Fundamentals of Fluid Mechanics for Chemical and Biomedical Engineers

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Lecture 15

Macroscopic Balances / Integral Analysis

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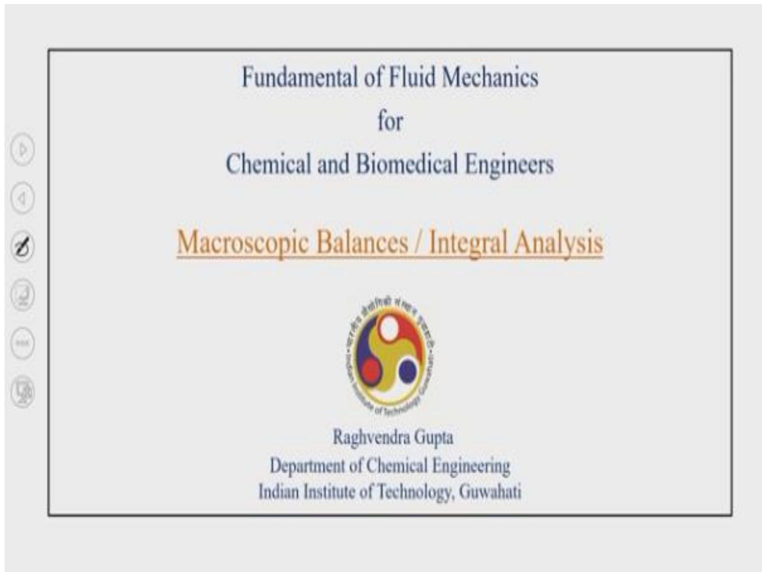

Hello, so, in this module, we are going to talk about Macroscopic Balances or Integral Analysis. During the start of this course in first few lectures we talked about the approaches that we can take to solve problems of fluid mechanics. So, we looked at main three approaches that one can take to solve fluid mechanics problems.

The number one was dimensional analysis, so where we can list down all the parameters on which the flow depends and then using dimensional analysis, we can find out the relevant non dimensional groups and identify the independent parameters on which a dependent parameter will depend on. Now, it can be used to design experiments or to give directions to our experiments, the next approach.

So, the dimensional analysis approach is a very primitive approach, but it is very useful to give or to get some insight in the physics of the problem, but we cannot get much quantitative analysis about the problem from the dimensional analysis. The next approach we talk about is macroscopic analysis or integral analysis. So, in this approach, what we will be looking at that on a macroscopic scale what are the forces or what is the energy exchange through a control volume.

So, when we identify the system or when we identify the flow domain, we can find out using macroscopic balances, the force being exerted on a particular body or the force exerted by the fluid on a beam, on the pillar of a bridge or on a car etcetera. What we will be able to get is the integral of this force, we will not be able to get the distribution of the flow field or the distribution of the force on the surface and that can be obtained using differential analysis. So, what we are going to talk about is macroscopic balances in this module or we also know it as integral analysis.

Now, as we discussed in the introduction that the fluid behavior can be analyzed using Lagrangian approach or Eulerian approach. So, Lagrangian approach is that when we identify a fluid particle and move with it. So, we are looking at the motion of a fluid particle or motion of a number of fluid particle, individual fluid particle when we are following those particles and looking at their velocity field, then that approach is Lagrangian approach.

Now, in solid mechanics when we are analyzing the motion of rigid bodies, we usually use this approach. So, for example, we are analyzing the motion of a car, analyzing the motion of an aeroplane or motion of a rocket we follow their path from Earth all the way to Mars for example, if we are talking about a rocket. However, when we are looking at a fluid, we are generally interested, let us say if we talk about rocket and when we are analyzing the motion of a rocket we of course need the force that the rocket will experience because of the fluid surrounding this rocket.

So, we are interested in the fluid motion surrounding up in a region, surrounding this rocket. So, our region of interest is a particular volume surrounding the rocket, we are not interested in the motion of a fluid particle that was surrounding the rocket at the earth, we will be looking at the motion of the fluid particles that may be present at any time, we will be looking at the motion of fluid particle that will be present around this rocket at any particular time. So, that approach what we call Eulerian approach, that we are looking at the motion of fluid in a particular region of interest and the fluid can come and fluid can go out of it.

So, using these two approaches Lagrangian approach and Eulerian approach, we can solve fluid mechanics problem but as I said generally we solve a conventionally all the fluid mechanics book or most of the theory that has been developed for solving fluid mechanics problem is developed for Eulerian approach, because that suits well for fluid mechanic problems from application point of view as well as from an analysis point of view. So, to do this, first we need to look at what its system and control volume.

