum conservation equation to find the forces. Now, if one would be asked to calculate the reaction force for example, there is a support on which the reaction force, then you need to consider the pressure forces, whatever have been given and from that pressure forces you need to find the reaction force. So, we will stop here and continue.