Course Name: INTELLIGENT FEEDBACK AND CONTROL

Professor Name: Leena Vachhani

Department Name: Multi-disciplinary primarily for Mechanical, Electrical, Aerospace and Chemical engineering streams

Institute Name: Indian Institute of Technology Bombay

Week - 04

Lecture - 22

Hello. In this video, we will look into control and learning. Essentially, we'll be looking into how learning-based frameworks can be used for control applications. Learning-based framework as in AI ML methods, machine learning methods to be very specific. In order to understand that, I will be covering again some idea about what the dynamical system is, and non-dynamical system is, and how the learning methods have been evolved for non-dynamical systems and how we can apply it for dynamical system for the control purposes.

All right, let's begin by understanding the dynamical system. Some part of it, we have already addressed dynamical system or non-dynamical system in the earlier video, but let's revise this for the sake of completion here. By definition, the system where a function describes the time dependence of a point in a geometrical space is called dynamical system. In essence, we will be looking into the dynamical systems for which the state of the system depends upon its initial state and how it has evolved to a particular time instance. Not only the initial state, but how the trajectory has evolved to reach to that particular time instance.

So the state will be dependent on the time as well as the initial state from where it has started from rest. We can take some examples here. For example, the solar system, the distance of the planet to sun is just not enough, but also it depends upon the time at which this particular state, the state of the solar system is to be considered. Similarly, weather patterns, which is ever evolving and changing. The stock market is a very large system where many, many different parameters or variables to be considered to understand what the market is and how the market has evolved from a particular time instant to this particular time instance.

As I mentioned, the dynamical system has characteristics which evolves and exhibit behaviors dependent on the initial conditions. We also characterize the dynamical systems with the help of differential equations by showing the changes over time. So, for example, we have inverted pendulum shown here. So, inverted pendulum as in it is pivoted at the bottom and pendulum is inverted and its equilibrium is or what we want is that this particular pendulum should stay in the upright position. But what happens is little bit of disturbance into it.

If you have applied small force, it will come down and settle over here. In any case, when we have whatever is the condition of this particular pendulum at a particular time can be described by this differential equation given here, which is given by the second derivative of theta here. With respect to time equals G by L sine theta where theta is the angle of the pendulum with respect to the vertical axis and L is the length of the arm. So, we can see that we cannot simply say that the position of this particular pendulum which is given by theta, angular position theta is just not being given based on what is the length of the arm and the gravity component G, but also it depends upon from where it has started. So, depending upon from where it has started, this particular solution of this derivative, this differential equation is going to be giving the state or the angular position of this particular inverted pendulum.

All right, let's consider non-dynamical systems. As against the dynamical systems, we will say that, okay, these systems do not exhibit changes in state over time. So whatever is the input applied, we have a certain relationship which describes what should be the output. Depending upon, so in essence, the output depends only on the inputs being applied, and it does not change with respect to time. We can give certain examples such as the static forces on a bridge.

So we have this infrastructural bridges or infrastructures, civil bridges, civil infrastructures where we calculate its static forces and depending upon its structure is being designed. Similarly, shape and size of the mountain ranges. They are not changing with time. Of course, they change over a small change happens over the period of, say, a thousand years or whatnot. So at least for my lifetime, I would say that shape of the mountain ranges is fixed because its changes are very, very small with respect to time.

And that's the reason I can always consider that it's a static system for this particular duration of time. Something similar even we do it with the dynamical systems too. We can always consider within the particular sample time it is not changing. But that is when we are solving that particular dynamical equation. Here in the static case we will say that okay it is for particular time instance the entire time period of the assessment it is completely static.

There's no time concept when I'm describing the output of that system. Similarly, circuits with no current flow can be considered under static systems. They are, as I said, they are characterized by static or equilibrium systems and they remain unchanged over time. Let's take this example of strain gauge. The strain gauge is a sensor which gives the change in the resistance at the output lids based on the strain applied.

