

Course Name: INTELLIGENT FEEDBACK AND CONTROL

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Week - 01

Lecture - 04

Hi, this video is about disturbance models and PID. Some features of PID characteristics and how this is helping in rejecting the disturbance is what we will look into this particular lecture. And it is important to understand these disturbance models and how PID is helping us in rejecting disturbance in order to apply in the practical systems. Once we know that this is a possibility of this kind of a disturbance, then accordingly the control objectives can be set. If we look into typical nature of the disturbances, one is the disturbance that is appearing at the input side and this is given by D .

The other is the noise which gets added at the output of the plant. So typically if we represent it in this way, the analysis is based on this particular representation, mainly because the noise is prevalent at the output when you are measuring through the sensors. Similarly, disturbance is prevalent at the input side because that's where your control inputs you are applying. But in general, disturbance can be appearing at any other place in the system block and this particular signal flow block diagram and so on. But we can always understand and its analysis when it is prevalent at that particular place and look forward for these summers being placed anywhere else and so on.

So the idea here is to see where in actual plants these disturbances or noises are prevalent, and see what could do better in order to design the controllers for rejecting the same. If we see where are the changes happening into the system. We happen to change this particular signal set point. So this particular set point change is something which is desirable. And it is typically needed in case of many applications.

So this is desirable factor. Fine. That is something the set point change needs to be reflected at the output is what we would definitely want to. The load disturbance is something that is appearing, as we said, at the input of the plant. This is typically a low frequency signal.

And the measurement noise, which is typically appearing when we are taking the measurements, is a high frequency signal. And that's the reason for the nomenclature disturbance and noise. Just to distinguish that this particular signal is a low frequency signal and we characterize it as a disturbance, measurement noise because it's a high frequency signal. And these load disturbance and measurement noises need to be rejected as compared to the set point changes that those are desirable factors, desirable changes to appear. So typically the characteristics of disturbance, if you look at, we can say that sometimes this can be set as impulse or it can have some kind of a step, changing steps and so on.

And it could be a ramp or some kind of a sinusoid changing the frequency at a slow rate, phase, frequency or amplitude at very low rates and so on. Whereas if I look at similarly this particular look at the signal how the noise appears to be with respect to time, it is changing very frequently. You can see the difference between the disturbance which is appearing now and then at a very low rate as compared to the changes in the signal when it is a noisy signal. So this is typically high frequency signal and represented as a spectral energy and represented as a random variable. When we look at the PID control, the PID control is given by the terms called proportional control, integral control, and the derivative control.

And yes, the gain term is something what turns out to be common, and then we can represent in terms of the integral time constant and the derivative time constant. Here, we will try to understand this particular PID structure in a slightly different way. Now, let us consider the proportional action. Now, what happens in case of the proportional action is that if it is just the K times E of t , then error is never able to be 0 because if error is 0, then U of t is 0. So in order to make this particular input non-zero when the error needs to be zero, our desired part is the error should be zero because this is where the output signal is equal to the set point value.

This is exact following of what the input is given and the set point is given and exactly what I'm getting as an output. Now, if the error is to be made to 0, then this is not possible with the help of the proportional control. And that is the reason let us consider some bias to the input value given such that when there is E of t equal to 0, there is some constant input being given to the platform. Fine. So this particular bias is nothing but we can consider a nominal value, which is between some maximum input value, maximum and the minimum input values and average of the same.

So this particular E of t becomes zero at a set point. All right. Let's see the effect of this particular bias term. So we already know that I have the particular plant output X and I can write the signal flow, signal values and so on and so forth. So X turns out to be K_P times U plus D because this is my plant gain is what we have.

That is in terms of the steady state gain. Output Y is X plus N , whereas X is equal to, we already have said K_P is equal to U plus D , and the controller output U is given by K times Y_{SP} minus Y , where K is my proportional gain. K_P is my plant gain in this, and K is my proportional gain. And what we are applying is a proportional gain with a bias term U_B . when we substitute and resubstitute and bring the X in terms of the output Y_{SP} , we get a neater form given by this particular equation.

Now, it is interesting to see, since this is an LTI system, this particular form is easier to understand that the change in the input Y_{SP} and the change in the noise input are opposite to each other because of the sign change and because the again term turns out to be same. Whereas the bias term that we have added as the control output term has the same effect, bias term which we have added as the term in the control input form has the same effect as the disturbance term, okay? With this particular observation, we see that what should be the selection for this loop gain k times k_P . This loop gain turns out to be giving a trade-off.

