Fundamental Fluid Mechanics for Chemical and Biomedical Engineer
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Pressure Distribution in a Static Fluid

So, in this module, we are going to look at fluid statics. So, fluid statics refers to those problems or that branch of fluid mechanics in which the fluid is not under motion and it has a number of applications. So, let us look at the content first.

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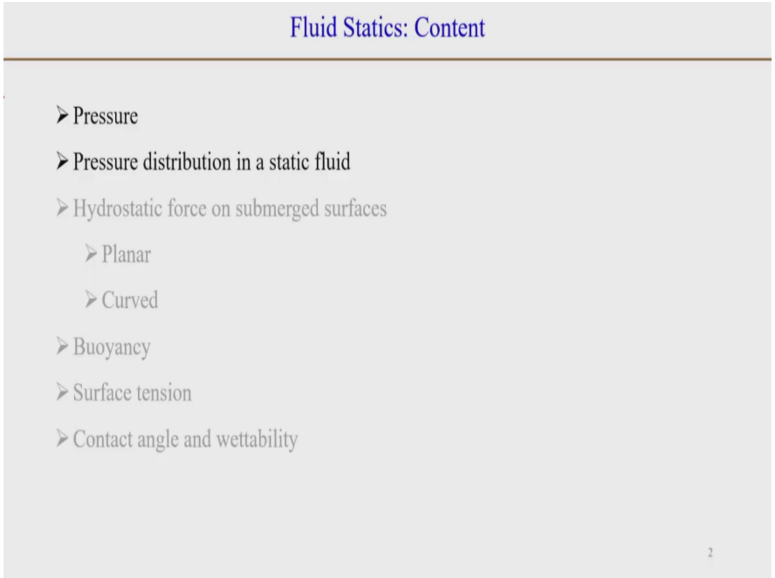

We will be talking about pressure and then pressure distribution in a static fluid. All of us know that p = ρ gH in a static fluid. So, we will derive this relationship and look at different applications of this. Then, we will also look at in the subsequent lectures, how we can calculate the hydrostatic force on submerged surfaces. These surfaces might be planer, or these might be curved. Then, we will look at how we can derive using the principle of fluid statics, how we can derive the principle of Archimedes or Archimedes' Principle which gives rise to the concept of buoyancy.

Then, we will look at fluid-fluid interfaces, capillarity effect, surface tension between two fluids. And when there are 3 point contacts, solid-gas-liquid, or solid-liquid-liquid immiscible liquid on a solid surface, then the concept of contact angle and wettability. So, in today's lecture, we will look at pressure and pressure distribution in a static fluid.

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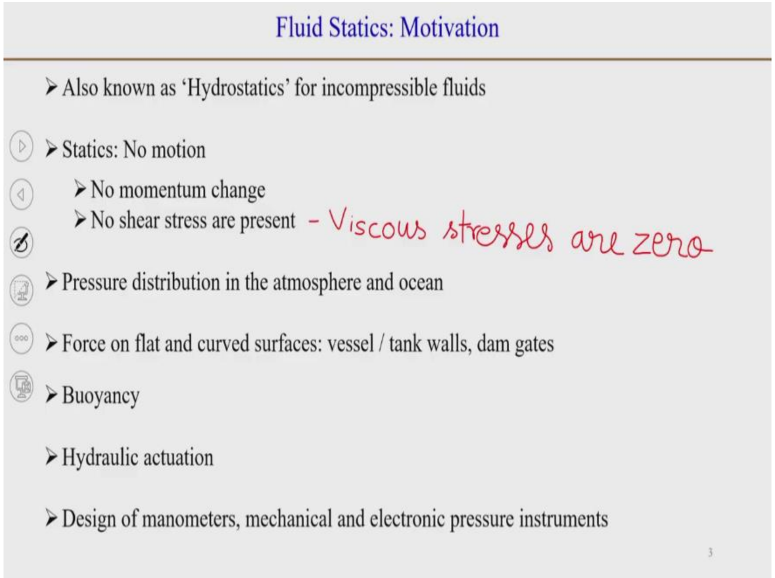

Now, as the term suggests, that the fluid statics, so statics refers to that branch of mechanics in general, where the body is not in motion. So, when we are talking about fluids, fluid statics will refer to that branch of fluid mechanics in which there is no motion of fluid involved. Now, as we know that fluid can be gases, or liquids, gases in general, they are compressible in nature. So, we can have compressible fluids or incompressible fluids. Now, when it is incompressible fluids, then, the fluid statics is also known as hydrostatics.