So these are being typically used on a metal pipes or the bridges or a civil infrastructure such as such that even if there is some compression or expansion of that particular structure, they will be able to gauge the changes in the resistance over there and the change in the resistance is then captured through some circuit and so on. What, as a system engineer, we are interested in, then what is the input-output relationship? So, the output is the resistance, which depends upon the length of the strain gauge and the cross-sectional area of the strain gauge. There could be resistive types and capacitive types. We are not interested into understanding its working part of it.

What we have is now understanding that if it is a strain gauge system which is I know that this is falling under static systems because the output R is not changing with respect to time. Whereas, this changes with respect by input L and input A. If there is change in the length or change in the cross sectional area, the output resistance R is going to change. All right. So, of course, in these cases, what we have is the model available means input output relationship was available.

But in certain cases, certain systems getting this input-output relationship is not easy or coming up with the differential equations in case of the dynamical systems is not easy. So, in that case, what we will consider is learning the systems with the help of the set of inputs given and corresponding changes in the outputs. For example, in the static case, for example, I know that for the strain gauge system, strain gauge example, that it depends upon inputs L and A, but the exact relationship or the mapping from input output mapping is not available. So I want to understand what should be this particular function F look like. So then what I will do is I will take multiple measurements depending upon, I will consider L and A as inputs and I will consider getting the output R recorded.

So I will consider different measurements taken and corresponding the output values being recorded. But I can do this if it's very easy to understand if I have only one input and one output. So for example, I have only one input and one output system. And let's assume that cross-sectional area A remains constant all the time. And for the strain gauge, we have only one input L and output R to be considered.

So when I plot the L versus R curve, this could be linear or nonlinear. Linear means I will get some kind of a straight line. For different values of L, I can plot this, what should be the value of R. And then try mapping, try getting, fitting it as a straight line, then it's a linear relationship. It could be possible that it is for example, the fifth point turns out to be somewhere here.

So, I would like to, I know that there is some non-linearity in the relationship between R and L exists. So, then what I will do is I will consider saying that R is some function of maybe a polynomial of L. So, then I will consider fitting a polynomial L, AN minus 1, l power of n minus 1 plus a power of n minus 2 l power of n minus 2 plus a naught l power of.... Okay, fine.

So this polynomial fit is more or less like an optimization problem. We would be able to solve it. But what if it's not a polynomial form? For example, here our coefficient a n was actually we knew that this relationship is l by r, right? So this coefficient is further a function of, say, F2 of A, some other parameter A, some other input A, all right?

So, then it is nothing but a neural network that we are trying to solve, right? So, instead of a constant coefficient, it is actually another function of the inputs and so on. Fair enough? Now, coming up with the differential equations form, which solves the dynamical case. Dynamical systems, we describe our state as X dot equals AX plus BU and Y is equal to CX.

And in case of the pendulum equations, we have theta dot equals G by L sine theta. So what I am considering here as the state of the system as double dot, theta and theta dot, then I will be able to represent this particular system in this particular form. All right. So my x is two dimensional and the state is given by theta and theta dot. All right.

Even though even when I say this, I have said I know that this is not a linear relationship anymore. So what I'm saying here is that the relationship is some nonlinear in terms of X and U. And at the same time, this particular change in the state is captured with the help of this differential equation or a nonlinear state form. So this we have been doing it considering the state space realization for describing a process or a system. And this process has multiple input, can have multiple inputs and can have multiple outputs, multiple measurements coming up.

These measurements could be directly related to output or maybe some combination of outputs. At the same time, the system is also being under the certain disturbances. So in case of the learning framework, we will consider the system is acting under an environment here. So this environment is to be modeled in some cases or maybe what should be my output of the environment is to be controlled. What is the output of this environment are the set of observations and input to the environment is action.

This action is directly or not directly or indirectly related to inputs may or may not be right. So, that is something we will look into it. Here it is important to understand the terminology that is being used between learning and control. When we say state in terms of the system state, it's the state of this particular process. Whereas when we consider in terms of the learning language, ML language, and we describe a state, the state is for this entire environment.