And if the noise is zero, the bias is zero, then it is desirable to choose a higher K times K_P , so that this term turns out to be almost unity and X follows your Y_{SP} . And at the same time, higher loop gain means your disturbance term is getting rejected. U_B is influenced same as the disturbance. And if n is not equal to zero, then we will not be able

to use a very large loop gain because then the noise will also be added into the output of the plant. And therefore, there is a tradeoff between the higher selection of the loop gain.

Higher loop gain is desirable for reflecting this change in the set point, whereas if there is a noise in the system, then higher loop gain is not desirable. And also we know that system becomes normally unstable at high loop gain. So there's only a cap over there in order to select a particular loop gain. So design of the controller, or the design of the loop gain in this case, or a bias term in this case, will be dependent on which objective is more important. Maximum loop gain is, of course, determined by the process dynamics, which is coming up from the stability of the system and so on.

All right. It is possible that PID is not able to achieve all the control objectives, but what is more priority is what we have to look at. All right. Let's look into a few more observations from the proportional control here. When we have, let's say, the system is very nice, it is disturbance and the noise and the bias term is equal to zero, then what happens if I'm having varying values of K , the step response is particularly plotted in this particular figure for a transfer function with three poles at minus one, multiple poles at minus one.

So we can see that when I am increasing k value, this k equal to 1, we had a very large steady state error. But as we increase k from 1 to 2 to 5, the steady state error is decreasing. And this is where even the error plot is given. But as we increase k , there is an introduction of oscillations. And with the increase and further increase in the oscillations, the transients die out at a very later date, very later time, and the system response becomes slower in this case.

Now, if this particular steady state error is given by $YSP - Y$, which turns out to be $YSP / (1 + K B)$, so increasing the value of K is definitely decreasing the error. But now what happens if my bias term UB is not equal to 0? Then this steady state error is given by $YSP - K UB$. Now, for a zero steady state error, we will have to consider giving UB equal to YSP / K . Let us see what comes up with the integral action in this.

Now, integral action, the zero steady state means the process variable is equal to the set point. Y process variable or process output variable is Y is exactly equal to the YSP. So how is this getting achieved? Let's say there is a constant control input U_0 at a steady state given. And let's say this is corresponding to a constant error E_0 .

Let's assume that. We are assuming in this way. Now with PI control, what happens? PI control is given by gain term, proportional term like this, and with an integral time constant and an integral value. At a steady state, U_0 , which is the control input U_0 which is constant as we assumed here, is given by proportional gain times E_0 plus E_0 by T_I times T .

Now, this term is growing with time, which means U_0 is not a constant. Our assumption was that there is a constant error E_0 and for which we are giving a constant error, constant control input U_0 . So, this particular assumption is contradicting because now U_0 is increasing with time. It means for an integral control, proportional and integral control, we cannot have a constant error E_0 if a constant U_0 is given. We have already reached a steady state means the constant control is given as U_0 .

It means E_0 is definitely has to be zero in order to reach to this particular reach to the constant control U_0 value. All right. OK, let me go back to the previous slide. The idea about this particular bias term added to the gain term is giving us more and more insights how the integral action is taking place. So something like this bias term idea is very old and it has been applied in very nice industrial control methods.

If I implement the PI controller with this particular way, you have a proportional term coming out here and the integral control term is implemented like a low-pass filter in a feedback loop. Of course, you can derive this particular signal flow. You will get that. You will get a PI structure itself. Now, the advantage here is that this particular integral term is appearing as a bias term. And this particular bias term is being driven by the output U instead of a constant bias.

And this particular implementation is that's why I call automatic reset in in many different industrial controllers that are available in the market. At the same time, digitally also it is easier to implement because this term is representing your low-pass filter. So

any higher frequencies which are captured through the sensor measurements, this low-pass filter is rejecting them automatically and it's getting added as an integrator. So this particular structure of implementation is that's why is very, very popular for implementing the PI term instead of simply a cascade form of proportional and integral terms. Let us all further look into the properties of this particular integral action.

If I decrease the integral time constants from infinity to values equal to 5, equal to 2, and then 1, what we observe here is that for t_i equal to infinity, this is simply a proportional action, and that is why we have a steady state there. For larger values of T_i , the output response, output signal grows gradually and reaches to the final state in a very slow manner. But for smaller T_i values, we can see that this particular one reaches the zero steady state error or the final value very fast. After a certain time, of course, if we further decrease the integral time constant, then it oscillates and then approaches this. Output reaches set point almost exponentially with time constant T_i by K times K_P .

