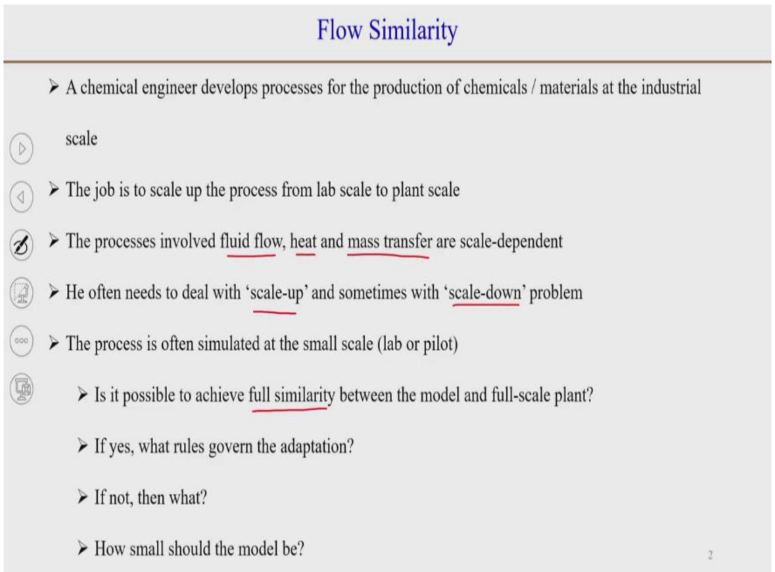
Fundamental Fluid Mechanics for Chemical and Biomedical Engineer
Professor. Dr. Raghvendra Gupta
Department of Chemical Engineering
Indian Institute of Technology, Guwahati
Similitude or Scale-up

Hello. So, in today's class, we are going to talk about Similitude or Scale-up. What we talked about in the last class was dimensional analysis.

(Refer Slide Time: 0:48)



So, as a chemical engineer, we need to develop processes which have been developed, or which have been invented at the lab scale. And the role of chemical engineer primarily is to produce materials or chemicals or biological materials, which have been produced at the lab scale or the process for their products and has been demonstrated at the lab scale. And the goal of a chemical engineer is to scale up such processes. So, scaling up is a general requirement for a chemical engineer.

Now, when we come to that the job of a chemical engineer to take a process from lab scale to the plant scale. So, the processes that are often involved in taking these from lab scale to plant scale are fluid flow, heat and mass transport, which we combinedly call transport processes. Apart from that, there are heterogeneous reactions etcetera that also needed to be scaled up, other unit operations also.

So, as a chemical engineer, we often need to deal with scale-up and sometimes scaling down problem, so the problem of scaling down will come into picture. For example, there is already an existing chemical plant and what you need to investigate some problem in the plant. And that may not always be possible to do in the plant scenario. So, what an engineer does, he develops a scale down model of the plant, and then look at the things.

Whereas scaling up generally refers to before the processes being developed in order to test various hypothesis in order to understand more or know more about the process, one needs to develop first lab scale process and then take it to a pilot scale what you might have often heard the word pilot plant. So, which is the intermediate between lab scale and the full-scale plant. So, that once all the experiments that need to be done, they are generally done first in the lab scale and then at the pilot scale.

So, in all these cases, one need to look at these questions that he need to simulate the process at the small scale. So, he need to answer the questions that is it possible to obtain or to achieve full similarity between the model that he is going to test or on which he is going to test his process and full-scale plant? So, is there full similarity between the two? If there is full similarity, then, what are the rules that govern this adaptation or this change in scales? So, what should be the flow rate? What should be the velocity, etcetera, so that the same conditions or similar conditions in the two can be achieved?

Other question, if they are not, then what should he do? Can he work with partial similarity and if yes, how? So, that often a common question in chemical engineering applications. As we will see in an example later on that it is not always possible to achieve full similarity between the full-scale plant and a model. And then, if this can be done either for full similarity or partial similarity, what should be the size? So, how he should calculate the scale-down size of the model.