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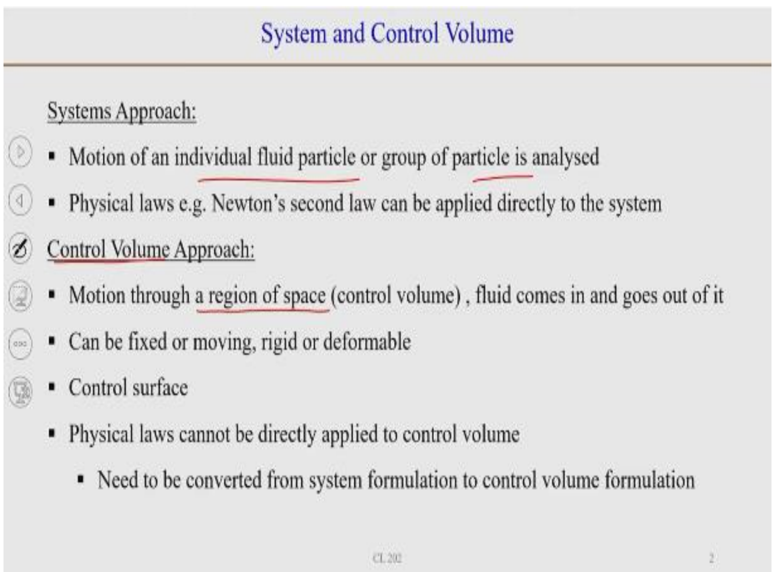

So, what we define in system when we talk about Lagrangian and approach we are talking about the motion of a fluid particle or a group of fluid particles. So, the system is basically a fixed mass and this mass or the system may be rigid or a deformable mass, but, there is no transfer of mass from the boundary of the system. So, system is a motion of it can be an individual fluid particle or group of particle and that is what we analyze when we talk about Lagrangian motion.

So, most of the laws that we talk about because that is what we have been using for rigid body analysis, we take a system, we take a car, we take an aeroplane or for example, we take a balloon, which is a deformable body or piston cylinder system, there again we have a change in the volume, but the mass inside the system is fixed. So, when we talk about rigid body motion, we generally use systems approach. So, all the laws that we have been studying applicable for rigid body motion because they are laws of mechanics and they are applicable for any matter, they will also be applicable for fluids.

Now, we want to use those laws for fluid mechanics or analysis of fluid motion however, those laws has been written or has been developed using a systems approach or using Lagrangian approach where the system is fixed. Now, in the fluid mechanics we use control volume approach or we look at a fixed region in a space and analyze the fluid motion. So, the fixed region in a space is what we call control volume.

So, this control volume in this we take a particular region of space and the fluid can come in and go out of this fluid, of this specified region. Now, this region it can be fixed or it can be moving, the region that we talked about or this control volume can be rigid, we can have a fixed boundaries of this control volume or this can be deformable. So, we should remember this control volume is an imaginary volume that we talk about. So, it is not necessarily that it will always be a enclosed volume where we will have some doors and windows etcetera from which the flow can come in or go out.

When we analyze the problem most of the time we will see that the control volume is an imaginary volume and depending on the problem, we might need to assume that the, this control volume is fixed or it is moving with a constant speed or it is moving with or it is accelerating or the control volume is fixed or its boundaries are fixed or control volume may also be having flexible boundaries or moving boundaries. So, the boundaries of this control volume, we will call this the boundaries of the control volume as control surface.

So, in today's lecture, what we are going to look at, we already know what are the laws of mechanics applicable in systems approach or in the Lagrangian approach, and we will try to relate these or try to identify that how we can write down knowing the laws for a systems approach how we can relate them with an Eulerian approach.

So, how can we write down these laws for an Eulerian approach and use it for fluid mechanics problem that is what is going to be the crux of this module in macroscopic balances. But before we do that, we will derive a general relationship which can relate the formulations or the laws, laws of mechanics with system formulation to control volume formulation. So, before we do that, let us look at the main laws of mechanics.

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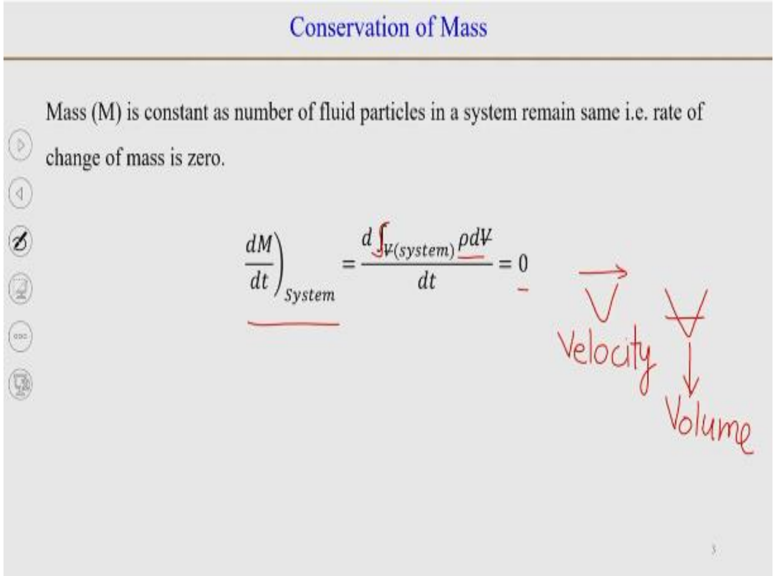

The main laws of mechanics that we will be using or that we need to use is, the first one is Conservation of Mass. So, this we might not be using very frequently to solve problems in rigid body mechanics because sometimes it is implicitly assumed that in a rigid body the mass is of course, constant, but in fluid mechanics we need to look at this. So, the first law is that in a system where the number of fluid particles is a constant, so the mass remains constant.

