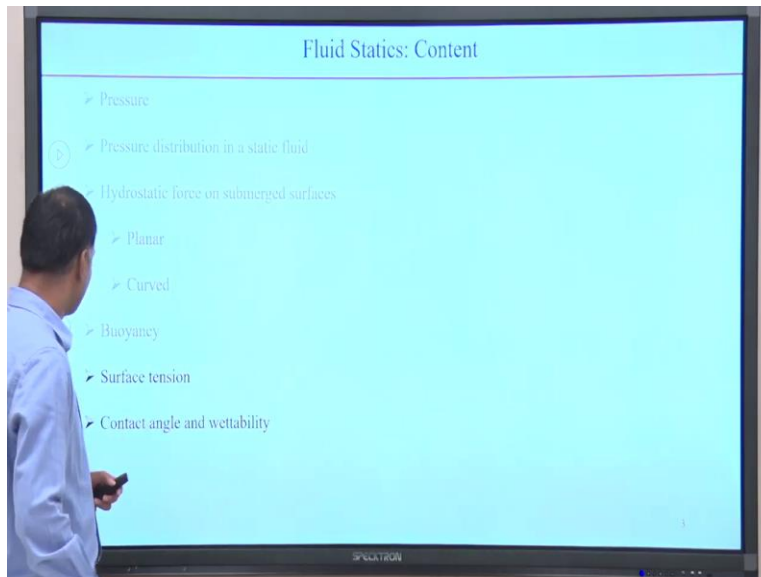**Fundamental Fluid Mechanics for Chemical and Biomedical Engineer**
Professor Doctor Raghvendra Gupta
Department of Chemical Engineering,
Indian Institute of Technology, Guwahati
Lecture 14
Surface Tension

Hello. So, in this course, we are going to talk about surface tension. In this module, where we are talking about fluid statics, first, we talked about the hydrostatic pressure, how does it vary with the depth. Then, we looked at the forces on submerged surfaces plane as well as curved surfaces, and using those force pressure distribution, we also looked at that that how we can obtain the buoyancy force on bodies.
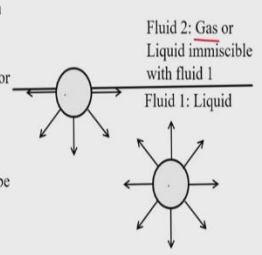
(Refer Slide Time: 0:55)

In this lecture, we are going to look at surface tension, which occurs at a gas liquid interface or at the interface between two immiscible liquids. We will also look at the contact angle and wettability, where a three-phase contact happens.

Surface Tension

- The molecules in a liquid attract one another.
- A molecule in the interior of the liquid experiences attractive forces from the molecules of the same liquid surrounding it.
- However, a molecule at the interface between the liquid and a gas or another immiscible liquid experiences a net cohesive force.
  - And minimize surface area.
- Surface tension (represented by σ or γ) is the energy that must be supplied to increase the surface area by one unit (Unit: J/m^2 or N/m).
- Surface tension at room temperature
  - Water 0.072 N/m, mercury 0.485 N/m, Ethanol 0.022 N/m
- Surface tension decreases with an increase in temperature or in the presence of surfactants.
- The surface energy between two immiscible liquids is known as interfacial tension.

Fluid 2: Gas or Liquid immiscible with fluid 1

Fluid 1: Liquid

4

So when we look at a liquid, the molecules in liquid they attract each other. Now the molecule, which is inside the liquid, the molecule is attracted by the surrounding molecules and it experiences in an average sense, the equal force from all the directions, whereas the liquid molecule that is at the interface, the molecule experiences the cohesive force from the liquid side, but from the other side where there is gas or different liquid present, it does not experience this cohesive force.

So there is a net force acting on the fluid because of the molecules present in the liquid. This molecular interaction at the macroscopic level is what we call surface tension. So surface tension is the property, because of which, all the liquids, they try to minimize their surface area. So if we want to increase the surface area of a liquid, then, we need to provide energy to the liquid. So the surface tension can also be defined or is defined as the energy that we must supply to increase the surface area by one unit.

So surface tension is surface energy per unit area. So the unit for surface tension can be J/m^2. Now the surface tension, it can be represented by σ or by γ. So in this lecture, we will use the symbol σ for surface tension. Now, as we will see in a few minutes, that surface tension is also represented as force per unit length. So the unit of surface tension is also M/m.

