**Course Name: INTELLIGENT FEEDBACK AND CONTROL**

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**Week - 03**

**Lecture - 16**

Hi, in this video we'll be looking into the concepts of decoupler and some idealistic methods for designing the decoupler. Let's start seeing what is this decoupler and why do we need it. In the previous video, we looked into decoupling the system for multivariable SISO method. Especially when we are looking into multiple inputs and multiple outputs, we looked into decoupling the system by pairing the input and output, which particular input is affecting which particular output and can we then consider them under considering multiple single input, single output systems so that we can have a very structured individual control loops sitting in for individual SISO subsystems. In this video, we'll first introduce what are right-hand side transmission zeros, and then look forward for decoupling kind of a controller, specifically the feedforward way, and next we'll look into the feedback way as well.

Let's see what are these zeros. The zeros for a single input, single output system limits the performance because especially when it is on the right-hand side, it gives you a non-minimum phase kind of a response. For example, there is a zero on the right-hand side and it is a first order system. Then for a step input, initially your output is going to go in an opposite direction of the input, and it picks up and settles down. So this particular non-minimum phase kind of characteristics, it not only adds the delay into the system, but at the same time, since it's a reverse action happening, while even going into the higher side, it first goes into the lower side and then tries to settle down.

So to a certain extent, this is undesirable because such kind of a characteristic can appear even for the disturbances as well. And the disturbance rejection further gets delayed in some sense. So, the transfer function for the single input, single output system, if it is of the N of S and D of S side, and we have been looking into certain techniques at times when we have model-based methods to be considered, we need inverse of these transfer functions. So further, these zeros appear as a poles in case we want to design it based on the inverse of the transfer function. So these RHP zeros are again an issue when one has to design the controller based on the inverse of the transfer function.

If it is a model inverse method, then it will certainly going to result in unstable controllers. So, we know when we have to consider the inverse model, what we consider, we factor out these RHP zeros and the portion of the model which does not have the RHP zeros is used for the inverse methods and so on, inverting the model concept. In case of the multivariable transmission, multivariable systems or the MIMO system, other than the zeros of individual transfer functions, there are transmission zeros as well. Let us see what these transmission zeros are. These transmission zeros are at the value of S for which the transfer function matrix loses rank.

So, the value of S for which the determinant of G of S is equal to 0, where G of S is a transfer matrix form. So, this particular one then becomes the pole for the inverse system, additional pole for the inverse system and that is where we should be worried about these transmission zeros as well. For example, if this particular transfer function G of S has S plus 3, 2, 3, 1, whereas all the poles of all the four transfer functions of the system, by the way, this is two input, two output system, and that is why the G of S turns out to be of 2 cross 2 matrix form. But the first subsystem, first transfer function, which is between input U1 and output Y1, it is having a 0 at S is equal to minus 3, which is on the left-hand side. Fair enough.

But when we try to get the determinant of G of S, what we get here is S plus 3 minus 6 equals 0. When we consider this kind of finding out where the transmission zero is, we find the determinant of g of s, which is equal to s plus 3 minus 6. I am doing nothing but the cross multiplication over here and then subtracting it. And when I substitute this 0, we get s minus 3 equals 0 or s is equal to 3. Means this is what the zero, or a transmission zero, which is on the right-hand side.

So we can see that the transfer function in itself was not having a zero, the right-hand side zero. But the transmission zero, which is resulting from getting the determinant of G of S, is having a right-hand side zero, which is going to create issues when we are designing the system which involves inverting the model. So we have, as we said, there is a zero of G11, which is at S equal to minus 3. But the matrix transmission zero is at S equal to 3, which we should be bothered about, which was not visible clearly. But now when we took the determinant of G of S, now we know that this particular system has a transmission 0 at S is equal to 3.

All right. Now let's see what we are talking about a decoupler in this case. And why should we be worried about the transmission zeros in such cases? Let's consider this kind of a multivariable control system where we have a plant transfer function given by GP, the controller transfer function is GC of S. Since this is multivariable transfer functions, we can consider that each of this GP and GC is nothing but matrix transfer functions.

So, if it is a SISO controller, means multivariable MVSISO kind of a method, then the controller is nothing but a diagonal matrix. So in that case, your Y of S can be easily written in terms of GP of S U of S plus GD of S D of S. As of now, we are not considering this diagonal when I said GP of S, but at the same time, one can see that the controller design becomes very easy when we are considering the MVSISO method because then we have independent controllers to be designed and which are not interacting with each other. At the same time, all the interactive terms can be considered plugged into saying that, okay, there's a small disturbance, and so we should be worried about rather doing the disturbance rejection in this case. And those interactive terms, to a certain extent, is reflected in this GD form.