That is something we have to understand. So the entire importance is being given when we formulate a problem in terms of the ML method and that too having a control objective that is something we will look into it is to say what are my observations and what are my actions. All right. Then we should be able to come up with whether it's the entire observation-action pair to be to be considered into a database. Or if I have some some training data available, then can I come up with a policy as in the reinforcement learning methods?

All right. If I have to control, if I will consider this control of non-dynamic, whether it's a control or not, that we'll see. But at the same time, the learning-based framework for the non-dynamical system for which the model is not known, model is not available. So, what we can look forward is to identify this model which is finding the input output relationship. So, for example, I have this particular input and this output or a set of inputs as in case of the strain gauge L and A and R.

Of course, those relationships are already available. But if that relationship is not available, then what we will do? We will collect the data and corresponding R values. That is straightforward. So, in case of the strain gauge example, we have input length L and cross-sectional area A, whereas the output is resistance R.

And we are interested in getting this particular relationship F, which is the mapping from input to output. Are we interested in controlling this model? May not be because if I have this particular system described by input and output and this relationship is available, then I will change this input in order to get a desired output over here. That's the control problem. I should be able to control the output of the system.

If I want to do it in the context of the sensors, at max what I like to do is to get some controller, which is an open loop perhaps, and say, okay, I want to consider matching this particular output So if this since the relationship is not is already known, which is R equals L by A, I will have a controller, which is just the inverse of it, which is A by L so that what is my desired R can be achieved at the output R. That's all could be a control problem for a non-dynamical systems form. But that's actually not a control part, it's just an, okay, unit change or whatever you want to consider it in this case. All right, so control of dynamical systems which is very interesting to look at from the learning concept point of view.

We can consider having two approaches where we are mainly applying the dynamical systems under the learning framework when the model is unavailable, when the input-output relationship is not available to me. So what we can consider these two approaches as, first is we will learn the model and then design the control. In which case, we will consider describing this particular process under an environment. And then we will consider that, okay, we will consider some observation set and action set. We'll learn this.

And then, okay, once the process model is available, then we will design a controller which is giving the input to the system directly. This is one way. The second way is we will consider learning to control the system itself. In which case what we will consider that within the environment itself, we can have a controller already existing and which is giving the input to the system. And so the observations and actions may or may not consider the control parameters as a part of these sets.

So now we have two options. We can consider the system as the black box directly. So where my controller itself, this is my controller. Whether it's a controller sitting here or not is the second option rather. The first is we will consider that, OK, my actions are that I'm giving over here are to certain extent taking care of the control objective in the absence of these controllers.

So to a certain extent, my learning method, which is sitting here, which is taking the observations and the and the actions being given is acting as the controller directly. So we don't have this controller, but this controller is my learning-based controller. So what we are doing in this case is we are making sure that my ML algorithm is coming up with some kind of method which is acting like a controller. The second is that we will assume the form of a controller and learn to tune it. In this case, I will consider that there's a controller sitting here and which is giving an input to the system or a process or a plant here.

So then I will consider the form of the controller, for example, PID controller, and its gains are given by KP, KI, and KD. So my action commands are such that this is being given for a particular state of the environment The actions are being conveyed through the ML algorithm or the database, which is describing what should be the KP, KI and KD for that state. So, this is where we can also call it as that the ML algorithm is giving its objective is to tune the KP, KI and KD values. So it is not necessary that my system or a process should be linear system, but for a particular state which has evolved from an initial state should have a particular gain values.

So this Kp, Ki, Kd will again be functions of time t. So every instance we will have the value depending upon a state of the system, the KP,KI,KD value could be changing. And then we would be able to even address nonlinear systems or highly nonlinear systems in this case. So these are the advantages we can explore by giving the learning framework to the control objectives. Next video, we will look into designing a database entries for tuning the KP, KI and KD values.

See you.