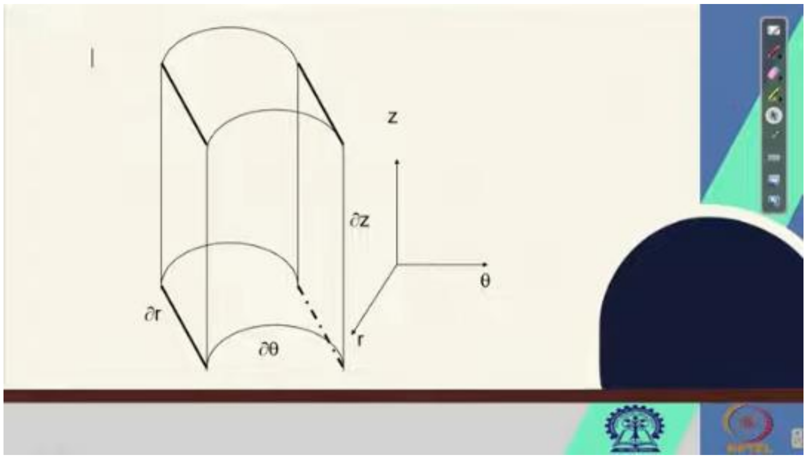
# IMPACT OF FLOW OF FLUIDS IN FOOD PROCESSING AND PRESERVATION

## Lecture09

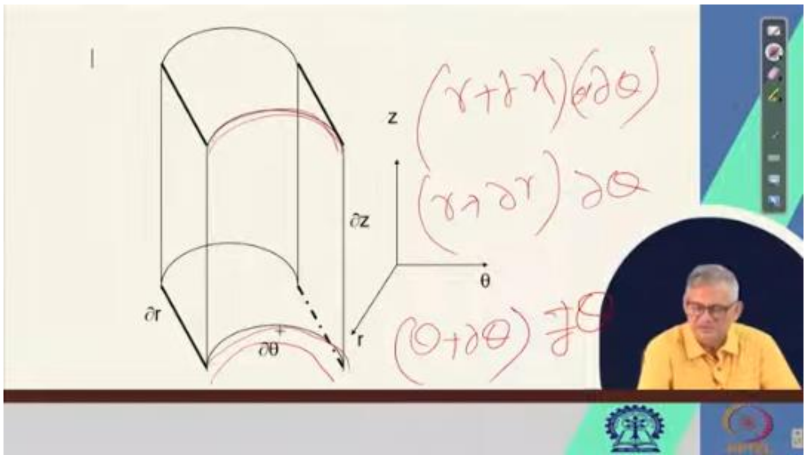
## LECTURE 9 : EQUATION OF CONTINUITY IN CYLINDRICAL AND SPHERICAL COORDINATES

Good evening my dear students and friends. We have developed the equation of continuity for Cartesian coordinate, right. We were we are proceeding to do the same for the r theta z coordinate that is cylindrical coordinate, but perhaps because of the, I mean constraint of time we could not do it fully. Now, we should do that derivation of equation of continuity in the cylindrical coordinate, ok.

So, here we start. that our cylindrical coordinate we are fixing it like this r theta z. Mind it that, this is the fundamental that here it is r del theta is the arc and here this is r plus del r del theta, right. this is like that and this is like this, this we have to keep in mind right, because if we take the other one that r plus del r that is this and del theta plus rather  theta plus del theta, if we would have taken, so many small things would have come up. So, we have taken that in consideration that theta plus del theta that is roughly equal to del theta, right, that is roughly equal to del theta, that we have taken because here our thing



r theta z this is the coordinate, ok. Now, this if we keep in mind and this if we do understand that what is r, what is r plus del r, what is del theta, what is theta, if we understand this, then it will be easy to remember, and it will be easy to follow, right. So, we start with, like the Cartesian coordinate, we start with the r direction, right. In the r theta z coordinate we start with r direction and we say in




mr in r rather. However, because earlier this thing we had done that in r minus out r plus delta r that was equals to minus del del r of r rho $v_r$ into del r del theta del z. Here we have said earlier sorry we have here we have said earlier also that this in minus out r plus del r in r minus out r plus del r in this form we got from the UV expansion and UV contraction both. Earlier we have shown UV expansion that was del del x of u v, right, that was equal to u into del v del x plus v into del u del x, right.