Let's understand what derivative action is helping us. Our objective with the derivative action turns out to be improving the closed loop stability. And how? We have also looked into that derivative action turns out to be giving you the predictions. So it is giving you the faster actions.

When we, let's see how it is helping in the stability form. It's because of the process dynamics. We take some time for the changes in the control to appear in the process output. And therefore it is late in correcting for errors, but with the PD controller, we are having some predicted process output and it is helping us in reaching to the final value faster. Predictions are definitely made by extrapolating error by tangent to the error curves, all right?

Let's see what is happening here in terms of the same transfer function with k equal to three and t_i equal to two. The derivative action in the PID control, how is it helping us in reaching the final value faster? If we see this particular plot, we have the time derivative, the derivative time constant T_D equal to 0.1 selected, then 0.7 and then 4.5. For the T_D equal to 0.1, we were able to reach the final value with oscillations in certain time, but with introduction of a little higher T_D value to 0.7, we are able to reach the system output

final value faster. But beyond a certain point, the derivative action is giving some more oscillations and some different kind of characteristics.

So there is an upper value of TD for which the system is going to behave for a particular proportional and integral constant. So there's a further reduction in the settling time possible with the introduction of derivative action. So this is exactly what I just said. Increasing TD initially gives the increased damping, but later damping decreases and then even the system response is no longer a sinusoid, but rather it is more oscillatory even while approaching the final value. When we implement the derivative action, this is one way of implementing it and in terms of the feed forward way and in terms of adding the bias to the proportional controller.

So this is what is the PD block showing up. The forward channel, this one, is simply a proportional control. And this part, which is implemented again with the low-pass filter, is giving you the final form in the form of a PD controller. So, this transfer function turns out to be equal to ST by $1 + ST$ which already has a low pass filter kind of a system already available. So, the derivative actions are notorious to pass the noises which are high frequency noises and that is going to give us a lot of trouble and that is the reason derivative action is typically used with the low pass filter.

And this is a kind of implementation is very, very, very popular again in the industry. Further alternative representations for the PID control implementations are discussed now. What we have is in the form of controller given by the proportional gain, integral gain, and the derivative gain. Standard or the non-integrated interacting form of the algorithm implementation is very straightforward, which is given by simply the summation of proportional, integral and derivative term and which gives you the control input to be given to the system or the controller output U in terms of the error E . Whereas the series or the interacting form is also common or is also called the classical form of it, where we have this particular derivative term being implemented as a feedforward term along with that PI term as a bias term.

And here what we get is a multiplicative form because here is a cascade of PD and the PI form. It turns out the controller can be represented, is given by the gain term multiplied

by proportional and integral term and proportional and derivative term. All right, so your problem is to now represent K , TI , and TD in terms of K prime, TI prime, and TD prime. What are these? K , TI , and TD are the standard or the interacting form of this.

One can always convert it in terms of K prime, TI prime, and TD prime. What is this exercise is helping us in understanding is that when we write this, again, opposite can also, the vice versa can also be written and we can find the condition between Ti and Td now. Now, with this we should be able to say that okay when we have the transfer function of non-interacting form we will be able to find the condition when we it has complex zeros and the complex zeros are useful for controlling the systems with oscillatory modes. So the systems which are the secondary or second order systems and has oscillatory modes prevalent, then we should be able to find its solution in terms of the non-interacting form. And this particular exercise will help you in understanding that when we do this, then complex zeros automatically appear and the complex zeros with the system poles, complex poles, we can have some kind of cancellations happening and we will be able to do the PID control of oscillatory systems.

The next exercise will help you to say that, okay, series form has interpretations and frequency domain as well. When we find its zeros and we will be able to show that they are real valued. Okay, then if I consider this particular series form and represent its parallel form, is it introducing complex videos? Do this exercise. When you write this K, TI, TID in terms of K prime, TI prime, TD prime and vice versa, you will be able to interpret these results which are given in the last two bullet points.

If we look into the interacting versus non-interacting form, each control loop has its own structure when we have the interacting form. So when both I and D terms are present, then these structures are different. And both forms can be implemented in terms of proportional, proportional integral, and PD control as well. But one has to look into whether I'm implementing an interacting and non-interacting form typically to address these oscillatory modes and so on and so forth. That's all for this.

Next we will look into, next video is talking about the tuning methods for the PID control. Thank you. Thank you.