

Course Name: INTELLIGENT FEEDBACK AND CONTROL

Professor Name: Leena Vachhani

**Department Name: Multi-disciplinary primarily for Mechanical, Electrical,
Aerospace and Chemical engineering streams**

Institute Name: Indian Institute of Technology Bombay

Week - 01

Lecture - 05

This is on PID tuning methods. I am going to introduce you to the different tuning methods here and look into what could be the best applied on a particular scenario. So this particular video is going to talk about very popular method called Ziegler Nichols tuning method. And of course, we will cover certain other variations of it and will help you understand how this particular tuning rules have been have evolved, and when to apply this. OK, let us see what the Ziegler-Nichols method is and what this particular PID design that we are talking about.

In this tuning method, when we talk about tuning, our objective is to find PID parameters. What are those PID parameters? These PID parameters are proportional gain, integral gain and a derivative gain. Why we look into designing the PID? That also is clear to us.

We would like to obtain the controller with restricted complexity. We would like to consider any system. Any system, we would like to first simplify the process model and then design a control for a complex model and approximate it with a PID control. So this way, if we are simplifying it in the way we saw it represented in a FOTD model, SOTD model, or whichever is suiting a particular system response, then we are fitting that into a PID controller method in order to satisfy certain control objectives. This way what we are attempting is simplifying the process of control design.

Why is this needed? Because finally, we would like this particular controller to be tuned by an unskilled person. That's the reason these tuning methods are important. And of course, now, since we are in an era of digitization and other things, so we should be able

to apply certain algorithms to autotune. Now, this autotune needs certain rules to be followed.

So, these tuning methods are coming handy for us. And that's where we will learn about the different tuning rules, which are being used since long, since decades and since the control concepts have brought into the industrial usage. So the very, very old method is Ziegler-Nichols. Ziegler-Nichols suggested this method in 1942. But the important is to understand that this particular Ziegler-Nichols method, tuning method, how did they evolve?

And after that, once they have evolved, what is the significance in terms of theory that has come up? So what they had done is, though it is designed in 1942, but still it is being used for auto tuning purposes for at least starting point for any machine learning method for tuning the PID parameters. So when we see here how this particular method got evolved? How these methods are evolved, this is based on the determination of some features of process dynamics. There, they did the control parameters, what the tuning rule comes up is then expressed in terms of the features by simple formulas.

And since these are simple formulas, unskilled person can tune it very easily. Now, these are used for typically for getting the good response for load disturbances, which is one of the control objectives that we have been stating it. So Ziegler-Nichols did a lot of simulations, but with manual assessment of these results, they attempted it and looked into, okay, this particular tuning is giving the best load disturbance and whatnot. The design criteria that they followed is quarter amplitude decay ratio. And what is that quarter amplitude decay ratio?

For example, I have a step response of a second order system. This is what I'm drawing it here. And we get an oscillatory response such that there is a first peak and a second peak comes in. So this particular decay response between the first and the second peaks is about quarter decay. And this is where the advantage comes up is that, okay, I'm able to reach to the particular response very quickly and it settles down also very quickly.

So the peak is kind of quarterly. There's a quarter decay in the amplitude ratio here. All right, later on there are many, many other variations of the ZN method or Ziegler-Nichols

method was suggested that we'll see some of them. So ZN method has two parts. One is this first method which is based on the step response.

So one is based on the time response and the other is on the frequency response. This slide covers the time response. Now, for example, your process is an integrator type. Now, if the process is of integrator type, then what you get here is a kind of a response that is plotted on the right hand side. So the response has a delay.

We can see that there is a delay and then it's an integrator type. Now, this delay is characterized. So, if you have a step input applied and then you get this kind of a response, you can characterize this in terms of the parameters A and L . Now, L is what is characterizing the delay and A is the intercept with the y-axis. Now, if this A and L is available, so this particular parameter a and l are available.

Then this table gives you the tuning rule. If it's only a proportional type controller to be designed, then the proportional gain is $1/a$. But if it is PI, which is proportional and integral gain, then your proportional gain is $0.9/a$ and integral time constant is $l/0.3$. Similarly, if it is proportional integral as well as derivative, then you need to get the proportional gain, integral time constant and the derivative time constant. Fair enough.