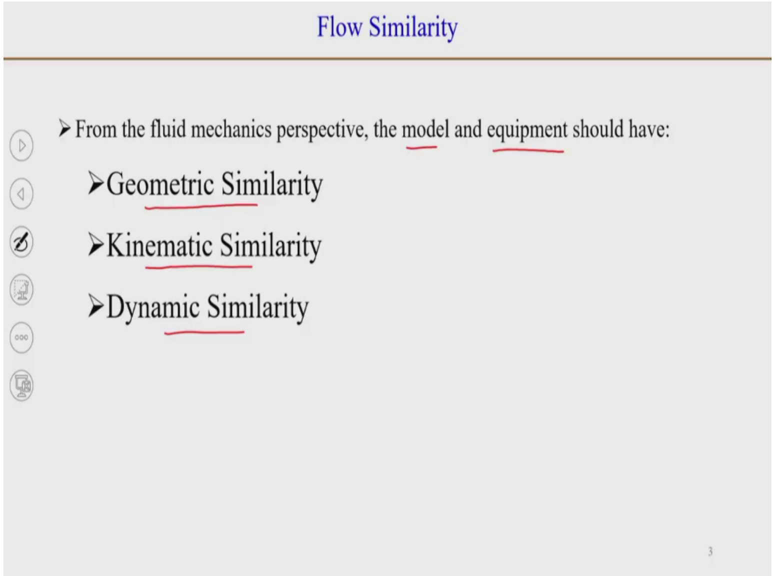
So, all these questions we are going to address today using dimensional analysis approach. Of course, with the advent of or with a lot of progress in computational fluid dynamics in today's era, the one can simulate the process either in a full-scale plant or a small-scale plant and can do one to one comparison between the things.

However, the dimensional analysis remains the most powerful approach as one can do back of the envelope calculations and try to understand quickly the processes involved or how one can scale things up, even for doing CFD simulations, one need to look at what are the scales that we should be working with, because ultimately to gain confidence in a process especially when there is a lot of money involved and a lot of safety involved where high pressure, high temperature processes are there.

So, in such cases, one need to work with experimental conditions or one need to work with a real plant scale, not only simulated ones. Of course, one need to first get a lot of experience by doing simulations using computer simulations or calculations. But before going for a full-scale plant, one need to test it in in a pilot scale. So, all those calculations, dimensional analysis can help us in doing scaling up and that is what we are going to look at today.

So, from a chemical engineering holistic point of view, one need to have similarity in all the processes. So, one need to have not only similarity in the flow, one need to have similarity in heat transfer or thermal similarity, one need to have chemical similarity and so on. However, this course is about fluid dynamics. So, we will be looking at flow similarity.

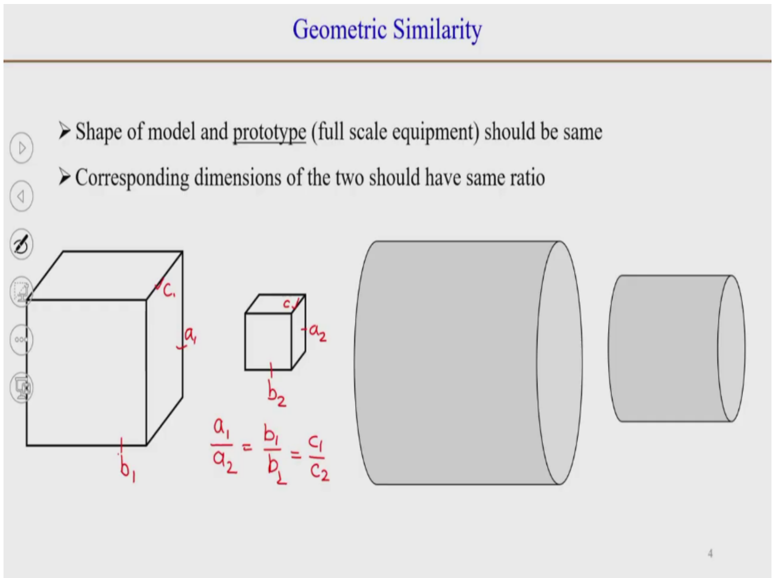
(Refer Slide Time: 7:20)



