Course Name: INTELLIGENT FEEDBACK AND CONTROL

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Week - 01

Lecture - 03

Hi, this video is about model reduction. We have seen that the system can be represented as a static and a dynamic model. But at the same time, there is a possibility that the system is a higher order system. In this video, we will look into what is one particular way to approximate the system to a simple systems with lesser order, but at the same time without compromising on the performance in the operating range of frequencies. So the step here is, of course, the objective here is to simplify a complicated model.

We should be able to tune PID controllers with the simplified models that we will get after approximation. For the systems that we are looking at to control with PID controllers, these transfer functions are either represented as FOTD model, which is first order time delay systems or second order time delay systems is what we are aiming to achieve after the approximation. So for approximating this, what we have to look into is defining the frequency range of interest. And once that is done, we will look into identifying what is the highest frequency omega star which will be used when we are operating this particular system. Identifying the highest frequency omega star for which the system is working is not difficult because we will be knowing that these are the input variations that we are going to give it for the system.

And this is what my highest frequency omega star is going to be helpful in order to understand how the approximations are done. For PI controllers, typically we use this 145 degree phase difference for considering it as in high frequency range as well. So from the control part of it also, we can consider what should be my omega star. For example, if I want to control the system with PI control, then omega star can be selected as omega 145, regardless of what going to be the actual frequencies of operations. So in order to approximate the system or reduce the system to a simplified system with lesser order, we can bifurcate the actual system in terms of lower order frequency components and the higher frequency components.

The pole zeros and time delays which are less or more than omega star are combined in G_l(s), whereas higher than them are combined in terms of G_h(s) when we are approximating this. So, what we will do here is that omega star will have relevance to where we are breaking this in terms of lower frequency and higher frequency. And this omega star part will take the role in identifying this 1/(1+sTs) Below this is my G_l(s) which we will retain from the actual model. Whereas, we will do the approximation for the higher frequency terms.

Let's see how we do it. So, this higher frequency factor is now my T_(arh), which is my overall effective average residence time. So, this particular T_(ar), for example, this is my transfer function. And this transfer function is having two zeros and three poles along with a time delay L. So effective residence time we have already seen for the cascaded system is nothing but the summation of all the time constants corresponding to poles plus the time delay, and then subtraction of all the time constants corresponding to zeros.

So Skogestad's half rule says that once you have identified this Ts of t, What we can do here in order to get this approximated transfer function, the rule can be, if it's a first order time delay system approximation, we can do it in this way by applying this kind of a formula. We'll look into the illustration of how to apply this particular rule shortly. Similarly, if we want to represent the system as second order time delay system, then this comes out to be the formula. So, when we are approximating this higher order terms, in that case, the model error is characterized by (T_(arh) + T_s)/2 because this is what your time delay system turns out to be.

And correspondingly at omega star which is the highest frequency of operation, this gives you the value equal to 0.1.. So neglected dynamics means what we have done? So, for example, this was the higher order term that we approximated it to this way, the first way, which is first order time delay system. then it is introducing the the phase lag of six degrees extra. That is something is fair enough if you are applying if you have enough phase margins available so then then in that case, the approximated system which has neglected dynamics introduced, neglected dynamics corresponding to this phase lag of six degrees is absolutely okay and one can actually neglect this dynamics and work with the approximated system and design the PID control for the same. Let's see now an illustration of how we are applying this skogasted's half rule.

Let's understand this with the help of this fourth order system, which has the time constants 1, 0.1, 0.1, 0.01 and 0.001 seconds. So for this system, omega 90 turns out to be 3 and omega 180 turns out to be 31.6. Let's say we have the highest frequency of operation selected as omega star equal to 3. So now equal to 3 means this is what, so s is equal to 1, we will have to retain. Because that is the time constant corresponding to something which is greater than 1 by 3.

So, this is a slower system. This is a lower frequency term that we will retain in G_l(s), and we will break at time corresponding to 0.1 second, which is just smaller than the frequency of operation for 3. So with this selection we consider Ts is equal to 0.1 so we are going to break down the system in terms of lower frequencies and higher frequencies corresponding and this breakdown happens at this particular 0.1 second. So initially what we did here is that we arranged this particular transfer function in the decreasing order of the time constants corresponding to the poles. Now, as we said, this particular lower term is corresponding to this higher time constant term.

So, my G_l(s) is Kp/(1+s) which we will retain. Our effort is in approximating this particular higher term, which is corresponding to time constant 0.01 and 0.001 seconds. So now the average residence time corresponding to the higher frequencies turns out to be the sum of these two time constants which is 0.011 There comes the approximate model now, which is given by Kp/(1+s(T+Ts/2)). So this was corresponding to the Skagasted's half rule for the first order time delay system, which is copied in this particular slide now.

Right. So in this particular slide. Here, you can see that what has been applied here in this formula is Ts is equal to 0.1, because that's where we have made the breakup, and time T is the corresponding lower order term, which is equal to one, and T_(arh) is what we found out, there is 0.011. And just a simple substitution of it gives you the approximate system given by Kp/(1+s(T+Ts/2), And this is my time delay. You can see that what we have done here is we have clubbed up all these higher order frequency time constants in this particular time constant itself or in the lag term.

OK, so what we have got now is when we apply this and see what comes out for omega star less than three point three, this omega 'T' terms is less than point two. OK, so this is introducing certain phase lag, which is corresponding to six degrees is what we what the claim is from this particular approach. Similarly, we can apply the breakup at a little higher frequency. So let's consider that we are intending to break this particular transfer function at omega star is equal to 31.6, which is saying that my highest frequency operation is 31.6 or somewhere close to it. So now my lower order transfer function corresponding values are (1+s), (1+0.1s) because now we are breaking at 0.01, and the higher order term is 0.001s.

So here now my T_s turns out to be 0.01. So, we are going to retain this particular time constant (1+s) as is, and this 0.1s is going to incorporate rest of the higher order terms values and approximate it there. Now, my T_(arh) is simple which is equal to this because this is having only one time constant which is 0.01. Putting this into the approximate values, what we get is (1+s), and the lag term. Since we are retaining this Kp/(1+s), the rest of the term is the approximated values and which turns out to be here.

So, this you can see now since we have made the breakup over here, all the higher order terms corresponding to time constant 0.01 and 0.001 are now getting reflected into the time constant 0.1 as 0.1005 and a time delay term corresponding to a very small time. So this way, when we are approximating the higher order system to lower order system, one can look forward for tuning it used with the help of PID controls. And this is why we can also give an answer that why the industrial control is dominated by the PID control. Because we are working in a particular operating range of frequencies. Given this particular method, we know what is my highest frequency of operation.

If we are confined our operations within this particular range of frequencies capped by this highest frequency, we can make an approximation something like this, and we can apply the PID tuning and we should not be worried about what is the dynamics for the higher frequencies because now we are not even operating in that range. The effects of those higher frequencies are just accounted into this with the help of an approximation, which is, of course, introducing certain errors into the modeling. But fair enough, those approximations and those errors are negligible, and we should not be bothered about it. And they are not introducing any stability issues for us because we typically keep a fair enough phase margin and the magnitude margins, gain margins. That is all on this model reduction.

Thank you.