So, this is very straightforward. No issues. We know that these are designed keeping in mind that these are designed based on the quarter amplitude decay. All right. Fine.

So, now let us see what comes up. In the next is if it's not an integral type and if it has a S shape, there's an S shape of the curve which means this is a first order kind of a system where the output response gets saturated to a steady state value. So then this particular table needs to be modified by considering L by T where T is the time constant of the first order system. All right. Now, the second method of ZN is your frequency response method.

Now, this frequency response method is based on the Nyquist curve. Now, what we will do is we have a process transfer function P of S , we plot the Nyquist curve of P of S . Now wherever this Nyquist curve intersects the negative real axis we know that this is called the ultimate point. We use this ultimate point to characterize the parameters k 180

and ω_{180} because this is crossing the negative x axis. With the help of K_{180} and ω_{180} , we find the ultimate gain K_u and ultimate period T_u , which is given by these equations.

All right. Now comes the problem. We are designing a control system, controller, and designing the controller for it, which is C of S . Let's say the transfer function of the controller is C of S . So we should be getting the K_u .

We need K_u and T_u which are the ultimate gain and ultimate period for P of S , C of S which is the open loop gain, open loop transfer function. But C of S is not yet designed. So what do we do? That's where ZN method comes into play by designing, saying that, okay, we have P of S only available. Then there is an iterative method that we can follow.

And this is the basis for finding the correct K_u and T_u . So what ZN method suggests is that you connect the controller to the process. Now your controller is only proportional controller. So you have a process. You have connected your proportional controller.

This proportional controller, you keep tuning it. So you keep increasing the gain of it. So as soon as you have your controller having the integral time constant as infinity and derivative time constant as zero, because your control action is only purely proportional action. Now, you increase the gain slowly. That's what is mentioned in the second bullet point here.

You increase the gain slowly until the process starts to oscillate. As soon as the process starts to oscillate, means you are at an ultimate point in the Nyquist plot. Now, this gain occurs when, so this point is the one where you will get the gain K_u and the period of the oscillation now becomes T_u . So once we have this particular K_u and T_u available, we will use the tuning rule separately, but let's understand what is happening here. When we have the, let's consider a particular Nyquist plot, some typical Nyquist plot which is crossing the x-axis only at this particular point, and this is what we call it as ultimate point.

Now this is what is the Nyquist plot for P of S , and now your controller is P , I and D kind or any combination of it. So what happens with P , I and P , I , so if you are only applying

the proportional control, then what happens? Your gain point moves radially outwards. So this particular curve is now going to go to move towards radially outward side whereas if you will apply only integral control then this particular point will move towards the I axis which is shown here which is orthogonal to the P axis which is nothing but the radially outwards at that particular point. We say the radially outward means this is what is the origin and the radially outwards is the direction in which the P point is this particular axis which you get it by joining the point on the Nyquist plot and the origin.

And orthogonal to this, these directions are I and D directions. Now, if you are applying only PI control, then your point on this will move in this particular quadrant, which is formed by the P and I axis, okay? Now, if you are having only P and D, then it will move towards this quadrant, which is formed by P and D axis, all right? Whereas now, if you are applying your PID then your point will move in terms of this half plane along this line. I mean you have the half plane given by I and D axis, and your point moves towards the radially outside or anywhere in between this half plane.

So with the help of proportional control, I control and D control and the combination of them, you can see that the point on the Nyquist curve can then move towards any half plane side. All right. Now comes the tuning rule. Now, we have found the K_u and T_u points by simply adjusting the proportional gain and you had the process block with you. With only P of S, applied the proportional gain, increased the proportional gain till we got the ultimate point.

That's what was your K_u . Now with this K_u , you set the proportional gain to half of this K_u . So now you are operating point, your gain, proportional gain is half of the ultimate gain point. Similarly for the tuning the PI controller, you consider the K_u as $0.4 K_u$ and $0.8 T_u$, whereas PID, there is a different rule that has been given. So, this method turns out to be very effective method that you start with a particular P of S and then you look forward for designing your proportional integral and derivative control starting from getting the ultimate gain point.