Statics, as I said, it refers to no motion. So, that has an implication that when there is no motion, the velocity is zero and the momentum which is the product of mass into velocity is also zero. So, there is no momentum change involved. And by the definition of fluid itself that, if fluid deforms continuously under the application of a shear stress or shear force. So, if the fluid is not in motion, that means the fluid is not deforming, then there is no shear stress. So, the only stress that will be present and viscous forces, which causes shear stress, so viscous forces in a stationary fluid will be zero.

So, we can say that in a stationary fluid, viscous stresses, as we know for a Newtonian fluid from Newton's Law of motion, that viscous stress is proportional to shear rate. And when the shear rate is zero, viscous stresses are also not present. So, the only surface force present in a static fluid is pressure force. So, there are a number of applications, where these principles of a fluid static are needed to be applied.

The first one that I have listed here is pressure distribution above the earth surface and below earth surface. As we know that P is proportional to ρ gH, where H is the height or it may be depth. So, when we go up in the atmosphere, the height increases. When we go down in the ocean, the height decreases or the depth increases. So, in both cases, the pressure is going to vary. And, any device or any person or any animal who is going in the atmosphere, well above earth surface or deep in the ocean, the pressure at the depth or pressure at an elevation needs to be calculated.

Then, all the storage vessels or dams, tank walls, they have flat or curved surfaces. So, each of these will experience a force because of the hydrostatic pressure. So, we need to also know how to calculate this force on flat and curved surfaces. So, we will look into this problem. Then, we all know the principle of buoyancy, Archimedes' principle that when a body is submerged in a liquid, then a force acts on it opposite to gravity if the body is lighter than the liquid or the density of the body is lighter than that of liquid, then a net force will act opposite to gravity.

But in any case, there will be a force acting on a body, which is a buoyancy force, and that will be equal to the weight of the fluid displaced by the body. So, that is what Archimedes' principle is, we will look into it. And then, in hydraulics, there are a number of applications. So, for example, hydraulic actuation and design of pressure measurement instruments, it can be a simple manometer or mechanical pressure measurement instrument, or electronic pressure measurement instruments. So, we will look at some examples of manometers.

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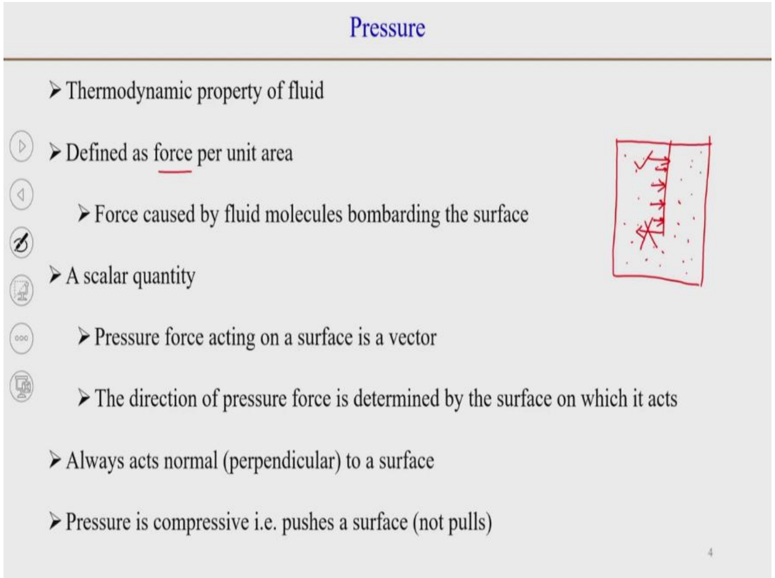

So, before we go into further details, let us look at or let us remind ourselves, what is pressure? So, when we read thermodynamics, or when we studied thermodynamics, we talk about kinetic theory of gases, and there are the thermodynamic property of fluid pressure comes into picture. So, it is defined as force per unit area. Now, what is this force? The force is caused by fluid molecules bombarding the surface.

So, when you have a surface, you can have a real surface or an imaginary surface inside a fluid, then, because of the molecular motion, all the fluid molecules will collide with the surface. And the net force that acts on this surface is what we call this force when we define pressure as force per unit area. So, of course, the net force will be normal to the surface. So, that is why the pressure force always acts normal to the surface on which it acts, and it is a compressive force. So, it is acting on the surface, not away from the surface.

So, you have a, let us say a vessel and in this vessel if there is gas filled into it, and there is a surface. So, the molecules in this vessel will be distributed randomly. And these molecules will have their random motion. And during this motion, they will collide with the surface and cause a net normal force normal to this surface. So, this force per unit area is what is called pressure force.

