

Course Name: INTELLIGENT FEEDBACK AND CONTROL

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Week - 03

Lecture - 14

Hi. So, in this video we will look into simplified control of MIMO systems which will result in a multivariable control kind of a structure. In my last video, you have seen how we can design the controllers like PID controllers for nonlinear system in an adaptive way. Either you use the adaptive way or the gain scheduling way and under what conditions the gain scheduling should be used. Something similar, we will now work with MIMO systems, multiple input and multiple output systems, and look into whether we can use control structures like PID.

So first of all, we'll have to understand, are we going to consider MIMO system as multivariable SISO systems, multivariable single input, single output system? You have, for example, you have three input and four output system. So one, four outputs are something like this, right? So now, can I consider this as three systems, three separate independent systems with some kind of one input. So three different systems with one input and one output kind of thing.

All right. So, I am going to approximate this MIMO system as multiple SISO system, single input and single output system. And at the same time, towards the end, we will frame up what are the methods for decomposing to MVSISO system. All right. So to begin with, let's understand what is the control loop interaction.

It deals with the MIMO systems, definitely deal with multiple input and multiple outputs, also deal with multiple disturbances appearing in the system. We also look into multiple measurements taken from this process MIMO system. So our question always is whether we will be able to use these powerful SISO methods, which are more or less giving a

simplifying design in terms of the PID controllers. All right. If yes, if that is the possibility, then we have to figure out which input is to be paired with output for the control system design.

All right. So the way we want to decompose, we decompose into multiple systems, and each of the system is having single input and single output. But which input from this MIMO system is to be paired with which output is what we have to figure out. I'm sorry. So then if this particular multivariable control system design is to be considered, the underlying problem now becomes the control structure selection means which input is affecting that particular output more significantly more importantly than the actual controller design and the tuning methods that needs to be that that need to be used for these multi-variable SISO system, all right so now we want to consider that the MVSISO controllers as we have understood

At the same time, we have to very much understand that this selection of input to output is important. And therefore, it will result into multiple SISO control loops because now I am going to have multiple systems So each of the system will have its own control loop. So basically we will have to look forward for selecting an output variable that is getting affected by a particular manipulated variable is what the control structure selection problem is. Output variable in such circumstances is mostly measured.

And we are also looking into considering the SISO system, SISO based approach for two main reasons. And one of the main reasons is that the SISO design is simpler, which I have been always saying in this lecture series that we will be looking into control structures as simple as proportional integral and derivative or their combinations. And at the same time, the control blocks and its softwares are more or less available in the market. So their hardware and software are more or less available. So one should be able to design the SISO system, multi-variable SISO system, without making any major hardware change or any major software change into it.

All right, so now our job is to define the exact problem, this problem of control structure selection. This nothing but boils down to selecting which measured output variable is to be paired with which manipulated input, or it is also called variable pairing problem. All

right. Let's understand this variable pairing problem from a given example. And this is a very classical example of static mixer.

The static mixer considered two streams of flow, and tries mixing it and finally what we have to achieve is a flow control as well as composition control. So this is typically used in blending systems like diluting the juice concentrates, mixing in flavors and colors and blending syrups into the milk, evenly distributing jelly within molten peanut butter. Imagine you have ordered a hot chocolate and you would like that particular brand to be followed. And every time you are trying to grab this hot chocolate from a given brand, from whichever machine, the mixing of the chocolate and the milk, the amount of the chocolate syrup and the milk needs to be fixed. And at the same time, it should dispense out exactly one cup of the hot chocolates.

So the two flow streams is the chocolate syrup and the milk. So chocolate syrup is deciding the amount of chocolate in the final composition. But at the same time, both the flows of chocolate syrup and the milk is deciding the total flow rate at the output. So, the problem in general sense is formulated here given that there are two flow rates of stream, two streams of flow given by U_1 and U_2 and at the same time the output total flow rate is given by Y_2 and the composition is given by Y_1 . So, F_c is your Y_2 and composition is your Y_1 .

All right. So if the problem statement is that I need to blend a stream of 40% juice with the pure water in order to produce a product stream of 30% juice. So I am reducing the amount of the percentage composition at the output by mixing water into the juice. And I want to control this particular 30% composition at the same time the flow output of it. So how do I pair the input and output now?

I have two options because there are two inputs and two outputs. I land up into two options. One is output-input pairing of Y_1 with U_1 and Y_2 with U_2 . So Y_2 is your F_c , which is flow control, is paired with U_2 or composition is paired with U_1 . So, what is it?

The flow control is paired with the, so your juice flow rate with composition means your U_1 is your juice flow and U_2 is your flow rate in this particular diagram. So, your juice rate is deciding the composition. Whereas your water rate is deciding the flow control or

the amount of the flow rate going at the output. We have the second option that the water rate, the water rate is deciding the composition and the flow and the juice rate is deciding the flow here. All right.

So here the pairing is with Y1 with U2, Y1 output is paired with U2, whereas Y2 output is paired with U1, all right? So now let's look into the intuition here. What intuition we have because we have the 40% juice flow that may be deciding the output composition here, which is 30% which is expected. So, the flow rate of juice U1 is deciding the composition while the water rate is deciding the flow. Or saying it in an opposite way, the flow rate is controlled by the water rate while the composition control is given to the juice rate here.