All right. Now, the criteria here turns out to be that if I follow this particular tuning rule, for example, we follow PI controller and we get this particular, we use these gains,

proportional gain as $0.4 K_u$ and T_i as the $0.8 T_u$. Then what happens is if we look at the controller transfer function here, this transfer function turns out to be K_u times 0.4 minus $0.08 T_u$. Okay, it's actually written in terms of the polar coordinates. Now, this particular controller introduces 11.2 degrees of lag at the ultimate frequency.

And this we should keep in mind that this ZN tuning method is introducing approximately 11.2 degrees of phase lag, if you follow the frequency tuning method. All right. So this is where we have the question that we can always look forward to answer. We follow the same method and we can say that, OK. We follow, we can design the PI controller with a given phase match.

So, we will have to go backward in this direction. All right. Backward, if we say we need a phase lag of only 5 degrees instead of 11.2 degrees because it is disturbing the stability, stability margins a lot and so on and so forth. Stability margin is not at all available. This much stability margin is also not available.

So, then we can always go back and do the P, PI tuning or PID tuning based on this particular methodology. Okay, now if we summarize this ZN frequency response method, we see that it is an empirical tuning procedure. Experimentally very well, very easily you will be able to get the ultimate point. At the same time, how did we get this particular ultimate gain? We simply do the P controller, the gain of the P controller.

We first apply the P controller and then we increase the gain until we get the sustained oscillations. And getting those sustained oscillations is also a wonderful idea because we don't get into getting any kinds of errors there. So we get this particular oscillation. With the help of these oscillations, we can get the ω_u as well, which is the ultimate frequency here. And then we'll reduce gain by half.

So the errors are not at all introduced in this particular experiment method, and which is the strongest point of the ZN frequency method. Because for a particular integral type of process or a first order type of a process, we will always get some of the other kind of a curve because your particular system may not be exactly like a first order system. It's like a system, like a first order system or like an integrator kind of a system but always there is going to be some nonlinearity associated. So every time when you do the step response

method, these particular values of A and L that we calculated will be erroneous somewhere. But the frequency response method is very powerful because first of all, you are getting everything in a very obvious manner.

You are getting the proportional gain and you are simply applying only the half of that particular proportion. So the chances of errors introduced in this particular tuning is very low and that's why it turns out to be a very powerful method. Now let's look into the step response method versus frequency response method. What is the difference? And once we know the difference, we will be mindful of that.

So let's look into this particular kind of a process transfer function, which is the integrator with a lag, which we say that it is described by two parameters, A and L . Now as soon as we have we are we are kind of considering that this is an integrating kind of a process we will say that okay ultimate frequency and the ultimate time period and the ultimate gain is turning out to be this. If we know the parameters we will be able to get these ultimate point parameters very easily. Experimentally, we know that if I apply this method, these particular values, ω_U , T_U , K_U , and apply it on the frequency response tuning, then we get this particular gain and this particular integral control. Whereas the step response was anyway in terms of A and L values.

So if we compare between these values, what is the gain that you got with step response and what is the gain we require for the frequency response for this kind of an integral type with lag kind of a process, then what we have here is that we can see that you got the integral time constant as $3L$ whereas in the frequency response method you got integral time constant as $3.2L$. So there is a 10% difference between the two methods. Fairly okay. In practice, we should not be very much equal kind of things. Almost equal is just that what we look at.

That's why things are working. That's the reason PID control, you don't need to tune it to very precise values and so on and so forth. In certain cases, step response method will work. In certain cases, frequency response method will work. But at the same time, we should know, we should be mindful of it that when we are designing with respect to the

step response method, the proportional gain is approximately 40% more than that of the frequency response method.

So what are the implications that could arise from it? Since the proportional gain is higher, the stability margins are compromised. All right? Fine. Now let's look into the PID control.

The equivalent PID control tuning rules for the step response as well as the frequency response. What we have is similar values. We can see that integral time constant and the derivative time constants are turning out to be same if you follow step response or the frequency response. Whereas the differences between only the proportional gains And these are approximately 25% higher for the step response.

Fine. So we are again noting down that the step response method, if we will follow for ZN tuning, the proportional gain is slightly higher than that of the frequency response for the same process. All right. OK. Now I'm taking a pause and we will move to to the new method.

Thank you.