Now, the surface that we talk about, it may be a real surface or an imaginary surface. So, pressure, we can say that pressure can be at any point in the fluid, it is not only at the surface. But it is defined with respect to a surface. Now, pressure is a scalar quantity in itself. When we talk about the direction, it is the pressure force that acts on a surface. Now, the direction of this pressure force is determined by the surface on which it acts. So, when we are defining pressure, we need to have a surface also on which it acts.

Now, it always acts normal and it will be compressive in nature. So, it will be acting on the surface, not pulling or not away from the surface. So, this will be the direction, whereas it can never have a direction, so this is not correct, whereas this is correct. Now pressure on this surface, if we say pressure on the left-hand side, from left to right, whereas pressure from the molecules on the right side will be acting from right to left on this surface.

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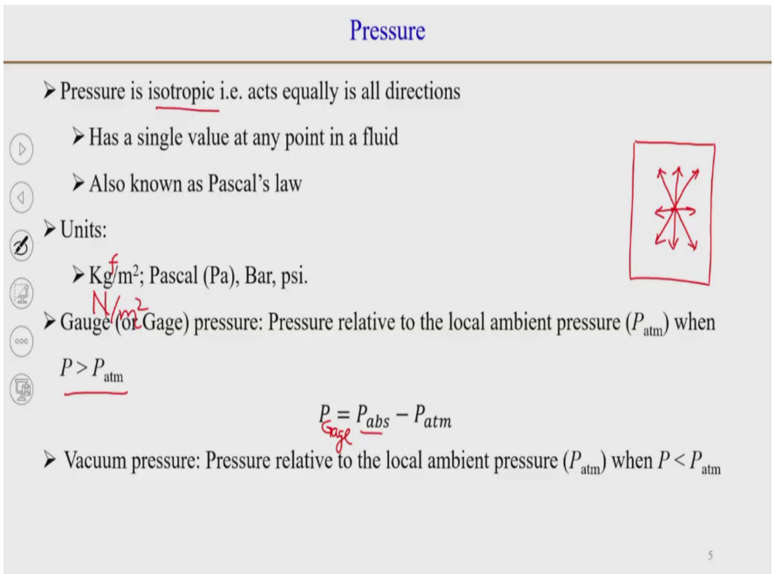

So, another property of pressure is that it is isotropic. That means that it if you take any point in a fluid, if we again draw the same image and take a point in a fluid, then, you can have a number of surfaces that pass from this point. So, on each surface, the pressure will act normal to the surface. In order that this fluid molecule is static, there is no pressure difference in any particular direction. So, the force at this point or the pressure at this point has to be same across all directions. So, that means pressure is isotropic.

So, pressure in all the directions will be same at a particular point in a fluid. So, the term isotropic means that any property that it is same in all directions, then, we call it this property to be isotopic.

So, pressure is isotropic. It, of course, has a single value at any point in a fluid. And this principle is also known as Pascal's law or Pascal's principle. Now, we know that pressure is force per unit area, so it can have different units. If you talk about kg force, so kilogram force/m^2 or Newton /m^2, it may have units of Pascal, bar or pound per square inch and so on.

Now, because pressure, when it is measured, it is measured with respect to a reference pressure. In most of the cases, when we are doing things in, on the earth surface, then, it is measured with respect to the atmospheric pressure. So, it is often reported in respect to or with respect to atmospheric pressure. So, if the pressure is above atmospheric pressure, then, we represent it as a gauge pressure. So, P_{gauge} is equal to absolute pressure or thermodynamic pressure what we call - atmospheric pressure is called a gage pressure. So, this term gage pressure is generally used when we have pressure above atmospheric pressure.

If pressure at a point or in a vessel or in a reactor is less than atmospheric, then, it is known as vacuum pressure or if we talk in terms of gauge pressure, then, pressure will be negative. So, we say that 10 mmHg vacuum, that means the pressure is 10 mm less than that of atmospheric pressure. So, this we need to remember that whenever any pressure is being reported in an experiment or in any problem, then, we need to know, or we need to ask ourselves, is this the absolute pressure, is this the gauge pressure or is it vacuum pressure?