It's a simple, same SISO way of analyzing and finding the transfer function, the overall transfer function of the system of this particular controller block. We can say E of S is equal to Y of S times R of S, which is my error function over here y minus r sorry this is minus part over here y minus r so we can write in the similar way  we substitute y of s equal to all these y in terms of U and R in terms of U sorry when we just simply write in terms of the signal flow type we can say this is turning out to be similar as the SISO system form so we get one in the SISO form we say one plus GH where this was H and this is G and it gives you Y of S equals GH R of S plus GD D of S. So, therefore, the control system's transfer function becomes GH by 1 plus GH. Its counterpart in terms of the matrix transfer functions are being considered in this form now. All right.

Now, we have instead of 1 for the matrix form, 1 becomes your identity matrix. And when we are dealing with matrices, we have to be very careful, especially when it is an inverse form coming up. In the case of SISO part, it was simple division by 1 plus GH. Here, it becomes 1 I plus GP GC inverse GP of S GC of S. Similarly, over here, this is going to be left multiplied first.

That also we have to be very careful about that this inverse which is coming from the left hand side over here is now left multiplied with the rest of the terms over here. We also know in the matrices form, we have to be very careful that GPGC is not equal to GCGP. So when we are multiplying, when we are seeing the signal flows, which particular transfer function matrix comes first is very important. So here we have to make sure that we are making no mistake in terms of reversing it specifically when it is not a square matrix. Then multiplication anyway has to respect the matrix loss.

All right. So for the decoupling controller, now what we have here is that one method, which was pairing the variables. Now, for example, those pairing variables are not coming out possible. Then what is the possibility that can I have one more system, one more block, which is like a transformation, which is giving me a pairing function. So that's why the block is called as a decoupler and will help us in using the MVSISO methods

So what we have is an interaction which is resulting in a manipulated input affecting more than one control block. If that is what is the case, then we have decoupler, which will have this D block sitting here and which is nothing but giving U star and Y such that it affects only one process variable now. So this synthetic input variable or manipulated input variable we need to figure out what it is first of all. But at the same time, this synthetic manipulated input is chosen such that now I'm able to do the pairing. So U1 star is, say, affecting most the Y1, U2 star is affecting Y2 and so on and so forth, which was not happening if it was the true manipulated input over here.

So here comes the design for the decoupler, which is, again, once we see it, we should be able to appreciate that it's a very straightforward design. But there are many more other things that we can consider while designing it to make it a simplified D block or a decoupler block that is appearing here. All right. So now in a general way, we have a process which is comprising of four transfer functions, G11, G21, G12, and G22. So a transfer function form, this is G11, G12, G21, and G21 is what we are talking about.

So G21, for example, is a transfer function between Y2 and U1, Y2 and U1. All right. Now, these are these manipulated inputs U1 and U2. We figured out that they are not able to pair it well with any single Y1 or Y2. So, in a way, when I am changing U1, Y1 and Y2 both are changing.

Similarly, there is a possibility that U2 is also affecting both Y1 and Y2. Now, I want to design a decoupler over here. So, this particular decoupler is again of the form of D11, D12, D21 and D21. So, my decoupler block is again D11, D12, d21 and d22, but now it is between u1 and u star. So, instead of the output y1, now my output is u1 over here and this is a synthetic u1 star that we get it.

So, overall we would like u1 star to affect only y1, u2 star to affect only y1. So can I come up with this particular decoupler D such that I have a pairing option available? And that's what we would like to take it up. Since this is two-input, two-output system, it is easier to visualize. For any multi-input, multi-output system, the decoupler design is a wonderful way of coming up with MVSISO methods, and we should be able to design multiple SISO control systems.

All right. Now, let us see what is the decoupler design turning out to be. Now, I have Y1, Y2. Since this was since we can see that now decoupler is sitting before the process block. So it is in the feedforward way.

And this particular method is my feedforward way of the decoupler design. Now, Y1, Y2 output is nothing but GP of S, earlier it was like GP of S, U of S, U1 of S and U2 of S, All right. Now, this U1 and U2 is being coming from the U1 star and U2 star. So, and passing by the decoupler block D of S. So we have multiple possible choices for GP of S and D of S because I can have many ways by which I would be able to consider connecting U1 star with U1 by 1 directly.

So there are two choices. One is ideal decoupling, another is simplified decoupling. Let's look into the ideal decoupling first. Now, under the ideal decoupler, what we want finally is a GP of S, D of S should result me a diagonal matrix form now. So, that my y1 and y2 are related to u1 star and u2 star in a one-on-one basis.