So, to achieve flow similarity, from the fluid dynamics perspective, the model and the full-scale equipment, which we will also call as prototype they should have a geometric similarity, They should have kinematic similarity, and they should have dynamic similarity. So, as these terms suggest the geometric similarity is that the geometries of the model as well as the equipment, they should be similar. Kinematics similarity suggests that the kinematics in the two, the velocities in the two equipment or in the two models or in model and prototype, they should be the same.

Whereas dynamic similarity suggests that the forces, they should also be similar in the two models. So, the question comes or first thing we have understood that they should have these 3 kind of similarities. And now, we need to see that how we achieve these similarities and how these are related with dimensional analysis.

(Refer Slide Time: 8:37)

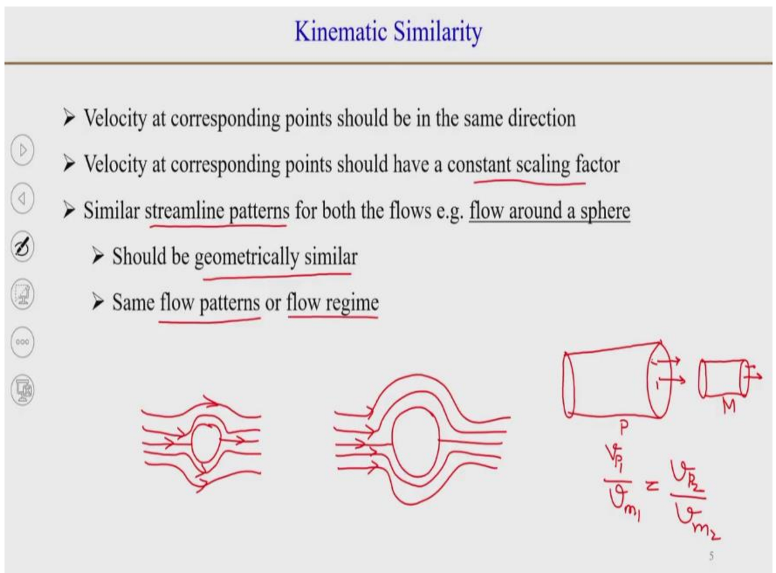


So, let us talk about first geometric similarity. So, in the geometric similarity, as the name suggests, the geometries of model and prototype should be same. So, prototype is generally referred the first equipment or the first vehicle or the first aeroplane that is developed, it is called prototype. So, the full-scale model or the full-scale prototype and model their shapes would be same, as well as their aspect ratio should also be same. That also suggests that the corresponding dimensions of the two models or of the two geometries should be same.

So, here are two few examples. So, what you see here, a cube of geometry. So, the corresponding dimensions they should be same in both the cases. So, this, let us say name this a and a they should be same. Similarly, the third dimension, let us say this is b, they should be same. And the first dimension that we looked at, so let us name it c. So, a_1, b_1, c_1 a_2, b_2, c_2 they should have same ratio.

So, what we should have in this case that a_1/a_2 is equal to b_1/b_2 is equal to c_1/c_2. This also suggests that the ratio a_1: b_1:c_1 is equal to a_2: b_2:c_2 and that is when you will have the same shape. Now, another example here is of a pipe. So, again for pipe, the height and the dimensions or the radius of the pipe should be similar.

(Refer Slide Time: 11:15)



Now, coming to kinematic similarity. So, for kinematic similarity, the kinematics is governed by say velocity or acceleration. So, to achieve kinematic similarity, the velocity at corresponding points should be in the same direction. So, velocity, if you take a bigger model, let us say a bigger pipe and a smaller pipe. So, velocity at the center of the pipe, the corresponding points that should be in the same direction in the two cases. And the velocity at corresponding points should have a constant scaling factor.