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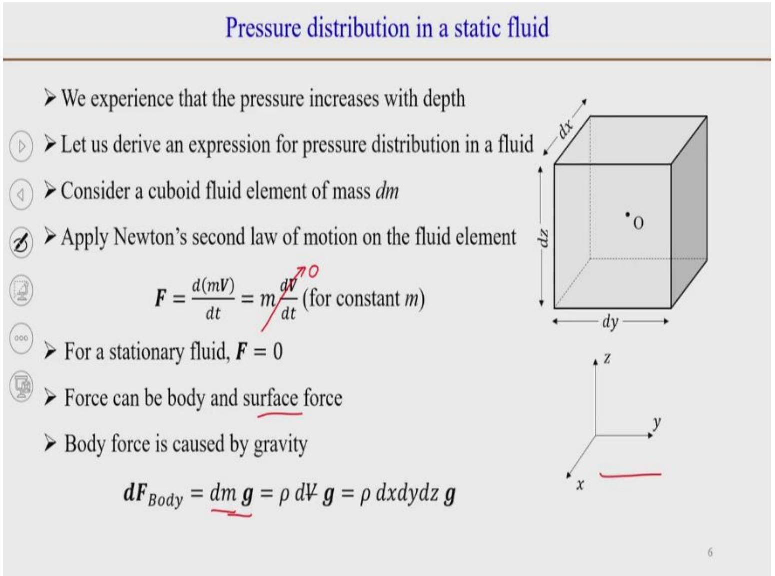

So, now we will look at the relationship that we have known for quite some time that P = ρGH, where H is depth in the fluid. So, we know that the pressure is proportional to H, where H is depth, so P = ρ GH. It increases with depth. So, let us derive this expression for pressure distribution in a fluid.

So, we will consider a fluid element, which is of cuboid shape. It has dimensions dx, dy and dz. We have a Cartesian coordinate system x, y, and z. And the length of this cuboid fluid element along the x direction is dx along the y direction is dy and along the z direction is dz. So, we will apply Newton's second law of motion on this fluid element, which we already know that force applied on a fluid element on a body is equal to the rate of change of its momentum. So, F = d by dt of mV. And if we say that mass is independent of time, mass is constant, then you can write it F = m dV by dt or more commonly we know that F = ma, where a is acceleration.

Now, because the fluid is not moving, so this term is going to be zero. So, mdV by dt will be zero. So, we have that, for a stationary fluid, the net force on the fluid will be zero. So, the only thing we need to obtain is what is the net force on this fluid element? Now, we talked about earlier that on a fluid element, we will consider two kinds of forces, body force, and surface force. So, in general, in fluid mechanics problems, when we talk about body force, it will be gravity force. And when we talk about surface force, they are pressure and viscous forces.

As we said earlier that in a static fluid, the viscous force is zero because there is no motion. So, the only surface force we will have is pressure force and then only body force that we will have is gravity. So, we need to calculate what is the body force and we need to estimate what is the surface force on this fluid element. This fluid element has 6 surfaces. So, we will need to obtain the force on each surface and then sum it up together.

Now, body force as I said, it is caused by gravity. So, we will consider that body force is equal to, if the mass of the fluid element is dm. So, body force will be dm into gravity and dm is, the mass = density into volume. So, if fluid element volume is dV, then we can write dm = ρ dV into g. Now, we can see that for this cuboid, dV = dxdydz. so, multiplication of the three sides will give us the volume of this fluid element. So, this is equal to ρ dx, dy, dz. so, this is the body force.

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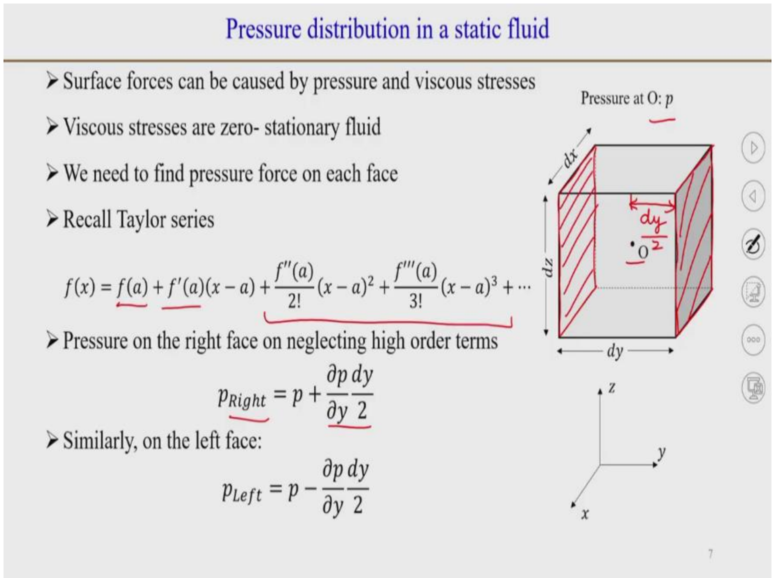

Now we need to calculate the surface force on this. So, on this, when we talk about surface forces, as I said that the viscous stresses are going to be zero for a stationary fluid. Now, we need to find pressure force on each face. So, let us assume that the pressure at the point, at the center of this fluid element, which we call point O is p, small p. Now, we need to obtain pressure force on different surfaces of this body. So, we need to consider, or we need to obtain the pressure on all six faces of the body, multiply it by the area or area vector. So, we will get the force on six surfaces, and then we sum it up, sum it up together. So, then we will get the total surface force that is being exerted on this fluid element.