And as we will see, after a few minutes, that all the things that we are looking at we need to write in terms of a rate. So, we will write that mass is constant or in terms of rate what we can write that the rate of change of mass of the system where capital M is mass of the system is equal to 0, that is mass is constant. So, mass does not change with time, we can also write this in terms of density and volumes, because we will frequently encounter the velocity vector $\vec{V}$ and V volume.

So, to differentiate this, we will use that capital V as velocity vector and a horizontal line cutting across $\forall$ we will use for as a symbol for volume. So, the mass we know that is density into volume, so if we have a small elemental volume dV, then the mass in that will be ρ into dV, if ρ is changing or if ρ is not constant throughout the volume, then we will need to integrate this over the control volume. So, d integral over the control volume ρ dV/dt will be a differential form where we can consider the space dependent density in the system. So, that is basically the conservation of mass.

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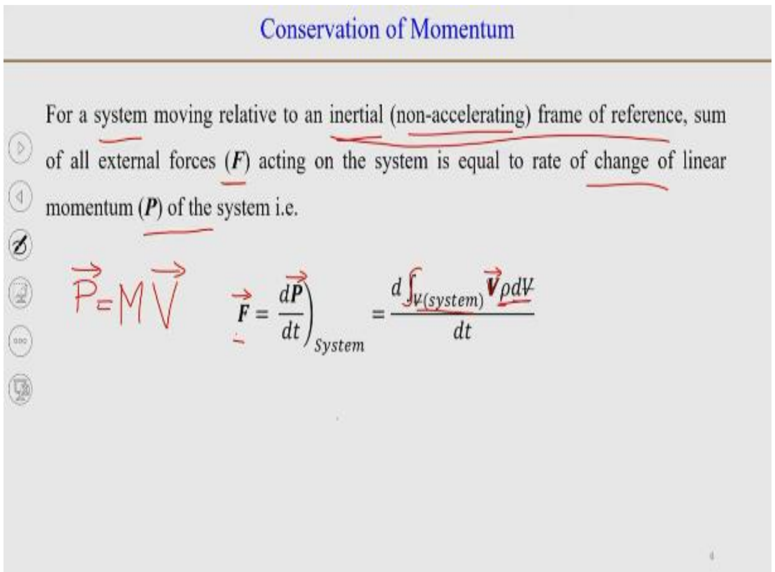

Now, the next important law for mechanics is Conservation of Momentum. We can have a linear momentum and angular momentum, but in this course, we are going to restrict ourselves for conservation of linear momentum only. So, conservation of linear momentum is basically what we know popularly as Newton's second law of motion, which states, we already know that F = m a, popularly F equal to m a is the expression for Newton's second law of motion.

So, in words we can say that for the system, moving with or moving related to an inertial frame of reference, so we will many times encounter this term, inertial frame of reference. So, to remind ourselves that what is inertial, inertial frame of reference mean that the frame of reference is either stationary or it is moving with a constant speed or constant velocity, it is non accelerating, so when the frame of reference or our coordinate system if it accelerates, then it will be a non-inertial frame of reference.

So, Newton's second law states that for a system moving related to an inertial frame of reference sum of all forces that is acting on the system that will be equal to the rate of change of linear momentum of the system. So, linear momentum P = mass of the system into velocity. So, F, $\vec{F}$ = dP/dt.

So, the rate of change of momentum of the system that is equal to the force applied on the system, when you consider it in terms of differential quantities or in a differential volume d$\mathcal{V}$ if ρ is the density and V is the velocity vector, then force is equal to dP/dt that will be equal to d/dt in the integral form, integral over the system volume M into V where or dM into V, V is the velocity vector and dM is ρ d$\mathcal{V}$. So, that is conservation of momentum.

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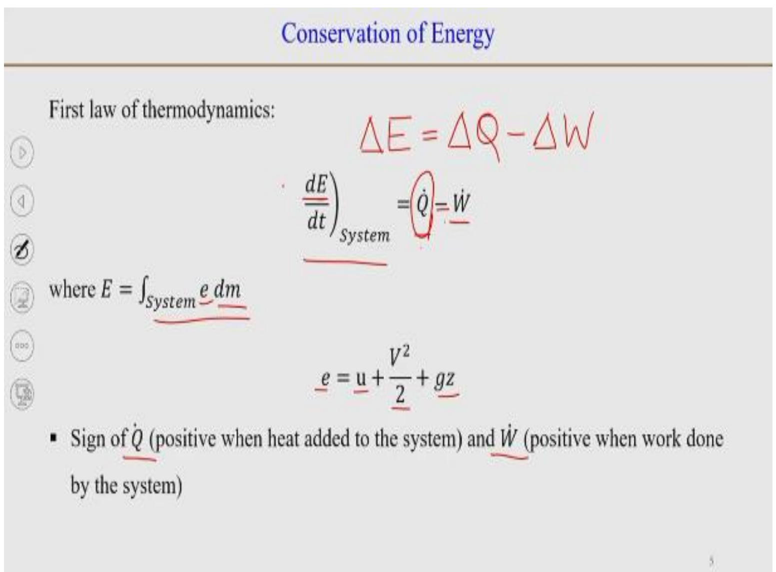

And the third law which is Conservation of Energy, so when we talk about incompressible flows and if the flow is isothermal we do not need conservation of energy until we are talking about problems related with heat transfer. But when we talk about compressible flows, so in compressible flows, the density is not a constant and we need to relate density with the properties of the fluids. So, we need to use a law which can relate ρ which is density of this fluid with the system properties or the fluid properties such as pressure and temperature.

