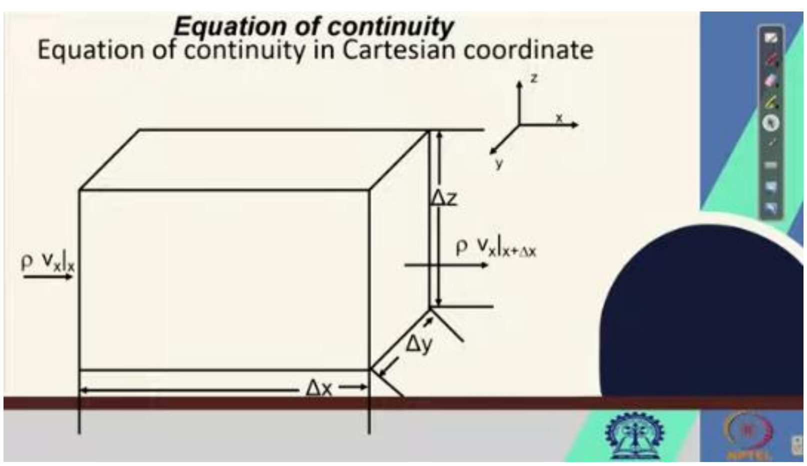
IMPACT OF FLOW OF FLUIDS IN FOOD PROCESSING AND PRESERVATION

Lecture08

LECTURE 8 : DERIVATION OF EQUATION OF CONTINUITY

Good afternoon, my dear students and friends. We have shown in the previous class what the different forms of the equation of continuity are, right? But we did not derive it. We have shown it from the principle of conservation of mass. Now, we will go to the derivation of the equation of continuity, right?

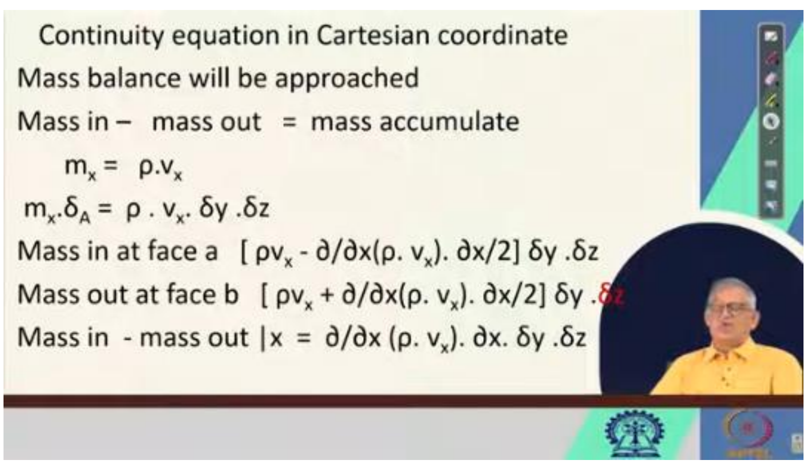
First, let us do it in Cartesian coordinates, where the coordinates are x, y, z. This is in the Cartesian coordinate system. We mentioned earlier, if you remember, that we take a volume, which is as infinitesimally small as possible or you can think of it. So, such a volume element we take as Δx, Δy, and Δz in the Cartesian coordinate system, right. And we also said, if you remember, that for derivation



or for the conservation of mass, we are not interested in what is happening inside the volume element. We want to see that whatever input of mass there is, the same output of mass is also there, then we are saying it to be the conservation of mass. So, in the same way, we take this volume element, Δx, Δy, Δz, and we take a momentum, that is ρv_x at the face x if we tell that to be x, and the other face if we tell that to be x plus Δx, then, momentum in is ρv_x at the face x, obviously, with area; the area will also come, and that is at the face at x plus Δx is ρv_x at x plus Δx. If that is true, then the continuity equation in the Cartesian coordinate system, we can do it on the basis of mass balance, right.

That means whatever mass is in, that mass is also going out; some mass is going out. So, mass in minus mass out should be mass accumulation, if there is any, right. So, mass balance says whatever mass has gone into the volume element, minus whatever mass has come out from the volume element. So, this must be equal to the mass accumulated in the volume element, and the volume element in the Cartesian coordinate system, we have taken as Δx, Δy, Δz, right.

So, mass in, we can write Or as such mass, we can write m_x is equal to rho into v_x, right. Rho is in kg per meter cube, v_x is in meter per second, then it should have been m dot x, but since it is very difficult, not difficult rather, it may be easy for you. For me to go into that, putting a dot on the top, m dot x, right, that is kg per meter square per second, right, that is mass flow rate per unit area or mass flux, right. So, we can say that m dot x or m_x, as we have written, whatever it be, is rho v_x, that is kg per meter square per second times the area, if it is there then it becomes kg per meter square per second into meter square, that is kg per second, that is rate of mass flow, right.