So, at a distance dy/2, if we consider, if we consider, the surface on it is right, then, this surface is at a distance dy/2 from point O. So, we can obtain pressure at this point using Taylor series. So, let us remind ourselves what is Taylor series, that using Taylor series, we can obtain fx knowing f at x is equal to a, we can obtain f at a point x, which fx = fa + f ´ a, f ´ a is derivative of f with respect to x. So, f ´ is $\partial f/\partial x$. So, f ´ a into x - a + double derivative of f with respect to x at point a divide with a factorial 2 into $x - a^2$ and so on.

So, because we are taking an elemental volume, so dy is small. So, that means dy^2 will be small. So, we will neglect the higher order terms, second and higher order terms. So, if we apply a Taylor series expression, then we will obtain the pressure on the right surface, as I have shown in the figure. So, pressure on the right surface will be p + $\partial p/\partial$ y into dy/2. Now, somebody might say that, why are we not considering the variations of pressure at the top and bottom above y, along the x direction and the z direction?

So, if we are considering a linear variation of pressure, then, we will see because, when we are considering only the first order term then we will see that even if we consider the variation of pressure on this surface in the x direction, or in the z direction, then, the terms will cancel out. So, on this surface, we will consider pressure p right = p + $\partial p/\partial y$, which is pressure gradient along the y direction into this distance dy/2. So, distance from point O to the surface on the right that is dy/2, because O is at the center of this cuboid.

Similarly, we can obtain pressure on the left face. So, the left face will be this face. So, this face will be again at a distance of dy/2, but it will be - dy/2, because it is in the negative Y direction. So, pressure on the left face will be p - $\partial p/\partial y$ dy/2.

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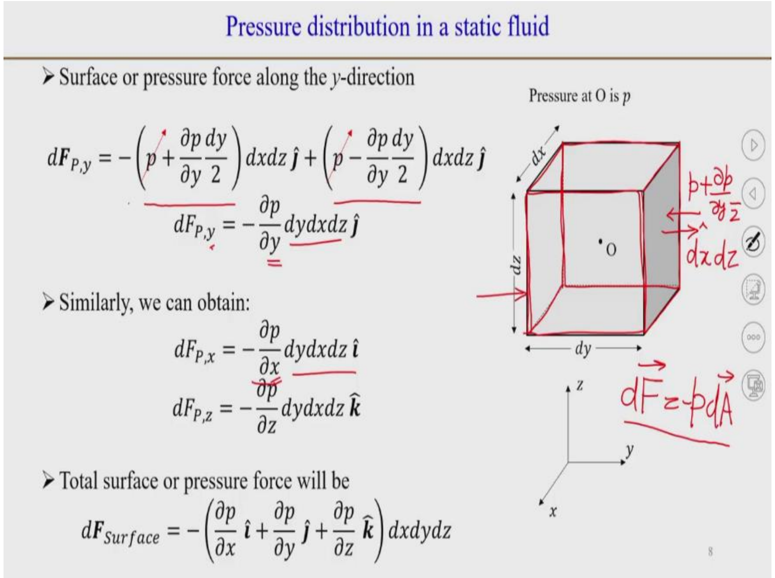

So, now we know what is the pressure on the two surfaces, surface on the right and surface on the left. And by looking at it, we can find out that when we talk about pressure along the y direction, or the pressure force along the y direction, then the contribution will come for the surface forces from the left and right forces, because that is where the normal will be along the y direction.

So, let us look at the pressure force along the y direction. We multiply this force. So, if we look at the surface on the right, the pressure here is p + ∂p/∂y, dy/2 and the area of this surface will be dz into dx. So the area is dx dz and it is the area vector will be always outward normal on this surface. So, when area vector is outward normal, then, we will have force on this will be the pressure multiplied by area and the direction of force will be negative. So, the area is outward normal, but the pressure will act in the negative direction.

So, if ever we write pressure in terms of p into dA area vector, we will need to write dF = - pdA. So, if we write dF on a surface, it will be - pd area vector. So, the area is positive. So, this is positive $\hat{j}$ direction or the unit vector $\hat{j}$. But the force that will act on this surface will be in the negative direction. So, we can see here that the pressure multiplied by area dx dz into $\hat{j}$ and negative sign.