So, the simplest law relating pressure and temperature with density is ideal gas law; $\rho = PM/RT$ for an ideal gas where M is the molecular mass of the gas. So, if the flow is compressible, then we need to solve for energy equation because then we need temperature and the temperature field may be changing because the velocities are high. So, we need to essentially solve conservation of energy always, but for incompressible flow, we may need to solve conservation of energy only when there is heat transfer involved or the flow is not isothermal.

So, the conservation of energy is basically given by first law of thermodynamics, which states that the change in energy of a system that is equal to $\Delta Q - \Delta W$. So, ΔQ is heat supplied to the system and ΔW is the work done by the system. Now, because the other two laws we have written in rate form and when we talk about the flow of fluid, it will be when the fluid comes in or goes out, the energy will be coming in and going out.

So, in place of the, what is the energy coming in we will be interested in what is the energy coming in time Δt or in other words, we will be talking about the energy coming in per unit time, work done per unit time and the change in energy per unit time. So, we will be talking about in terms of rates. So, the conservation of energy will be in terms of rates that dE/dt of system that is equal to Q rate of heat input to the system minus work done by the system on the surroundings.

So, when we talk about systems, we always have surroundings to it that anything that is around the system, because when the system the mass is fixed, so around the system what is presented is what we call surroundings. So, a system interacts with surroundings and that interaction results in terms of forces that we saw in Newton's second law of motion or conservation of linear momentum. And here, we are looking at the energy exchanges with the surroundings for a system.

So, this capital E or the energy or total energy of the system will be integral small e, where e is the specific energy and dm is of course, the differential mass of the small volume. And this energy will be sum of specific internal energy $u + V^2$, where V is speed and gz, where z is the height from a reference position. So, it is sum of the total energy, it is sum of specific internal energy, kinetic energy and potential energy, all of these are of course, in terms of specific energy, so, the units for all of these will be joule per kg.

Now, just to remind ourselves that the sign of $\dot{Q}$ which is that when heat is being added to the system, then it will be positive and sign of $\dot{W}$ when work is being done by the system. So, this we can understand, by looking at this expression itself that rate of change of energy will be positive, so energy dE/dt will be positive that means, the system will get energy if there is some energy being supplied from outside.

So, if heat is being added to the system from the surroundings, if the system gets some heat, then only ΔE will be positive. So, $\dot{Q}$ is positive or heat being supplied is positive, when heat is being added to the system. $\dot{W}$ will be positive because we have a negative sign in this equation, so $\dot{W}$ will be positive when the system spends some energy to do work. So, when somebody does work or when the system does work, it needs to spend energy.

So, because we have a minus sign here, so there will be a decrease in the energy of the system when work is being done by the system, so sign of $\dot{W}$ will be positive. Now, these signs depends on what kind of, what form of equation do we choose, so it is always important when we are looking at the any form of conservation of energy or first law of thermodynamics, we should always remember that what sign we are going to use for Q and W.

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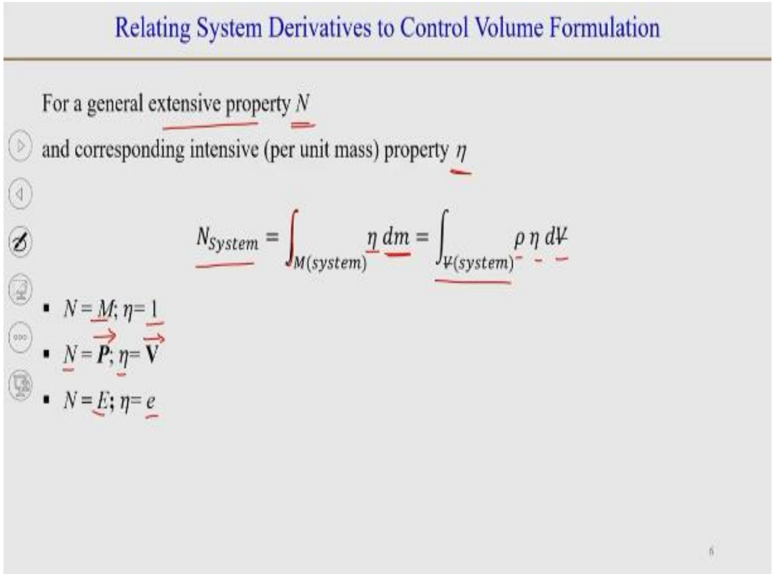

So, if we look at all three of these equations, conservation of energy, conservation of momentum and conservation of mass all of those had dM/dt which is rate of change of mass, dP/dt or dM V/dt which is rate of change of momentum and dE/dt which is rate of change of energy. So, all these were rate of change of a property of the systems.