Now for different liquids, the interaction forces can be different. For example, for oils, it is the Van der Waal force that is the interaction force. In water, hydrogen bonds are present. So the interaction is relatively stronger and in a liquid metal, such as mercury the interactive force or the
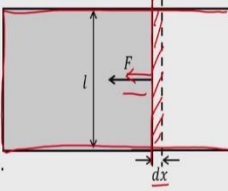
molecular forces are further stronger. And that is what we see in the values of surface tension. So these values are at room temperature. So the value for water is 0.072 N/m.

Whereas the value for mercury is about one order of magnitude higher, which is 0.485 Newtons per meter. And for ethanol, the value is 0.022. So it is significantly lower than that of water for organic liquids. The surface tension depends on temperature. So the value of surface tension decreases with an increase in temperature as well as the surface tension also decreases in the presence of surfactants. When we have in place of gas liquid, if we talk about molecular interaction between two immiscible liquids, then, this surface energy per unit area is called interfacial tension.

(Refer Slide Time: 6:20)

Now, this the phenomenon of surface tension or force caused by surface tension can be easily understood by an example. If we take a U-shaped wire and connect with it a movable rod, it may be a glass rod or another wire, which can move freely on this wire and dip it in a liquid or in soap water. Then, a liquid film will form. When the liquid film forms, and if we take it out from the liquid, the glass rod will start moving in this direction so as to reduce the surface area of the film.

And this is happening because of surface tension. As we said, just now that the surface tension, it is responsible for the reduction in the surface area of liquids. So if we assume that a force F is being experienced by this rod, then, the work done on this rod will be for a small displacement dx. The work done on the rod will be F into dx. From the definition of surface tension, this work will be equal to the change in surface area or change in interfacial area multiplied by surface tension.
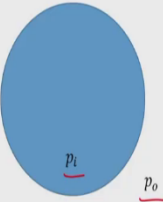
So the interfacial area for this small change will be this area. And this area will be dx multiplied by l. So γ into ldx will be the work, but note that this interface, or this surface has two interfaces, one on one side and on the other side. So the work done Fdx will be equal to 2 γ ldx and dx and dx will cancel out. So we will get F = 2 γ dl. So we can say that γ = F/2l. So basically that the force per unit length is surface tension and this force acts in the plane of the surface, and it is directed towards the liquid.

So surface tension is force per unit length. And it is, this force is tangential to the interface and directed in the liquid. Now, when we talk about bubbles and droplets, we know, or we would have heard, of that there is excess pressure in the droplets or bubbles because of surface tension. And this pressure is often known as or called Laplace pressure. So this pressure is caused by surface tension, and we will see, that how we can quantify this pressure.

(Refer Slide Time: 9:40)

So if we take a spherical bubble of radius R and the pressure outside this bubble is po and pressure inside this bubble is pi because the pressure inside the bubble is higher than the outside pressure. So the pressure difference or excess pressure will be p_i - p_o.
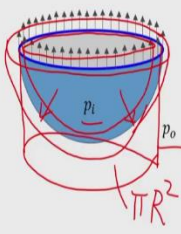
(Refer Slide Time: 10:09)

Laplace pressure for a spherical bubble

- The pressure difference across the surface of a stationary bubble (Δp) can be related to surface tension (γ) and bubble size (radius R).
- Consider a control volume surrounding the hemispherical interface.
- Forces on the control volume in the vertical direction:
  - Surface tension force on the hemispherical shell= $2\pi R\,\gamma$ (acting upward)
  - Pressure force = $p_i\pi R^2 - p_o\pi R^2$ (acting downward)
  - Weight of the interface can be neglected: negligible thickness
- Force balance gives:

$$2\pi R\,\gamma = (p_i - p_o)\pi R^2$$

Pressure difference across the bubble interface $\Delta p = (p_i - p_o) = \frac{2\gamma}{R}$

Now, if we take a control volume, which is the hemispherical shell, which is surrounding the interface. So if we take the control volume as this hemispherical shell and balance the forces in the vertical direction, so, we will have different forces acting on it. The one force is because of surface tension. So on this interface, the force caused by surface tension will be equal to σ or γ, which is surface tension multiplied by the length of this interface, which is equal to 2πR. So 2πR γ will be the surface tension on this hemispherical shell, which will be acting upward.