Similarly, on the left surface, the pressure force will act in the positive y direction, whereas the area will be outward normal. So, the area vector will be pointing in the negative y direction. So, you can either just directly look at consideration of the force direction or you can use this formula dF = - pdA, and area is vector. So, when you use this, the pressure on the left face is p - ∂p by ∂y into dy/2, and the area is again, dx by dz magnitude, and it will be - j and you multiply it with the -, so then you get finally +.

So, when you sum these terms, you see that the term containing p that will cancel out and the net force or net surface force in the y direction is - ∂p by ∂y, dxdydz. So, you see that dxdydz is the volume of this fluid element. If we do the same exercise for the surfaces which are say along the x direction. So, if we take this surface and the surface at the back. And when we sum up the terms, then we will get pressure force along the x direction will be - ∂p by ∂x, dxdydz.

So, you see that the only difference from dFpy, we have here is from y is replaced by x. Similarly, the third force will be in the z direction, or the third component of this force will be in the z direction or the third component of this force will be in the z direction so - ∂p by ∂z dxdydz.

So, next, let us look and put together all the terms. So, the net surface force, and because pressure force is the only surface force now. So, net surface force, when we combine all this together, dx, dy, dz is common in all this. So, we will have - ∂p by ∂x $\hat{\imath}$ + ∂p by ∂y $\hat{\jmath}$ + ∂p by ∂z $\hat{k}$, where $\hat{\imath}$, $\hat{\jmath}$ and $\hat{k}$ are unit vectors along x, y and z directions. Now so, we have obtained the surface force.

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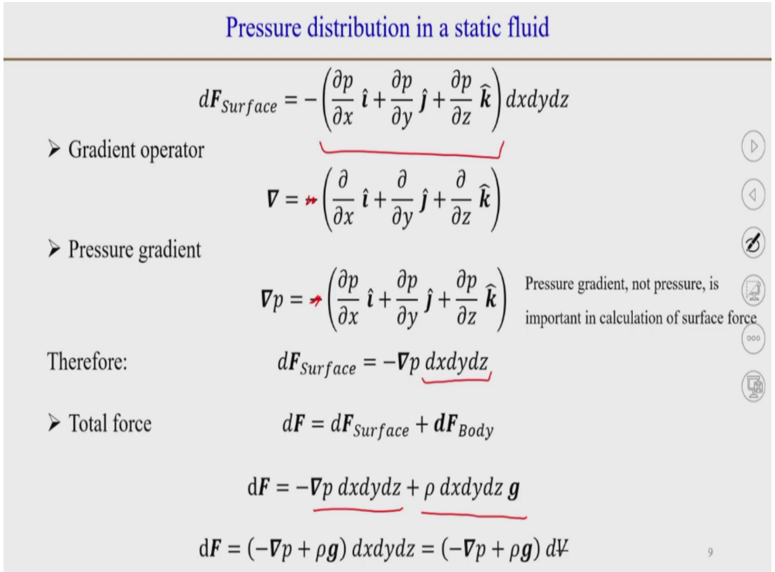

We can see here that this term in brackets basically gradient of pressure. So, let us remind ourselves what is gradient operator? Gradient operators, which is operated on any scaler quantity, and you get a vector quantity. So, gradient is - ∂/∂x, $\hat{\imath}$ + ∂/∂y $\hat{\jmath}$ + ∂/∂z $\hat{k}$ so, it is not true, we do not need - here. so, gradient operator is simply ∂/∂x $\hat{\imath}$, ∂/∂y $\hat{\jmath}$ + ∂/∂z $\hat{k}$. So, there is no - here that is a typo.

Now, pressure gradient when we apply, so we will get ∂p = ∂p by ∂x $\hat{\imath}$, + ∂p by ∂y $\hat{\jmath}$ + ∂p by ∂z $\hat{k}$. So, finally, we see that the surface force dF is - ∂p dxdydz where this is dx, dy, dz is the volume of the fluid element. So, the surface force is negative of pressure gradient. So, we can see that in the calculation of surface force, it is not the pressure itself, but the pressure gradient, which is important in a static fluid.

So, now we can combine the body and surface forces and obtain total force on this fluid element. So, the total force, we might recall from the previous slide that the total force is m, now total body force is mg, which is ρ, where ρ is density into dv, dv is dxdydz into g. So, this is the body force and as we saw just above, the surface forces - gradient of p dxdydz. so, the total force, if we take dxdydz out will be - ∂p + ρ g, or we can replace dxdydz by the volume of this fluid element.