So, we will be talking about now or in this lecture what we are going to do is how do you relate rate of change of property in a system in a general sense, this property can be mass, this property can be momentum or this property can be energy or for that matter, it can also be angular momentum or entropy of the system. And we will relate it with the rate of change of this property in a control volume.

So, before we do that, we will just define a general property which is an extensive property N and then corresponding to each extensive property, we will also define an intensive property which is extensive property per unit mass. So, to remind ourselves what is extensive and intensive property, the properties of the system or properties of a material, which depend on the quantity or on the size of the system or the volume of the system that is present that those properties are called extensive property.

So, for example, the temperature of a system does not depend on what is the size of the system or what is the mass of the system. So, temperature is an intensive property, similarly, density is an intensive property because it does not depend that how much fluid is present or how much a particular matter is present, whereas the volume or the momentum they are all extensive property because they depend on the mass of the system.

So, for each extensive property all the three properties that we talked about in the three laws mass, linear momentum and energy M, P and capital E they are extensive properties, but corresponding to each of these if you divide them by mass, we can find an intensive property. So, we define in general an extensive property which we call N and the corresponding intensive property is what is η which is N/m or in differential terms, we can define that capital N of system = integral over the system η dm where m is of course mass or if we write dm in terms of $\rho\, d\mathcal{V}$, then this will be equal to integral over the volume of the system $\rho\, \eta\, d\mathcal{V}$.

So, let us just see, if we are talking about mass of the system, then η will be of course, mass/mass which is 1, if we are talking about momentum of the system which will be a vector quantity, so momentum is equal to capital N extensive property is momentum P, then corresponding intensive property η will be P/M which is well basically velocity vector and if N = E which is total energy of the system, then the corresponding intensive property as we also saw in the previous slide that η will be a specific energy, small e.

So, when we will use the principle that we derive today what we call Reynolds Transport theorem, we can use or we can substitute E and e for N and η or P and V for N and η or M and 1 for N and η, depending on we are using energy conservation, momentum conservation or mass conservation.

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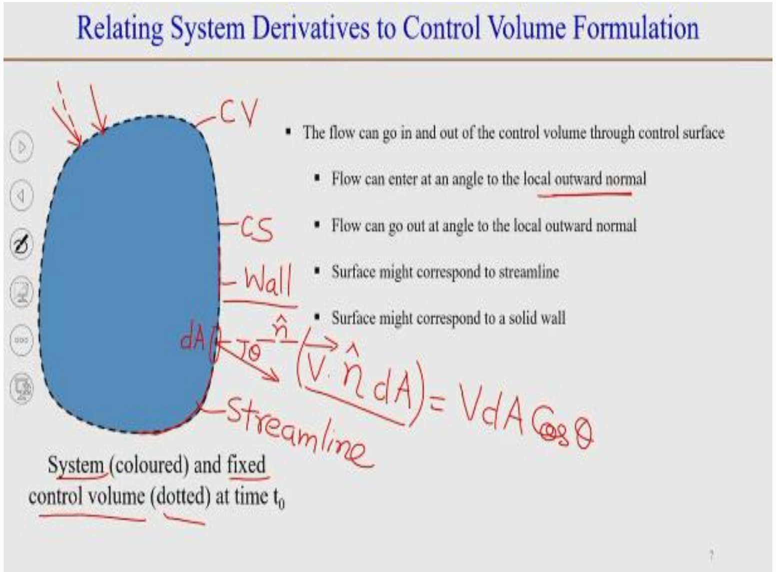

So, let us define first a control volume. So, if we consider a system or if we consider a small control volume fixed in space, so in this picture what we see is a control volume which is shown by a dotted line. So, this is our control volume as is written here that it is shown by dotted lines. Now, at time t^o the fluid surrounded by this control volume is what we consider as system. So, if we consider a system which is shown inside this control volume, this is our system. So, the system is going to move if the fluid is moving, but our control volume we consider that the control volume is fixed.

Now, because the flow is free, it can come in and go out of this control volume, so the surface of this control volume, all this dotted line is what we call control surface CS. Now, the flow can come in or it can go out, so flow may come in normal to the control surface or it may be coming in at an angle to the normal to the control surface and flow may leave the control volume either normal to it or at an angle to it or the flow might be moving along the control surface, so the streamline this, line might be a streamline or the this might be a wall.

So, all possibilities can happen that the flow may come in through the control surface or the control surface may be coinciding with the streamline. So, we know that the flow cannot cross a streamline. So, again there will be no flow through the control surface and it may have wall.

So, generally because we are talking about flow of fluid, so through this control volume there will always be some inflow coming in and some flow might be going out or you might have only flow coming in or flow might be going out, but generally when we solve problems, there will be at least one of these flows; the flow is coming in or flow is going out because then only we are looking at the motion of fluid.

So, when we talk about the flow coming in or going out, so, the net flow rate will be $\dot{V}\ \hat{n}$ dA, if we take a small area dA in this control surface and the unit normal to this area dA is $\hat{n}$ and there is an angle between the velocity vector and the unit normal, the flow is going out then the volumetric flow through this area dA will be $\dot{V}\ \hat{n}$ dA or that will be the V dA cosθ. So, that will be the flow that is going out of the system.