So, let us say velocity at center in one, or in prototype and velocity at center in the model that should have a constant ratio. The same ratio should exist, say a point in between in the middle of the wall and the center and corresponding point in this, they should also have the same ratio. So, if I say this point as 1 and point 2, point 1 and point 2, so we should have v_{p1}/v_{m1} is equal to v_{p2}

divided/v_{m2}. So, that means the ratio of velocities at point 1 and point 2 are same, where point 1 is at the middle of the section and point 2 is between the wall, at the middle of the wall and the center.

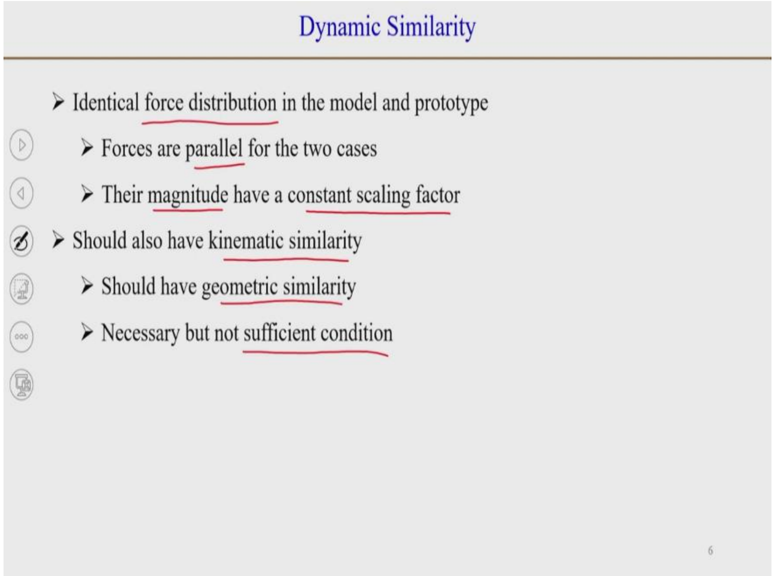
So, when we have the same direction and a constant scaling factor between the two, then, it will result in similar streamline patterns. Only thing is that the streamline pattern, it will be scale-down. So, the streamline patterns for the two flows will be similar. When the streamline pattern is similar, that will also necessitate in terms of streamed tubes, that the geometry for the two cases should also be similar.

So, an example, for example, a flow around a sphere, if we take a sphere and stokes flow around a sphere, a small sphere and a bigger sphere. So, for flow around a sphere, stokes flow which will be attached to the sphere, flow is from left to right. So, the streamline patterns should be same for the two cases. So, that also necessitates that this should be geometrically similar. So, that suggests that a necessary condition for flow to be kinematically similar is that the flow should be geometrically similar. So, we looked at kinematic similarity and geometric similarity.

Now the third come what is called dynamic similarity. So, before we go to dynamic similarity, another point that I would like to emphasize is that the flow patterns or flow regime in the two cases should be same. As we will see later on that for single phase flow, we can have flow regimes in terms of the fluids laminar or fluids turbulent. So, of course, when the streamline patterns are same, then, flow should be either laminar or flow should turbulent for both the cases.

Similarly, when we have a multi-phase flow, for example, a bubble column reactor. So, in a bubble column reactor, depending on the gas flow rate, you can have different flow regimes, a homogeneous flow regime or heterogeneous flow regime. Now, if your prototype and the model, they are going to have different flow regimes, then you will have very different conditions for the reactions. So, that should also be same in two cases. So, that also suggests that the flow patterns and the flow regime in the two cases in the model and prototype, this would be same.