Continuity equation in cylindrical coordinates

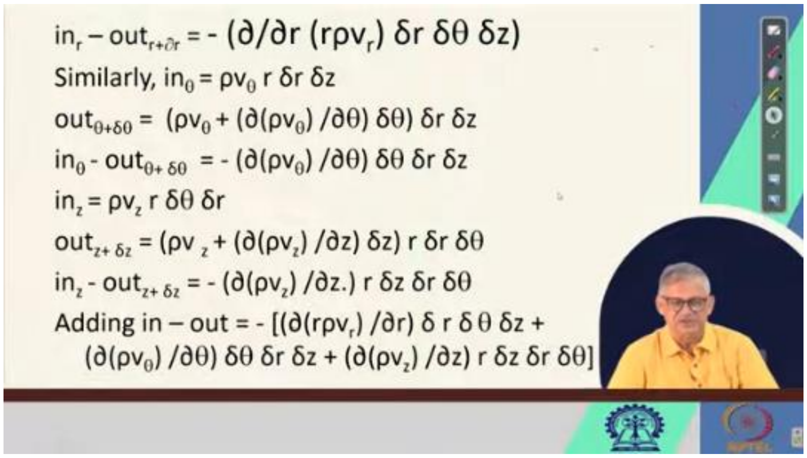
$in_r = m_r \, \delta_A = m_r \, r \, \delta\theta \, \delta z = \rho \, v_r \, r \, \delta\theta \, \delta z$ $\quad out_{r+\partial r} = (m_r + (\partial m_r/\partial r)\delta r)\,(r + \delta r)\,\delta\theta\,\delta z = m_r (r + \delta r)\,\delta\theta\,\delta z + (\partial m_r/\partial r)\, r.\, \delta r\,\delta\theta\,\delta z$

$= \rho \, v_r \, r \, \delta\theta \, \delta z + \rho \, v_r \, \delta r \, \delta\theta \, \delta z + \partial/\partial r\,(\rho v_r)\, r \, \delta r \, \delta\theta \, \delta z$

$in_r - out_{r+\partial r} = \rho \, v_r \, r \, \delta\theta \, \delta z - \rho \, v_r \, r \, \delta\theta \, \delta z - \rho \, v_r \, \delta r \, \delta\theta \, \delta z - \partial/\partial r\,(\rho v_r)\, r \, \delta r \, \delta\theta \, \delta z$

$= -(\rho \, v_r \, \delta r \, \delta\theta \, \delta z - \partial/\partial r\,(\rho v_r)\, r \, \delta r \, \delta\theta \, \delta z)$

$= -(\partial/\partial r\,(r\rho v_r)\, \delta r \, \delta\theta \, \delta z)$




$in_r - out_{r+\partial r} = -\left(\partial/\partial r\,(r\rho v_r)\,\delta r\,\delta\theta\,\delta z\right)$

Similarly, $in_\theta = \rho v_\theta\, r\,\delta r\,\delta z$

$out_{\theta+\delta\theta} = \left(\rho v_\theta + (\partial(\rho v_\theta)/\partial\theta)\,\delta\theta\right)\delta r\,\delta z$

$in_\theta - out_{\theta+\delta\theta} = -(\partial(\rho v_\theta)/\partial\theta)\,\delta\theta\,\delta r\,\delta z$

$in_z = \rho v_z\, r\,\delta\theta\,\delta r$

$out_{z+\delta z} = \left(\rho v_z + (\partial(\rho v_z)/\partial z)\,\delta z\right) r\,\delta r\,\delta\theta$

$in_z - out_{z+\delta z} = -(\partial(\rho v_z)/\partial z.)\, r\,\delta z\,\delta r\,\delta\theta$

Adding in – out = $-[(\partial(r\rho v_r)/\partial r)\,\delta r\,\delta\theta\,\delta z + (\partial(\rho v_\theta)/\partial\theta)\,\delta\theta\,\delta r\,\delta z + (\partial(\rho v_z)/\partial z)\, r\,\delta z\,\delta r\,\delta\theta]$