Similarly, we can use if the flow is coming in through the system there of course, the velocity vector and outward normal. So, conventionally when we take a normal to the surface or normal to the control surface we always take outward normal, so when the flow is going out $\dot{V}$ dA will be positive, but when the flow is coming in it will be negative. So, through this $\dot{V}$ dA vector or $\dot{V}\ \hat{n}$ dA, we can obtain the flow rate, volumetric flow rate and if we multiply it with density we will get mass flow rate. So, we have this control volume and system coinciding at time t^o.

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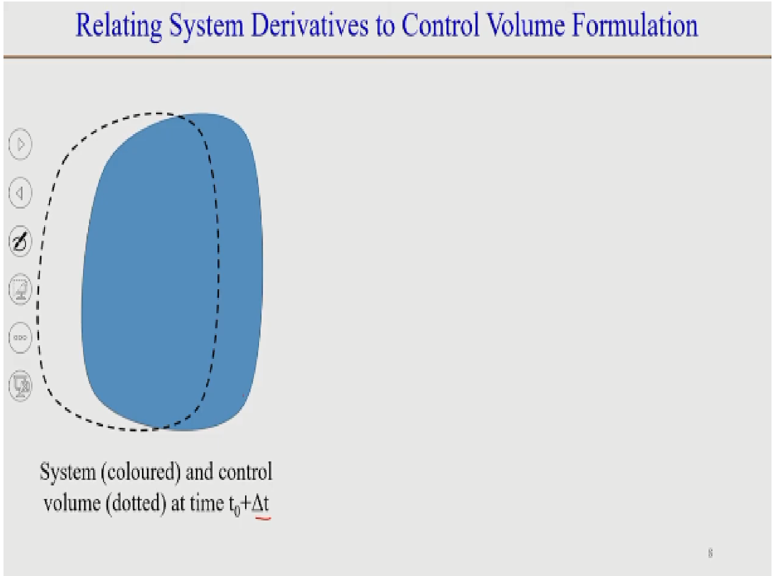

So, after time ∆t the system moves to a different position as we can see here, the system is colored in blue and the control volume is fixed as it was at time t^o. So, it remains in the same location.

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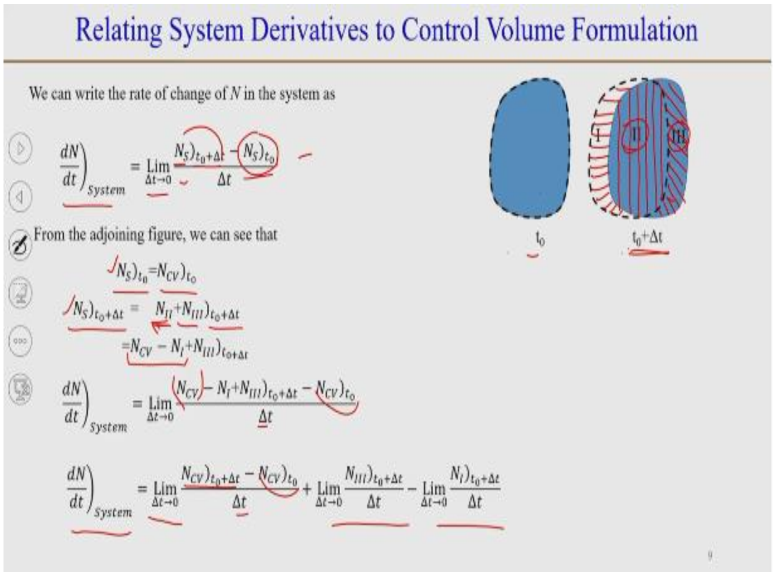

Now, we consider the motion of a particular intensive property N or what has happened to N when the system has moved from location, from the control volume from this position to this position. So, to do this we have divided the system at time t^o + Δt, in this figure we identify three regions. So, let us look at this region 1 is that is part of the control volume but the system has moved through it and we try to hatch it with kind of horizontal lines.

The region 2 is which is the intersection of the two, it is part of the system still and is also inside the control volume and region 3 is what has moved out of the control volume, but it is part of the system still. So, we consider these three regions 1, 2 and 3, what we need to find is basically the rate of change of intensive property of the system.

So, if we write the rate of change of N in system dN/dt of system from the fundamental definition of derivative, so we can write that in the limit ∆t tending to 0 dN/dt will be equal to N_S at t^o + ∆t which is the intensive property N inside the system at t^o + ∆t minus intensive property N inside the system at time t^o/∆t, so, that is dN/dt.

Now, we can try to see these two terms N_S $t^o + \Delta t$ and N_S t^o from what we saw in, see in these figures in terms of what is there inside region 1, 2 and 3 or control volume. So, we see that N_S t^o or the intensive of property in the system at time t^o will be equal to the intensive property N inside the control volume at time t^o because at time t^o the system and control volume coincide, so the intensive of property in the two will be same.

Now, let us look at the intensive property inside the system at time $t^o + \Delta t$. So, that we can see that this is equal to what is there inside the system N_2, N_3. So, $N_2 + N_3$ at time $t^o + \Delta t$. Now, N_2 what we can see here is N_2 is N inside the control volume at time $t^o + \Delta t$ minus N inside the region 2 because region 2 is still inside the control volume. So, N_2, that will be N_1, sorry, now, let us do it again.