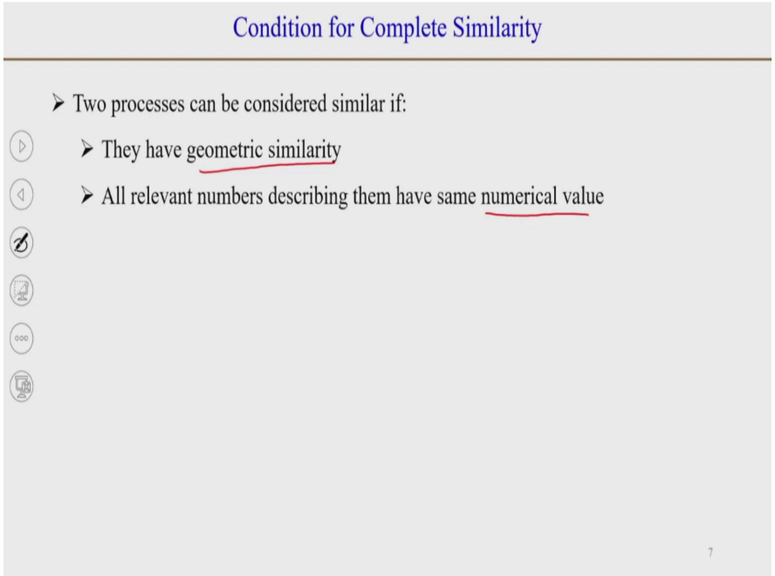
(Refer Slide Time: 17:36)



Now, the third case comes for dynamic similarity. So, dynamic similarity means the force distribution in the model and the prototype that should be identical. So, identical means, their direction should be same. So, the force in the two cases should be parallel, force on the model and force on the prototype, at the corresponding points, they should act in the same direction as well as their magnitude at the corresponding points they should have a constant scaling factor. So, that also required dynamic similarity required that the flow should have kinematic similarity.

And we have already seen that for kinematic similarity, there should be geometric similarity. So, dynamic similarity requires flow to be kinematically similar, flow to be geometrically similar, but that is not sufficient condition. We should also have the forces acting along the same direction at corresponding points, and they should scale by a constant factor. So, this is what we have in terms of, when we can have flow similarity in the model and prototype.

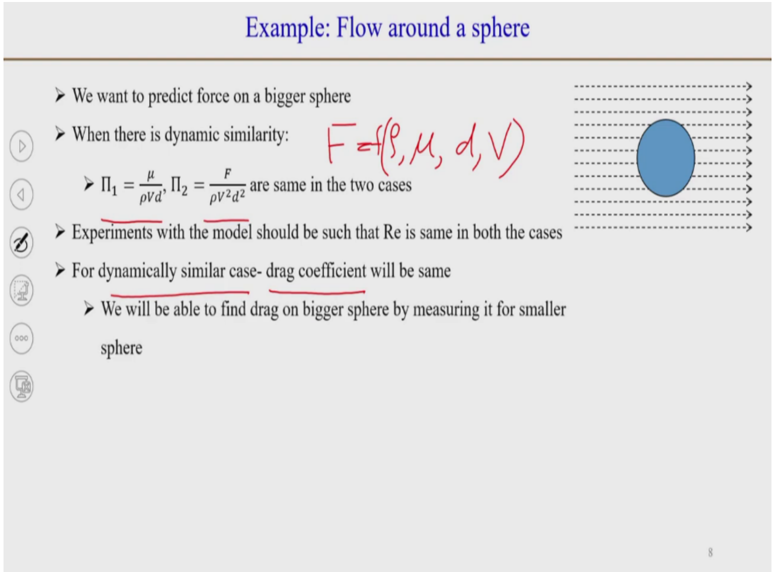
(Refer Slide Time: 19:04)



Now this complete similarity can be achieved if we have a complete geometrical similarity in the two. So, that is already a condition. Now, we can ensue, or we can make sure that there is complete kinematic and dynamic similarity if all the, there are dimensionless numbers or all the dimensionless groups in the model and prototype, their numerical values, they are same. So, all the dimensionless groups, which are relevant for flow if their values are same in the model and prototype, and two are geometrically similar, then we can say that there is complete flow similarity.