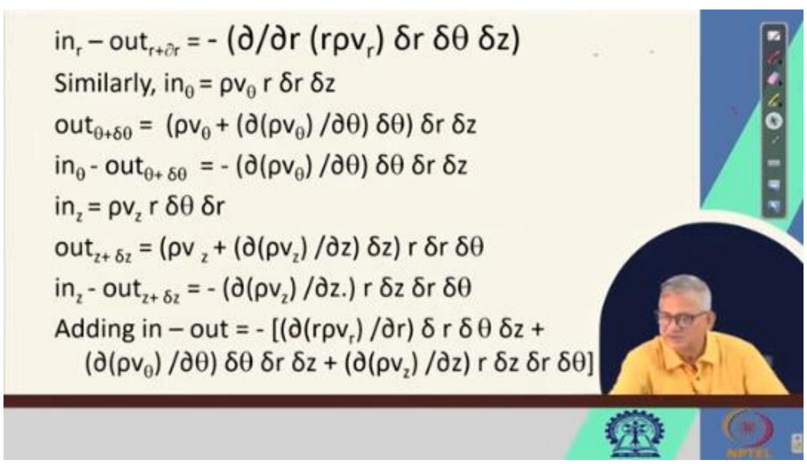
This was our expansion, but if we do the contraction if we have u del vx del x plus v del u del x then, that is contracted into del del x of u v, right. This we have applied to this form, right, if you look at the previous slide, in the previous class slide then you will be able to locate that we arrived at this r rho $v_r$ this contraction from that concept, ok. So, in a similar manner we can also write, in theta coordinate that in theta is, rho $v_{theta}$ r del r del $z_r$ Δr, Δz is the area,

area is r Δr, Δz. So, rho $v_\Theta$, right and out plus del , is rho v plus del del of rho v into del that is the distance, into Δr, Δz that is the area, right. So, in minus out that we can write equals to, because, this rho v and this rho v cancels out, right. So, we get minus del del of rho v into del theta Δr, Δz, right. So, that is for the theta coordinate.

Similarly, for the z coordinate we get that in at z is rho $v_z$ into r del  into Δr, Δ is the area out at r plus del z sorry out at z plus del z is rho $v_z$ plus del del z of rho $v_z$ into del z.  is the distance, right, into r del r del  is the area. So, again if we make, in z minus out z plus del z, then we come to this point that minus del del z of rho $v_z$ into r del z del r del  right. Now, if we add all

in minus out in all the three directions of r, ,  and z, then we get in minus out in all three directions is equal to minus del del r of r rho $v_r$. into del r del  del z that minus is common for everybody. So, it has gone out for all three directions. So, minus del del r of rho $v_r$ del r del  del z plus del del  of rho v del  del r del z plus del del z of rho $v_z$ r del z del r del , right.

Say here you see which one is the thing which will go out del r del  del z this is common del r del  del z this is also common del r del  del z this is also common. So, we can take out and divide So, it goes out right, then from this one. So, from this one if we go to the next one is accumulation, that was in minus out. So, accumulation is