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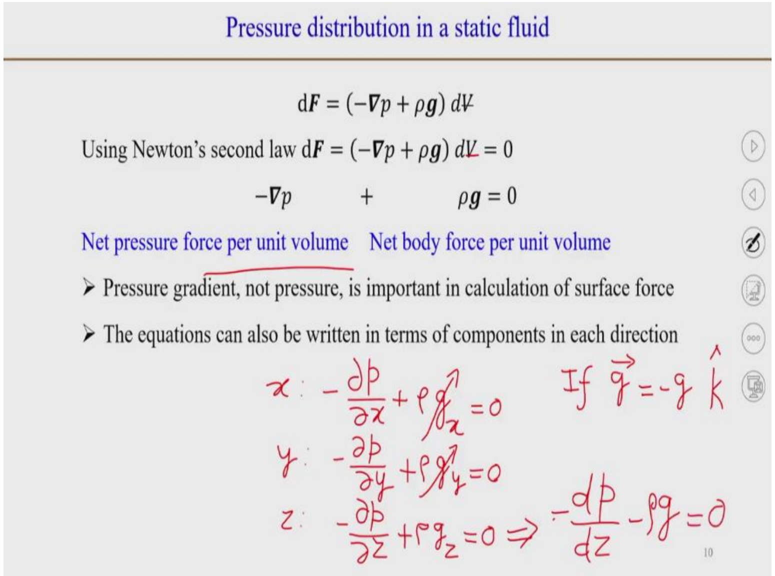

So, now, when we started driving this, we said that we are going to apply Newton's second law. And because there is no motion, so the acceleration is zero. So, we simply have that the force on this, the net force on the fluid element is zero. So, we have - ∂p + ρg within a bracket multiplied by the volume of the fluid element is equal to zero. This is volume. So, we cut by a horizontal line. So, we have - ∂p + ρg = 0, where ∂p is net pressure force per unit volume, because we have divided by the volume. So, gradient of p is pressure force, or the net surface force per unit volume, whereas ρ g is the net body force per unit volume.

So, the surface force and body forces, they will balance each other. So, we can write this equation, this is in the vector form. We can write in different component forms. If we write along the x direction, we will have $- \partial p/\partial x + \rho\, g_x = 0$. Similarly, along the y direction, $- \partial p/\partial y + \rho g_y = 0$. And along the z direction, we will have $- \partial p/\partial z + \rho\, gz = 0$.

Now if we choose our coordinate such that if g acts in the negative z direction, then, we will have this term to be 0 and this term to be 0. So, that means, pressure will be constant along the x direction, pressure will be constant along the y direction and we can write this is equal to this term will give us, because the pressure will depend only on z, so we can write this - dp/dz, we can write this is equal to k. so where g is the value of gravitational acceleration. So, we can have dp by dz = - ρg. If we write in terms of depth, if h is depth, then we can write p = ρgh.

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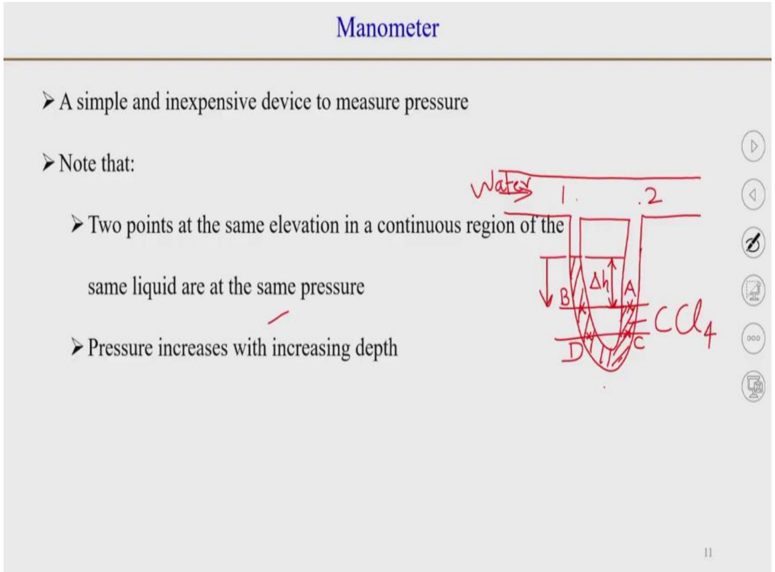

So, now we will look at an example for a manometer. So, manometers, we know that they are simple and inexpensive or cheap devices, which can be used to measure pressure. So, a very simple device, or a very simple manometer is what we call U tube manometers, so which has U shape and let us say there is flow in a pipe, and it can be used to measure pressure difference between two points, 1 and 2. So, the manometer is filled with a fluid. And the difference of the height of fluid in the two arms of the manometer will give us the pressure difference between the two points.