Now, this principle is even wider that if all that dimensionless numbers relevant for a process or relevant for a operation if they are same, it may be the dimensionless number may not be relevant to flow, it may be relevant to heat transfer, it may be relevant to mass transfer. If all the non-dimensional numbers are same, then, there is complete similarity. So, when we are talking about flow similarity, all the non-dimensional numbers relevant to flow if they are similar, then there is complete flow similarity between model and prototype.

(Refer Slide Time: 20:50)



So, let us look at an example. The example that we looked at when we were doing dimensional analysis, so we will look at this first, because we already have the dimensionless groups that we obtained there. So, we want to predict if we have a very big sphere, let us say few meters in diameter and we want to do experiments to predict the force on such a bigger sphere. So, we need to see the dynamic similarity.

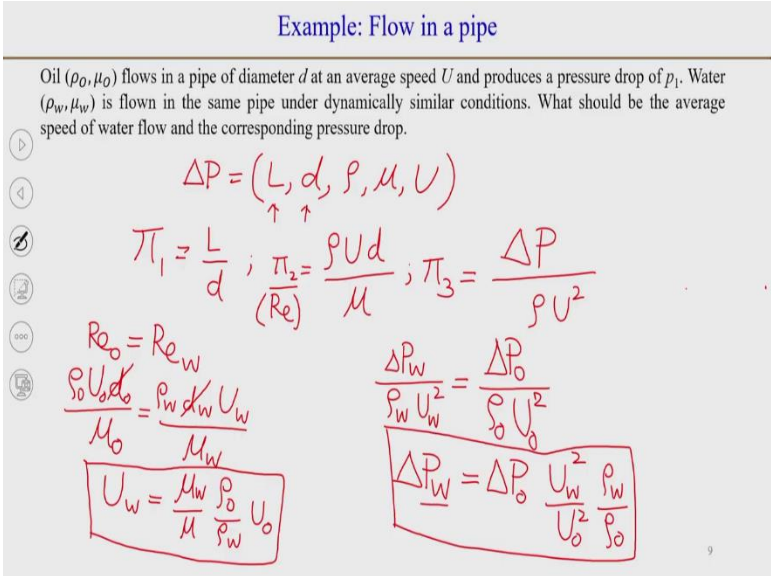
If you remember that the drag force on a sphere depends on ρ. It is a function of ρ, μ, where μ is viscosity, ρ is density, d is diameter of the sphere and V is the relative velocity between the fluid and the sphere. So, combining these dimensional parameters ρ, μ, d, V and F we obtained two dimensionless groups Π_1, which is a Reynolds number and Π_2, which is a drag coefficient.

Now what we need to do? We need to do experiments on a scale down model. So, we should make sure that the two non-dimensional groups are same in these cases. The geometry of course if they are spherical, then the geometry will be same, and it will be scale down by the ratio of their diameters or radii. So, we should make sure that the Reynolds numbers in the two cases are same. And if they are dynamically similar, then their drag coefficients will be same.

So, for example, if we want to achieve the same conditions, then from Reynolds number similarity or by equaling their Reynolds number, we will be able to find out what should be the velocity at which the experiments should be performed to simulate the conditions similar to that the prototype is going to face. Once you obtain the velocity, then, for flow conditions to be dynamically similar, if they are similar, then, you can safely assume that the drag coefficients will be same and you can calculate by obtaining the drag coefficients experimentally on the smaller sphere, you can safely calculate what will be the drag coefficient on the bigger sphere.

Of course, the assumption here is that the experiments are being done using the same fluid that the larger sphere is facing.

(Refer Slide Time: 24:06)



Now, let us come to a different example. Again, very often found in chemical engineering applications or also in our day-to-day applications, which is flow in a pipe. And when we look at flow in a pipe, we are looking at pressure drops. So, the question suggests that oil flows in a pipe of diameter d and the average speed for oil is U. And it produces a pressure drop of p_1. If in the same pipe, water is flown under dynamically similar conditions, then, the question is that what should be the average speed of water flow and the corresponding pressure drop?