For example, so basically I want this to be g11, g22 and these two elements as 0. So, then I would be able to write y1 is equal to g11 u1 star and y2 equals g22 u2 star. All right. Okay, so now if that is the case, then it becomes, I mean, we have many, many choices for G11 and G22. So we can result in not a unique design of a decoupler even in the ideal coupler itself, but we can always think about governing the way what is possible and what is not possible.

Finally, we will have to deal with G11 and G22 and the controller designs will be for G11 and G22. All right. So now my decoupler is nothing but D of S is equal to GPS of inverse G11 and G2. Now you notice this inverse, this inverse should not have, so GP of S is not having the RHP zeros or the transmission zeros, then this particular invertible transfer matrix can be used for designing the decoupler. All right.

As I mentioned, this is going to give me a diagonal form and in the form of Y1 is equal to G11 U1 star and Y2 is equal to G22 U2 star. What are the advantages and the limitations of this ideal decoupler? we will have independent SISO tuning parameters that can be used for each control loop. At the same time, we have the flexibility of designing what should be G11 and what should be G22 here. But what is the limitation?

We just spoke about if the transfer function has transmission zeros, then what to do, all right? So there are hacks available similar to the inverse model methods that we had. We can keep the transmission zeros part separately. We could design it for the transmission zeros separately. The main limitation is that it is sensitive to the model errors becauseour design D of S depends upon GP inverse of S.

And then, of course, G11, 0, G22 and so on. So one should know this GP of S very accurately in order to get its inverse and the matching poles and zeros over there. If there are slightest change in the position of the poles or there are model parameters changes because of which the poles are changing, then there is not an exact decoupling, and then these interactive terms will reappear and it is not going to give the solution for the single control loop way, multiple single loop ways. Since it is highly sensitive to the model errors, we should be bothered about using it under limited circumstances. But let's see what is further available to us is a simplified decoupler design.

There we were designing four transfer functions, D11, D2, D12, D21, D22. Now there's a possibility that, okay, Now, I should be able to, since there is a lot, we just have G11, G22 kind of things to be considered. So, we should be able to design it with lesser number of transfer functions, lesser number of transfer matrices, transfer function matrix elements, because that is finally is an add-on thing that you are applying over here. So if one some simple choice would be that the decoupler is having simple D12 and D21 terms in order to counteract the interactive terms G21 and G12 in the process plant.

All right, so let's take one example over here. Let's consider a G of S of this form, which is again two input, two output case. All right. So this is a transfer function between y1 and u1. This one is transfer function between y1 and u2, y2 and u1 and y2 and u2 in this case.

Now what happens if I consider converting rows to a common denominator in this case? So this turns out to be giving me a nice diagonal form anyway. And this part is something we have to take care of it in order to consider the decoupler design in this case, all right? Now the question here is of course we have a diagonal form but what do I do with this part and how do I design the decoupler. Let us take this over here.

Now we have to take care of this. So I am considering this as N of S. If I consider the inverse of this I get of this form. Fair enough. As soon as I have this inverse available, we can consider this itself as the D of S. All right.

Now, even if it is a D of S. whether it is sitting before the process control block or the after control blocks is to be considered very carefully. What we did in this case is matrix manipulation. So one has to make sure that we are not considering the associative property of this. All right.

So M of S, N of S is not equal to N of S, so M of S, N of S we have written it in this form. So now it is G of S times N of S inverse if I consider, then I'm getting a diagonal block, which is 1 by S plus 7, S plus 14, 0, 0, 1 by S plus 7, S plus 4. And this is a very simplified design. Right. Now, this particular N of S inverse, which is nothing but my decoupler D of S is sitting after the process control block.

Correct. Is that right? The output, I want this thing. Right. So, I am writing this in terms of Y of S.

Sorry, I will write it on this side. Y of S is equal to G of S and inverse of S, this is U star of S now. So, first multiplication is over here. You can see here that whenever I am writing this particular form, D of S is over here. But this D of S is sitting before this GP of S.

So though we have designed the N inverse of S, which was post-multiplied, but this D of S when we are designing, which is nothing but in that illustration as N inverse of S, this sits before GP of S. This sits before the process control block. This is something we have to be careful when we are doing the matrix manipulations. If the matrix is post multiplied in the transfer function block, it is actually in the is appearing before the before before the before in the signal flow. So in this way, this particular illustration that I'm considering is incorrect.

And one should consider D of S being connected before the process control block. That's all for this video. The contents are being taken from these two books. One can refer for more information. Thank you.