So, N_2 is equal to inside the control volume N_{CV} at time $t^o + \Delta t$, if we subtract N_1 inside the control volume at time $t^o + \Delta t$, then we can write that $N_2 = N_{CV} - N_1$ both at time $t^o + \Delta t$ + of course, N_3 remains same. So, we can substitute the two N_S $t^o + \Delta t$ and N_S t^o in this formula.

So, we get dN/dt of the system that = limit Δt tends to 0, we can substitute N_S $t^o + \Delta t$ that will be N inside the control volume at time $t^o + \Delta t$ $-N_1$ inside the control volume or $-N_1$ at $t^o + \Delta t + N_3$ $t^o + \Delta t$. So, N_1, N_3 and N_{CV} they are all at time $t^o + \Delta t$ in these respective regions region 1, region 3 or inside the control volume, minus in place of N_S t^o what we write is N_{CV} t^o/time Δt.

So, we can rearrange and write it again. So, dN/dt of system that will be equal to in the limit Δt tends to 0, we can take N_{CV}, which is at time $t^o + \Delta t$ - N_{CV} at time t^o. So, we can take those together divided by Δt and we can see that this will give us dN/dt inside the control volume + we have two more terms, which is limit $\Delta t \rightarrow 0$ N_3 $t^o + \Delta t/\Delta t$ -limit Δt tends to 0 N_1 $t^o + \Delta t/\Delta t$.

Now, if we look at what is N_3 and N_1. So, N_3 is the region, which has gone out of control volume and N_1 is the region, which has come inside the control volume. So basically, N represents the intensive property, so the intensive of property that has come inside the control volume is N_1, and intensive property that has gone out of their control volume in time Δt is N_3.

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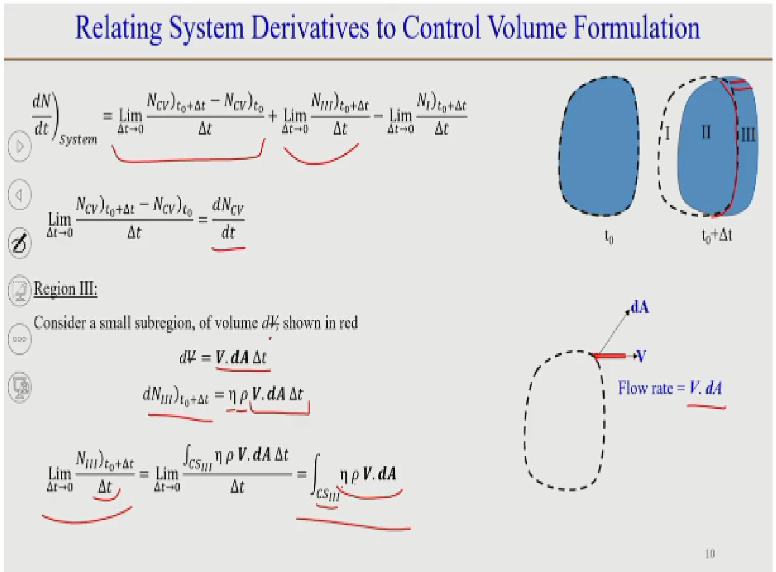

Let us simplify this, and we can write the first term = d/dt of N_{CV}, so the rate of change of intensive property in the control volume. Now let us look at this term. so if we take a small sub region, as shown here in the red and volume of this sub region is dV̶, if we say, so that basically represents that in this small volume, it represent what is the flow rate or what fluid has gone, so the volumetric flow rate will be $\dot{V}$ dA, as we discussed just now, that if the region, if V and dA they are not collinear or they are not parallel to each other, then the volumetric flow rate that comes out through this area dV̶ will be equal to $\dot{V}$ dA.

So, because this is the flow that has gone out of the control volume in time Δt, so through this area dA, the flow that gone out, that will be $\dot{V}$ dA into Δt, so in time Δt $\dot{V}$ dA Δt is the volume of the fluid that has gone out. Now, what will be the intensive property that has gone out through this, dN in region 3 t^o + Δt that will be equal to η ρ multiplied by the dV̶, so dV̶ we know $\dot{V}$ dA Δt. So, we can substitute this in place of d and 3, or in terms of integral.

So, if we integrate over this entire boundary what we call control surface 3, that is the boundary that intersects between the control volume and system. So, over this control surface 3, $\eta\ \rho\ \dot{V}$ dA $\Delta t/\Delta t$, and Δt is a small time which is constant or independent of this control surface boundary.

So, Δt will cancel out and what we get is this limit Δt tends to 0, intensive property in region 3 at time $t^o + \Delta t/\Delta t$, that = integral over control surface $\eta\ \rho$ V or dA. So that basically represents, the intensive property that has gone out of the control volume through this control surface. So, that is the intensive property that has gone out of control volume.