So, let us first find out what are the parameters that are important here. So, if we talk about pressure drop, let us say pressure drop Δp of course will be a function of length of the pipe, diameter of pipe, the properties of the fluid. So, density, ρ and viscosity μ and speed or mean velocity of the fluid. So, when we combine these in non-dimensional groups, when we combine these dimensional parameters, we will obtain the following geometrical parameters. I can see by observation that you have L and d two parameters, which have the same dimensions. So, one of the non-dimensional parameters, which will be a geometrical parameter that is L/D.

However, this does not concern us for the problem. When we're talking about the scaling up, because the pipe is same, what we are looking for, what happens or what should be the conditions to achieve dynamic similarity to use a different fluid. Now, another dimensionless group is, just by looking at these variables, I can see ρ, U, μ and d. So, I can combine them as ρ, U, d over μ which is a Reynolds number. If you do your dimensional analysis, you might end up with a variable, which is μ/ρ U d. But both of these groups are dimensionless because I know the definition of Reynolds number. So, I will use this form.

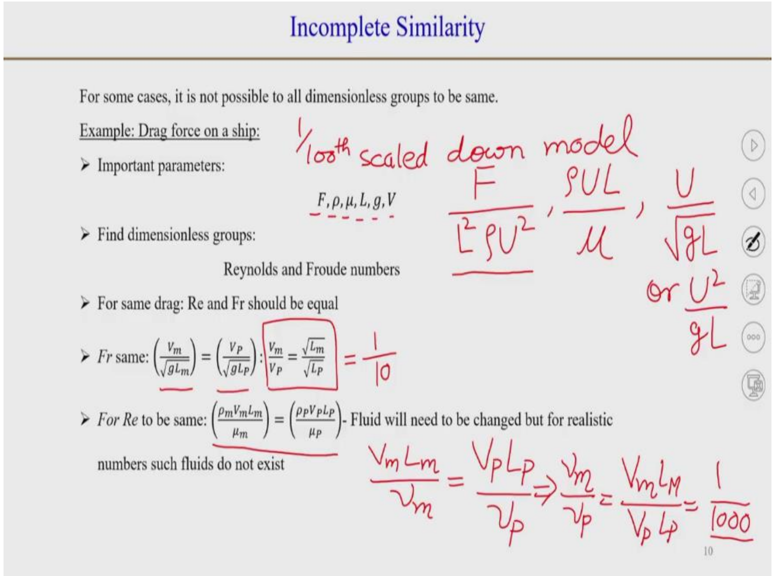
Now, the third dimensionless group I will get is ΔP divided by a length scale. So, ideally, one should have length here. But if you do dimensionless using d, ρ, μ as d, ρ, U as repeating variables, then, what you will get is ΔP divided by d, ρ, U^2 and that will be actually $\Delta p/\rho U^2$, we do not need d here. So, that will be your third dimensionless group.

Now, for dynamic similarity, you should have Reynolds number to be same or Π_2 should be same. So, we can say that this Π_2 is, Π_2 is Reynolds number. So, if Reynolds number in the two cases is same so we can say that the Reynolds number for flow of oil and flow of water, they are same. Then, we can write ρ, μ, d, ρ, U d over μ all for oil is equal to ρ_w, d_w, U_w divided by μ_w. Now, we know that the diameter is same, so d_o and d_w will be equal. And from this, we can find out what is U_w, which is the mean velocity for water flow, that will be Uw will be equal to μ_w/μ_{oil} into ρ_{oil}/ρ_w into U_{oil}. So, that will give you the velocity of water.

Now, finding the corresponding pressure drop, a more relevant or more important parameter is generally pressure drop per unit length. But because we are working with the same pipe here, so if the pressure drop, the pipe length in the two cases is same, so what, the ratio of pressure drop or pressure drop per unit length is going to be same. So, what we can write for the third dimensionless number that $\Delta p_w/\rho_w$, $U_w{}^2$ is equal to $\Delta p_{oil}/\rho_{oil}$, $U_{oil}{}^2$. So, Δp_{water} we need to find out.