So, when applying a manometer principle, we need to remember that the two points at the same elevation in a continuous region of the same liquid are at the same pressure. So, if we, let us say this a fluid water, and then, manometer can have a different fluid, let us say, this is CCl_4. The choice of fluid will depend on the magnitude of pressure and density of, so depending on what is your estimated pressure difference, or what order of pressure difference you are going to get, accordingly, you will choose a fluid, which is of the density. So, that the high difference can be significant. So, the accuracy of the manometer is good, or the sensitivity of it is good.

So, and coming to this point, that two points at the same elevation in a continuous region of same liquid are at the same pressure. So, that means if we take two points at this region, then point say A and point B, they are in the same fluid, so they will have same pressure. So, P_A and P_B will be same at this point. If we take another line, horizontal line and say point C and point D, they will be at the same pressure. So, we need to remember this. And the other thing is that as we go deep in a fluid, then along the depth the pressure will increase. So, the pressure increases with increasing death in a fluid.

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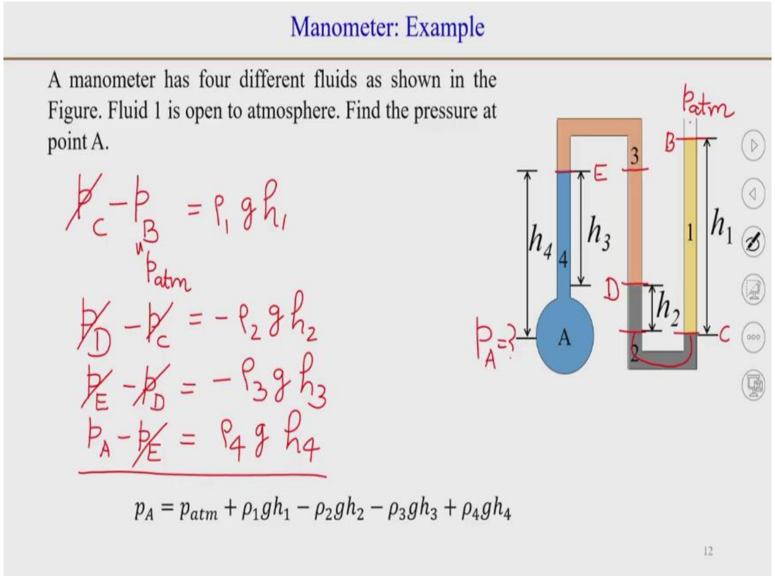

Now we will look at an example. So, if we have a manometer, where it is filled with four different fluids and the fluid 1 is open to the atmosphere. So, the pressure at this point, we can say that atmospheric pressure. So, what we need to find out is pressure at point A or the relationship between or the difference between P_A and $P_{atmosphere}$.

So, we can start with that pressure at point A will be equal to, so if we look at this point let us call this point we will name all the points. So, point B, point C, then point D, E and we already have A. So, we can write that P, we can write P_C - P_B, where P_B is $P_{atmosphere}$. So, that will be equal to $\rho_1 gh_1$. So, PC, where the pressure will be higher than at point B, because it is at a greater depth in the same fluids. So, $P_C - P_B = \rho_1 gh_1$.

Now, we will find out the difference between pressure at point D and point C, which will be equal to, so we know that the pressure at point C here and at the same location in this fluid it is same, because this is a continuous fluid region and at the same height, the pressure will be same. So, we can write that $P_D - P_C = - \rho_2 gh_2$, the - sign is because of the fact that D is at a height when compared with C. Now we will find the difference between point D and point E. So, we can write P_E - P_D is equal to again, the point E is higher than point D. So, we know that the pressure in fluid three at the two elevations will be same, so we can write $P_E - P_D = -\rho_3 g h_3$.

Now, finally, we can write $P_A - P_E$, that will give us A is at a depth, then, E so and the hight difference between the two is h_4. So, the pressure at A will be higher and we can write $P_A - P_E$ is equal of $\rho_4 g h_4$. When we combine all this, then, we will get PA = P atmosphere so this will cancel out, PD will cancel out, PE will cancel out and you will get PA - P atmosphere. If we take atmospheric pressure on the right hand side, so we will get $P_A = P_{atmosphere} + \rho_1 g h_1 - \rho_2 g h_2 - \rho_3 g h_3 + \rho_4 g h_4$.

So, it becomes straightforward. If we remember the two principles, the first one being that the pressure at two points having same elevation in a continuous region of fluid will be same. The second is that the pressure increases in the same fluid, the pressure increases along the depth. So, that is all for this lecture.