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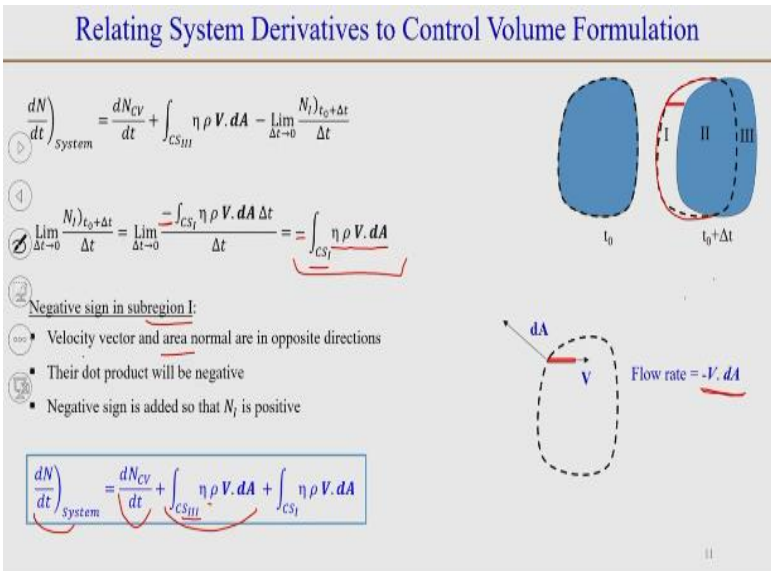

Now similarly, we do the same analysis for region 1, for region 1 the only difference that is going to come in that in this case the fluid is going to come inside the control volume, so if we consider a small region in region 1, then the flow rate here because outward normal and the velocity vectors they are in opposite direction, so the flow rate that will be coming in will be -$\dot{V}$ dA.

So doing the same analysis, but with a minus sign because the flow is coming inside the control volume, so we will get limit Δt tends to 0 N_1 at time $t^o + \Delta t$, that is the intensive property in region 1 at time $t^o + \Delta t$, that is what has come in, in time Δt, inside the system.

So that will be equal to minus integral over the control surface 1, which is this boundary that is an intersection of region 1 and control volume, so that = $\eta\ \rho\ \dot{V}$ dA, so this is the surface CS_1 through which the fluid comes in. Now we substitute this, so there is negative sign, this negative sign is because velocity vectors and area normal they are in opposite directions, and their dot product will be negative, so we add a negative sign there.

So, if we substitute all this, then our equation becomes that rate of change of N inside the system that is equal to the rate of change of N inside the control volume + the flow that is coming in CS through the control surface, CS_3. The intersection of control volume and region 3 η ρ $\dot{V}$ dA + integral over control surface 1 η ρ $\dot{V}$ dA through control surface 1. Now, because we have taken the negative sign into account while deriving this, we can combine these two.

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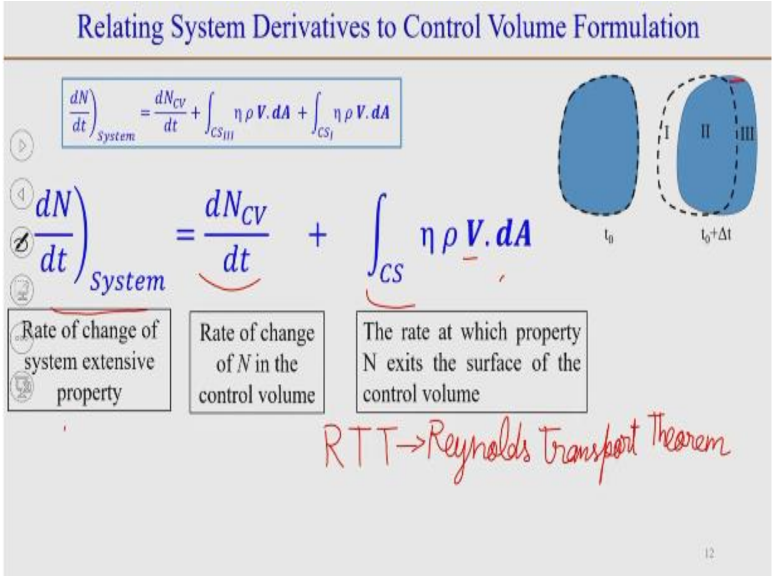

So, on combining we can write the general equation that dN/dt of system that will be equal to d N_{CV} /dt the rate of change of intensive property inside the control volume that is equal to the net outflow from the control volume. So that is, rate of change of extensive property N that is equal to the rate of change of N in control volume + rate which the property N exits the surface of the control volume. If it is coming in, then we have to take into the negative sign of V into account and that will be taken here of by the sign of outward normal and the velocity vector.

So if we look at, this is a mathematical expression and that will be very useful when we solve problems and identify that what mass, what is the momentum or energy that is coming in and going out of the system. But if you look at this system, this is a very simple equation which tells us that inside a control volume, the rate of change of intensive property in the control volume is equal to the rate of change of extensive property inside the system minus that has gone out of the controls, through the control surface of this control volume.

So we will stop here and in the subsequent lectures we will look at the application of this equation, which is known as Reynolds Transport theorem, so this is what we call RTT commonly or Reynolds Transport theorem, which will be used for converting the conservation of mass, conservation of momentum and conservation of energy formulations, from a system formulation to a control volume formulation. So in the subsequent lectures we will look at that, how we will do that and solve some problems, we will stop here. Thank you.