Now, the force caused by pressure so the pressure will be acting normal to this surface. And the magnitude of this force will be p_i - p_o into πR^2. Note that, we are looking at the component in the vertical direction. So the component of the force in the vertical direction will be equal to the projected area of this surface and the projected area in the vertical direction will be equal to πR^2. So the pressure force will be acting downward, and that will be equal to p_i - p_o into πR^2, because the interface is of negligible thickness. So the weight of the interface can be neglected.

So we have two forces acting on this control volume, the force caused by surface tension and the pressure force. So if we balance these forces 2πR γ, that will be equal to pi - P⁰ πR², π and π cancels out, one R cancel out, and we have the excess pressure Δp, which is equal to pi - p⁰ that is equal to 2 σ/R. So that is the excess pressure when we have a spherical bubble of or droplet of radius R.
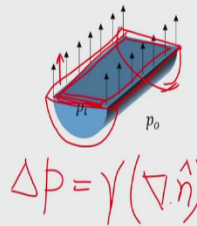
(Refer Slide Time: 13:14)

Laplace pressure for a cylindrical jet

- For an infinitely long cylindrical column of liquid of radius R and length l in air
  - Consider a control volume of length l around the semi-cylinder
- Forces on the control volume in the vertical direction:
  - Surface tension force on the semi-cylindrical shell $= 2l\gamma$
  - Pressure force $= (p_i - p_o)(2Rl)$
  - Weight of the interface can be neglected: negligible thickness
- Force balance gives:

$$2l\gamma = (p_i - p_o)2Rl$$

Pressure difference across the bubble interface $\Delta p = (p_i - p_o) = \frac{\gamma}{R}$

$$\Delta p = \gamma\left(\nabla \cdot \hat{n}\right)$$

Now, if we have a cylindrical column or cylindrical jet of liquid in a gas medium, then, we can similarly find the excess pressure inside a cylindrical jet. So if we consider an infinitely long cylindrical column of liquid, which has a radius R and for the analysis purpose, we will consider a control volume of length l. So, if we consider as before for the sphere, we will consider a control volume.

And in this control volume, if we look at the forces then again, force balance in the vertical direction. So surface tension force will be acting upward in this direction. And that will be equal to surface tension multiplied by the length of the interface, which is 2l. Note that, these 2, because this is not an interface, this is again extended towards liquid. So because this is an infinitely long cylinder, it is the only length l we have taken as the control volume. So the surface tension force, this semi-cylindrical shell is equal to 2l into σ, where 2l is the length of this interface.

Now, again, the pressure force on this will be p_i - p_o, which is the pressure inside minus pressure outside multiplied by the projected area. So the projected area in the vertical direction will be 2R, which is the diameter of this cylindrical jet, and l is the length. So p_i - p_o into 2Rl is the pressure force. And again, we can neglect the weight of the interface and, doing the force balance, we get that 2l σ = p_i - p_o, which is the excess pressure multiplied by 2Rl.

So 2 and l, they cancel out and we get excess force is equal to γ/R. So the pressure difference across the bubble interface, Δp = p_i - p_o that is equal to γ/R. Remember for a sphere, we had 2 γ/R,

whereas for a cylinder, it is γ/R. For an interface, which is of, whose curvature is varying the excess pressure can be given as σ into the curvature of the interface.

And the curvature of interface can be given as $\nabla.\hat{n}$. So $\hat{n}$ is the unit normal to the interface and its dot product with the ∇ multiplied by surface tension coefficient. So, which is, we are using the symbol γ. So we will use that for the sake of uniformity that is $\gamma(\nabla.\hat{n})$ is the excess pressure when it is, the interface is, the curvature of the interface is varying.

(Refer Slide Time: 17:34)

Now, what we talked about is when the interface is between a gas and a liquid, or between two immiscible liquids, we also come across cases, where the interface is in contact with a solid surface. For example, a droplet is sitting on a solid surface. You have a droplet in our hand, you see the droplets on a glass surface, on grass, on leaves, etcetera. And if you notice the shape of the droplet on different surfaces, you find that depending on the surface and the liquid, the shape of the droplet will be different. And that is because the different surfaces have different properties as well as different liquids also have different affinity towards surfaces.

And this is explained by what is called wetting. So, this comes under what we call contact between three phases. So wetting refers to that, how, when a liquid is deposited on the surface, how does it spread? And that depends on the contact angle. So what is contact angle? When you have a droplet deposited on a surface, a contact line forms. Contact line is basically the line or the intersection of the interface and the solid surface.