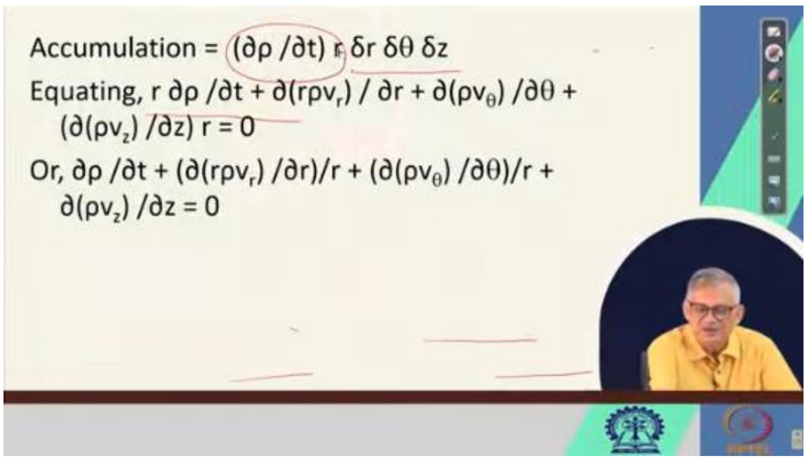

Accumulation = $(\partial\rho/\partial t)\, r\,\delta r\,\delta\theta\,\delta z$

Equating, $r\,\partial\rho/\partial t + \partial(r\rho v_r)/\partial r + \partial(\rho v_\theta)/\partial\theta + (\partial(\rho v_z)/\partial z)\, r = 0$

Or, $\partial\rho/\partial t + (\partial(r\rho v_r)/\partial r)/r + (\partial(\rho v_\theta)/\partial\theta)/r + \partial(\rho v_z)/\partial z = 0$

in the volume element, what is the volume element? r del r is one, del  is another, del z is another. So, these are the three r's or three boundaries in all three directions. So, volume element is r del r del  del z and the change in density with time is the accumulation that is del rho del t right del rho del t r del r del  del z right.

So, now, if we equate that in minus out equal to accumulation that was our basic equation, right. So, in minus out and dividing all by this term del r del  del z right both right hand side and left hand side, if we divide, then we get del rho del t into r, r del rho del t from here r del rho del t plus that minus has become plus. del del r of r rho vr plus del del  of rho v plus del del z of rho $v_z$ into r, right. So, if we divide all with r this r goes out this r goes out right and we can write del rho del t




plus del del r of r rho $v_r$ over r divided by r plus del del  of rho $v_\Theta$ divided by r plus del del z rho $v_z$ or del del z of rho $v_z$ is equal to 0 right. So, this is the cylindrical coordinate equation of continuity in the final form in r theta z coordinate. We have arrived at this point that del del t of rho or del rho del t plus del del r of rho $v_r$, r rho $v_r$ rather, del del r of r rho $v_r$ plus del del Θ of rho $v_\Theta$ divided by r plus del del z of rho $v_z$ this is equal 0 It is for r theta z coordinate, right.

Now, continuity equation in spherical coordinate where the coordinate is r Θ Φ, we are not going to derive it, we are giving you the final form and the final form is like this. ok. Before, again doing it, let me highlight on this that on the cylindrical coordinate, the continuity equation, we have arrived at equal to this equation that del rho del t plus 1 by r. del del r of r rho $v_r$ plus 1 by r del del Θ of rho $v_\Theta$ plus del del z of rho $v_z$, this is equal to 0. And this is the cylindrical coordinate continuity of equation form.

**Continuity equation in spherical coordinate**

$\partial\rho /\partial t + (\partial(r^2\rho v_r) /\partial r)/r^2 + (1/r\,\mathrm{Sin}ø).\ \partial(\rho v_ø\,\mathrm{Sin}ø) /\partial ø + (1/r.\mathrm{Sin}ø).\ \partial(\rho v_\theta) /\partial\theta = 0$

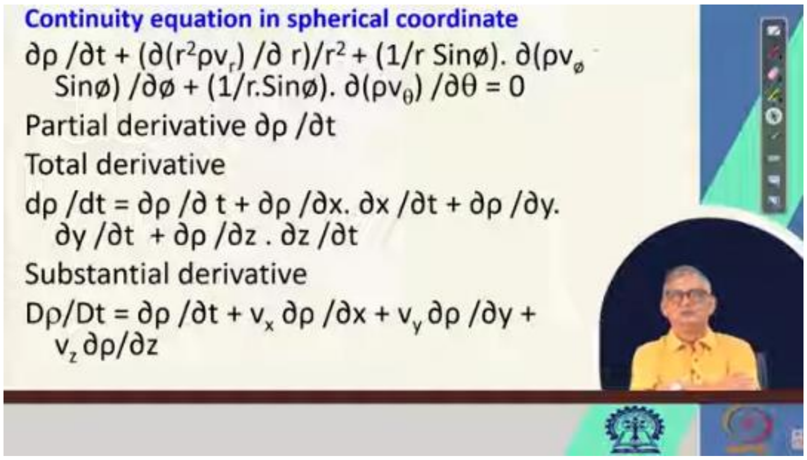
Partial derivative $\partial\rho /\partial t$