So, Δp water will be equal to Δp that is caused by the flow of oil, where the velocity of U_o multiplied by $U_w{}^2/U_o{}^2$ multiplied by ρ_{water}/ρ_{oil}. So, one can substitute U_w here and obtain the values. So, here we can obtain complete similarity and for complete similarity, what we have obtained that for the dynamic similarity, what is the corresponding velocity of water and what is the corresponding pressure drop.

(Refer Slide Time: 32:19)



Now, in some cases or in quite a few cases, it so happens that it is not possible due to the flow conditions or due to the materials that we use, that it is not possible to achieve complete similarity and we will look at one such example. So, this is a very common example for incomplete similarity that we see in the textbooks. And so, this is an example for drag on a ship. So, the important parameters for drag on a ship, where drag is F is drag force F, density of water or seawater, viscosity of water, a characteristic length scale of the ship, gravity, and, velocity of the ship with respect to water.

Now, when we look at these parameters, the group of parameters are same what we used for calculation for updating non-dimensional numbers when we looked at drag on a sphere. The only thing you can say is different that here we write a length scale L, there we might have written, the diameter, because we knew specifically what is the characteristic length scale there. So, combining this, we know that what are the non-dimensional groups that we will get, we will get a Reynolds number. We will get Froude number, this Froude number we might have not seen if the gravity is not an important parameter.

So, of course we will have, there are 6 parameters and 3 repeating variables. So, we will have 3 different non-dimensional groups, which will be F over L^2, ρ U^2, another will be Reynolds number. So, I will write in the form of Reynolds number ρ, U L over μ and the third one will be g over, the third group will be U /$\sqrt{gL}$ or U^2 /gL that will depend on what non-dimensional groups, or what repeating variables that you choose in your analysis.

So, the non-dimensional groups representing or the drag force that will be a function of Reynolds and Froude number. So, in order to achieve the same drag force on the model, we should have same Reynolds and Froude number for the model and prototype. Let us see this. So, if we have same Froude number, then V / $\sqrt{gL_m}$ for the model should be equal to V / $\sqrt{gL}$ for the prototype. Here, m refers first to the model, and P refers to the prototype. So, this, if your both model and prototype are on sea level on the surface of earth, then of course the value of g will be same. So, we will end up with this relationship.

Let us say that the scale down model is 1/10, 1 by hundredth scale down model, which is a reasonable assumption considering the size of the ships. So, if you look at this, then Lm/Lp is going to be 1/100, and for that, you will get a Vm/Vp is^2 root of that, so which is 1 over 10. So, for Reynolds number to be same, you need to have Reynolds number for the model and prototype to be same. Now, for that to happen, if we look at the Reynolds number, we already know the ratio of velocities and length scales and ρ, U and μ are properties of the fluid.

So, if we write down in a different manner in terms of kinematic viscosity, then we can write V_m L_m / ν of model, where new is kinematic viscosity or μ/ρ, that should be equal to V prototype, L prototype divided by ν prototype. So, this will give us ν_m / ν_P is equal to V_mL_m /V_pL_p and we know already these ratios, V_m/V_p 1/10 and L_m/L_p is 1/100. So, this ratio will become 1/100 into 1/10, that will be 1/1000.

So, this suggests that the experiments should be done with such a fluid, which has 1/1000 kinematic viscosity of that of water, because of course, the ship will be sailing in water. So, V_p is fixed, it will be water only. Now, if you want to the experiments for the similar conditions to achieve dynamic similarity, then those experiments should be done with a fluid which has 1 by thousandth kinematic viscosity that of water, but no such fluid exists, probably mercury is the fluid, which has dynamic viscosity more than water. But that ratio is also of the order of 10.

So, you do not have a fluid using which, where you can achieve such a ratio. So, in such a case, you will not be able to achieve a complete similarity. And then, there are different tricks that people use to achieve partial similarity and try to find out the different ratios one ways to say for example, somebody wants to in such case may want to decouple the forces in terms of form drag and the pressure drag, and try to find one by dimensional analysis and so on. But that we are not going to discuss in this lecture. So, that is all. Thank you.