So in this figure, the black line is what is contact line. Now this contact line is in contact with three phases, the gas phase, the liquid phase and the solid phase or the solid surface. So the angle between the tangent to the interface and the solid surface is contact angle. Now, one can think of that what is the contact angle, this angle or the angle on the other side. So for clarity, it is the angle through the liquid that is called contact angle. So in this case, this is the contact angle.
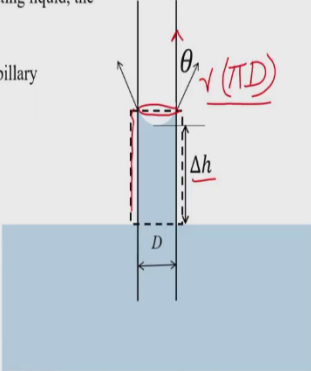
Now, if this angle is less than 90°, then it is called wetting surface, or the surface is known as wetting surface for that particular liquid. And if this angle is greater than 90° then the surface is called as non-wetting surface. Because in many applications, we need to design materials of surfaces, where it the requirement is that the liquid sticks to the surface or the requirement is that the liquid does not stick to the surface. So the hydrophobicity or hydrophilicity of the material is an important property of the material.

So when a new material is developed during its characterization, the contact angle is also an important factor, and that is how the material can be characterized, that if the contact angle is less than 90°, then it is called wetting or hydrophilic surface. And if it is greater than 90° then it is called non-wetting surface. You would have seen, if you go and see a lotus leave, the droplets on the lotus leaves, they are almost spherical shape, because, we say that the liquid does not wet the lotus leaf surface. And that is one natural material, which is very hydrophobic surface.

So, another phenomenon which is related and relevant is what is called capillary rise. So you might have often wondered that when you light a lamp or when you light a candle, the wax, or the oil or ghee that because of the capillarity action, it rises up, it goes against gravity and then rises in the wick, because of the capillary rise phenomenon. So the capillary rise phenomenon is that when a tube of small diameter D is dipped in a pool of liquid, then, the liquid rises up to a certain height in the capillary.

So if you take a capillary and the liquid is wetting for the capillary material, then, it will rise a height, let us say Δh. So if you do the force balance, the force at the interface, where a three phase contact line forms will be equal to γ into the length of this interface. So the length of the interface will be π into D, which is equal to the circumference of the capillary, because that is where the interface will be.

So taking a control volume, as you can see by the dotted line here, the force because of surface tension is $\gamma \pi D$. And if we take the vertical component of this force, that will be equal to this $\gamma \pi D$ multiplied by $\cos\theta$. Now, the other force on this liquid column of course because of gravity. So the force due to gravity will be m into g. So, g is the acceleration due to gravity and m will be density into the volume of this liquid column. So ρ is density and the volume of liquid column, because it is cylindrical column, we can approximate it as a cylindrical column, so that will be

equal to $\pi/4D^2$, which is the cross-sectional area of the capillary multiplied by the height of the liquid column.

Now, so, we understand that this liquid is being driven by the surface tension force. So if we balance these two forces, then, $\gamma \pi D \cos\theta = \rho g \pi/4D^2 \Delta h$, π and D will cancel out and what we get is $\Delta h = 4 \gamma \cos\theta / \rho g D$. So we can see that smaller the diameter, higher the Δh. So that is why if the capillaries, which are smaller, then the liquid can rise to a greater height. And the same phenomenon is also responsible for the transportation of water and nutrients from the ground to the leaves.

So in a way that also determines that to what height can a tree grow, because the tree can grow only up to the height, where the nutrients can be supplied by the roots. So, the capillary rise phenomenon can also determine the height of trees that can theoretically grow in a particular area. Now, this also depends of course on $\cos\theta$. So if $\cos\theta$, if θ is less than 90, then this value is positive. If θ is more than 90°s, then $\cos\theta$ will be negative. So $\cos\theta$ more than 90 means it is a de-wetting liquid. So in the de-wetting liquid, the liquid surface in the capillary will go down or the level of liquid in the capillary will go down that in the surrounding liquid pool.

(Refer Slide Time: 26:55)

So, then, we can list down some of the important non-dimensional numbers, which are relevant for surface tension. So, we can compare it with different other forces, which are important in fluid mechanics. So I have listed down three numbers here. The first one is known as bond or Eötvös

number, which is basically the ratio of gravity, or more precisely buoyancy and surface tension. So that is $\Delta\rho gL^2/\gamma$, where $\Delta\rho$ is the difference of the densities of the two fluids. It might be gas and liquid, or it may be liquid liquid. So that is called Eötvös number. And that gives us an estimate of the relative importance of gravity and surface tension forces.