Total derivative

$d\rho /dt = \partial\rho /\partial t + \partial\rho /\partial x . \partial x /\partial t + \partial\rho /\partial y . \partial y /\partial t + \partial\rho /\partial z . \partial z /\partial t$

Substantial derivative

$D\rho/Dt = \partial\rho /\partial t + v_x \partial\rho /\partial x + v_y \partial\rho /\partial y + v_z \partial\rho/\partial z$




And we said that for the spherical coordinate will not derive it, because, it is much more complicated, because, obviously, there it is r Θ and ø right. So, the equation is complicated and the figure is also complicated. So, for that we are taking the final form, that is del rho del t plus 1 by r square del del r of r square rho $v_r$ plus 1 by r sin phi del del phi of rho $v_{phi}$ sin phi plus 1 by r sin phi

del del theta of rho $v_\Theta$ is equal to 0 right. We have already shown earlier that the partial time derivative is del rho del t, total time derivative is d rho dt that is equal to del rho del t plus del rho del x dx / dt plus del rho del y dy / dt plus del rho del z dz / dt and substantial time derivative we have also shown earlier is capital D rho capital Dt. This is equal to del rho del t plus vx del rho del x plus vy del rho del y plus vz del del z right.

So, what we can say is that, we have already defined partial time derivative, we have defined total time derivative, we have defined substantial time derivative, and we have made equation of continuity. in the previous classes we have shown what are the forms of equation of continuity both in analytical form as well as in vector form, we have shown. We have also explained its significance, right and now we have gradually developed or derived the equation of continuity, first in the x y z component, that is in the Cartesian coordinate. Next, we have derived, developed the x y z, then r theta z, that is in cylindrical coordinate we have derived.

We have drawn also the volume element, right. If you remember, there we had said that the two arcs of theta, that is del theta and r plus del r theta plus del theta, that arc to be equal. Otherwise, it would be very very complicated mathematics where, again we have to neglect the small quantities, right.

This also can be done, I am not saying that is not, but that is a part of mathematics, not the part of either engineering or any application, right. So, we have shown that and the third one which we have not derived, again we have said the reason why? Because, the coordinate is r theta phi, that is the spherical coordinate, and as we have seen the equation is also very complicated, because it involves lot of sin phi sin theta etc, These components, and instead of deriving it again we have shown you, the final form of the spherical coordinate.

And we also have said that we repeated, rather many times, we have repeated that derivative of a variable. and we have taken the variable to be say density, then, what is the partial time derivative and what is its significance? We have said the total time derivative, partial time derivative, we have given the notation as del rho del t. We have also said about the total time derivative, that is d rho d t small d rho small d t right, which we have also said that when somebody is looking at a point, fixed point, x y z, what is the effect of the velocity components while counting the densities, right.

And the third one we have also shown that the substantial time derivative that is capital D rho capital D t and capital D rho capital Dt, we have said it to be the substantial time derivative. And also we have shown the limiting conditions, that, what happens to the equation of continuity, that, when the density becomes constant, or for an incompressible fluid, because incompressible fluid density is constant. And if density is constant, then the substantial time derivative that is capital D rho capital D t, that becomes equal to 0, right. And also we have shown, if the fluid is incompressible, then the partial time derivative is also 0 that is del rho del t is 0.

And for that the very simple relation which is widely applicable in that form also we have established. So, I hope or we hope that equation of continue in its full form we have developed derived and established, right. So, next we shall go to the equation of motion, right. Obviously, equation of motion is much more complicated than that of equation of continuity, but the equation of continuity, that form is also utilized in developing the equation of motion, that is even more complicated, because,

for that we will be perhaps taking only the Cartesian coordinate for developing and we will take the final form of the cylindrical and spherical coordinate. And from this equation of continuity sorry from this equation of motion, right, a very very world famous equation in fluid flow. will come up, right. This we will say in the next class. So, thank you for listening.

We will meet again in the next class. Thank you.