Fair enough, because we are kind of considering 40% juice here and which should get, the output you should get as 30%. Let's observe the behavior of control loop 1, which is my juice rate paired with the output composition. So when we have this particular control loop 2 is open, control loop 2 is open and we are only controlling the composition with this. so what we have the observation if I have the change in output y1 means if I want the composition to be changed from 0.3 to 2.31 it is a step change here the output y1 changes very nicely because u1 is changed but at the same time you see that the flow rate has changed from 4 to 4.2, it is no longer constant for flow because your u2 was constant here, it was control loop 2 was open So, total flow rate is deviating while our objective of composition control is achieved.

Fair enough, because our control loop 2 was anyway open. But now what happens if I close the loop 2 as well? But so we would like to maintain the total blend rate at a constant value. Means the output I want to dispense out is the one cup exactly. So I don't want to disturb the total flow rate here.

So now, input U2 is water flow rate and output Y2 is total flow rate and now loop 2 is closed. We have a question here, how do I tune the control parameters for loop 2? What we will consider that when we are tuning the control loop 2, this is tuned independent of loop 1 considering that the loop 1 is open. If that is the case, then what happens is as soon

as I had the change in Y1, so let's say Y1 was changed from 0.3 to 0.28, fine. So what we had was the Y1 has changed.

So Y1 has changed. So now we have given the disturbance. So sorry, the set point for Y2 is 4.2 here. So, U1 is like not making any changes. So, if I am trying to change this particular total blend rate, because of the U2, the change happens, but the Y1 is now getting disturbed.

So, this is the case when loop 2 is closed, but loop 1 is open. All right. So we have seen that, of course, the objective of that particular loop is getting achieved. But the second loop, the other output is getting affected. But in order to control both the outputs, we will have to close both the loops.

So, as soon as I close both the loops, then what happens is that the outputs are chasing each other, and it is not able to control each other. So, do we conclude that we are not able to do the multivariable SISO the way we had considered Y1 being controlled by U1, Y2 is being controlled by U2. There are no interactions at all. And that's the reason this particular two SISO systems method is not at all working. It's not a good conclusion because we have not tried the second option yet.

All right. So as soon as we try perhaps the second option, there's a chance that we'll be able to achieve the control properly. And this is rather the case when if I consider the bad variable pairing and I consider multiple SISO way, multiple variable SISO way of controlling a MIMO system, then there is a chance that the system may become unstable. So underlying part is that what characteristics of this control strategy cause the instability? Was it the control tuning or was it the choice of the variable pairing that we did?

So it boils down that the variable pairing is playing a very, very important role here in decomposing the MIMO system to MVSISO system. Because if I choose a different variable pairing, this kind of instability can be avoided and we may be able to achieve the control with the help of multiple control loops, which are of the structure PID itself, the simple control structure itself. All right. So now we have a pairing problem. Now, the

exact problem is how do we select the pairing for a control structure consisting of multiple SISO loops?

So, in general, if I have system with M inputs and N outputs, so my transfer function matrix can be written in this particular form. So, this individual G_{11} of S , for example, is a transfer function between Y_1 and U_1 , while we consider G_{12} of S is a transfer function between Y_1 and U_2 and so on and so forth. So, each of this since I consider LTI systems again here, this particular Y_1 output is nothing but the linear combination of $G_{11}U_1$ plus $G_{12}U_2$ plus $G_{13}U_3$ and so on. So each of this Y_i of S can be written as a linear combination of U_i s and their corresponding outputs through the corresponding transfer function values. Now, the question here is, this is LTI system, fair enough.

So now when we are designing this particular pairing problem, I will have to consider pairing Y_i output with input U_j , all right? So this is what my big question regarding variable pairing is, which output Y_i should be paired with U_j . So do I have any good technique which will tell me that I should do this kind of pairing? Because so far in the previous example of blending or a static mixer example, we just looked into, okay, we had two inputs, two outputs. We have only two options available of pairing Y_1 with U_1 or Y_1 with U_2 because I'm considering SISO, single input, single output.

I can always decompose in terms of the smaller numbers as well. But at the same time, if I'm considering SISO, then I need to consider one output to be controlled using one input alone. So that's the simplest way we are looking at. Now, if I'm looking at that, I had two outputs and two input systems. I had only two options available.

So I could have tried both the options, whichever works fair enough. But if the number of inputs and number of outputs are also large, beyond three, four, beyond four numbers, the number of options that I have in order to consider multiple SISO way, I will have those many combinations to work out. So do I be looking at all those combinations? No, there should be a better way of analyzing it, which is going to give us some way of information that, okay, you should try considering this output being controlled by this manipulated variable. So Y_1 should be manipulated by a particular U_i , U_j .

Or YI should be controlled by a particular UJ, so that the multiple SISO loops that we are creating are going to give you solutions as a stable control system using simple structures like PID structures. So in my next video, we will look into those methods, some simple heuristic methods that have evolved for considering MIMO system control with the help of MVSISO. So these methods also will give you an insight whether we would be able to solve the MIMO control problem using MVSISO probably, which is also a very nice tool for me. I should look into simplified methods of control only if there is a possibility exists. But then who will tell me this possibility.

These tools are going to give us the possibility. So in the next video we will look into relative gain error method and a condition number method. These are two popular methods. There are many, many more. And we will look into the relative gain array method and the condition number and their combinations to figure out what pair should be considered for the SISO implementation.

See you then.