So, in microfluidics, which deals with flow of liquids in millimeter size or smaller channels. Now, the bond number becomes smaller and the gravity and surface tension forces become almost of equal importance. And as the dimension becomes further smaller, say less than 1 mm for air water flow, then surface tension becomes a dominant force. So some of the effects, which we are used to see. For example, if we have gas and liquid flowing, and in large channels, we can see that there is a stratified flow under certain conditions, which means that the liquid flowing at the bottom and the gas is at the top.

Whereas in a micro channel, you may not see such phenomena because the surface tension force is important and the interface will be a circular one. So you will see in place of stratified, you will see annular flow in such cases. Another important number is capillary number, which compares viscous and surface tension forces, where V is velocity or the characteristic velocity of the phenomena. So $\mu V/\sigma$, when inertial effects are important, then, the Weber number comes into picture, which is inertia divided by surface tension.
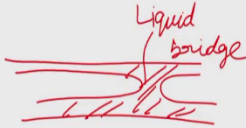
So as I said, that, surface tension effects because in microfluidics, in general, the surface effects become important because the surface area per unit volume or surface area density, or interfacial area density in case of two phase flows or in case of multi-phase flows, the interfacial area density, which is interfacial area per unit volume becomes high, because that is proportional to 1/D. So that becomes important in microfluidics.

Then, Marangoni stresses, so, we talked about surface tension effects, and we discussed that surface tension is dependent on temperature or the surfactant. When the temperature increases, surface tension decreases or the presence of surfactant is also responsible for decrease in surface tension. So if there is a gradient in temperature that will also cause a gradient in surface tension. And that will start, or that will cause a flow along the interface. And that is, that will result in stresses also along the interface. So these stresses are known as Marangoni stresses and such flow generated because of these stresses is called Marangoni flow.

(Refer Slide Time: 31:20)

Now some typical applications of surface tension. So the formation of bubbles and droplet that is determined by the surface tension and its relative importance when compared with the other forces such as viscous, gravitational or inertial forces. And depending on their relative importance, one will get the size of bubbles and droplets. And that is very important. Some applications, for example, in inkjet printing, what size of droplet is going to be generated, are there secondary droplets going to be generated?

How these droplets will go and deposit in the interface, how they will spread in the interface? That is all where surface tension and contact line and contact angle effects are important. The shapes of bubbles and droplets, again, when the bubbles are of very small diameter, then the surface tension is the dominant force and the bubbles are spherical or the droplets are spherical. But as the size of the bubble or droplet grows, other forces also become dominant.

So in that case, the bubble shape may not always be spherical and that we see it in our day-to-day life that sometimes bubbles are spherical and sometimes they become an oblate shape and so on. So, there also surface tension effects are important. The generation of droplets and bubbles are also important in an epidemic situation. So generally, the contamination or the disease spread happens because of the droplets generated when somebody speaks or somebody breathes or sneezes or coughs, and this is all happening, or this is all being caused by the droplets.

So what is the size of droplets that are being generated when somebody sneezes, coughs or breathes or speaks, the size of the droplets being generated. And then, how long can these droplets will travel will determine that what is the safe separation distance between the two people or social distancing what we call in. Then, in the chemical industry, we would like to have paints, which can make the surface hydrophobic or and the water and other liquids do not stick with it.

Insecticides, which can spread to a larger area and spread over the plants, etcetera. Then, there is another important effect in biomedical engineering, which is called Infant respiratory distress syndrome. So when we breathe air, there is a mucus lining in our lungs, in our airways. Now, when there is so basically where the flow of air is two phase flow, the air flows of a over a mucus liquid. And because of surface tension, a bridge can form. So if this is the airway, and then, the mucus may form a liquid bridge. So this is the liquid bridge.

Now, for the air to come and go, this liquid bridge needs to be broken. In adults and toddlers also, a surfactant is generated, which reduces the surface tension, naturally the surfactant is generated. But in premature babies, which do not complete the gestation period of 40 weeks, and if they are born prematurely, then the surfactant is not generated in them. And then they need to be given either ventilation so that they can breathe easily or a dose of surfactant. So this is known as infant respiratory distress syndrome or hyaline membrane disease. So surface tension is a very important phenomenon in chemical as well as biomedical engineering, having a number of applications. So with this, we will stop today. Thank you.