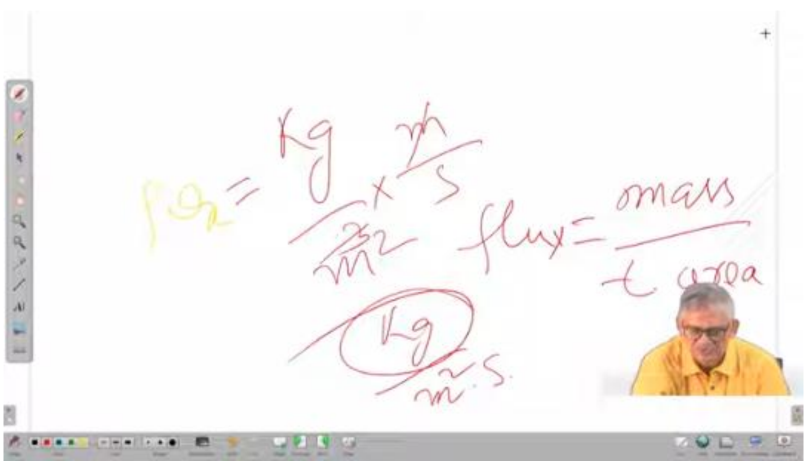
So, m dot x into delta A or m_x into delta A is rho v_x into the area of the x face. The x face area is Δy, Δz, right. That we can check by looking at the screenshot 1. This was, right, in the x. So, it is in the x, this is the face, ok. So, this face is del y del z. Similarly, this at x plus Δx, this face area is Δy, Δz, right.

delta A is rho v_x into Δy, Δz. Mass in at the face A, A is that which we have shown in the x direction, is rho v_x minus that was rho v_x has come in. Now, some has got accumulated. So, that minus that quantity, how much? Del del x of rho v_x into Δx by 2 if we consider, go to that point again. If we consider that this center is the point of interest.

Whatever it has come from this side to this point, whatever it has accumulated. So, this plus this accumulation plus whatever it is going out, right. So, mass in at the face A is rho v_x minus del del x of rho v_x into x by 2, because that is Δx by 2, times the area Δy, Δz. This is at the face a. Similarly, at the face out or b, that can be written as rho v_x plus what was the quantity, which it got at the face A deducted, that same direction is here as addition.

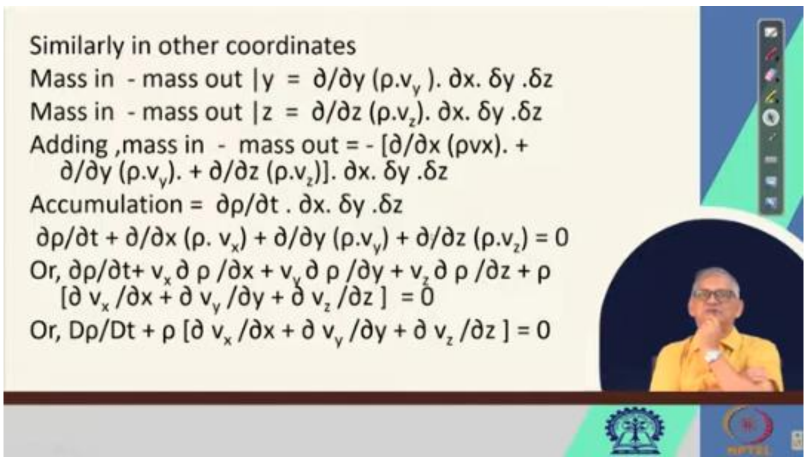
So, ρv_x plus $\partial/\partial x$ of ρv_x into Δx/2, times area that is Δy Δz, right. So, mass in minus mass out in the direction x we can write. This is nothing, but you see that $\partial/\partial x$ of ρv_x Δx/2, Δx/2 this goes out, right. Mass in minus mass out, no that remains, but ρv_x & ρv_x goes out, right, ρv_x & ρv_x goes out. So, we can write that.

Minus $\partial/\partial x$ of ρvx into Δx/2 Δy Δz from face A and this is also minus $\partial/\partial x$ of ρv_x into Δx/2 into Δy Δz, right. These two by addition we can write that $\partial/\partial x$ of ρv_x times. Δx/2 has made it Δx Δy Δz. There is a mistake which I hope you can identify and you can put it, right. I show it in another. Book that this mass in minus mass out, mass in minus mass out, this is equals to minus this sign is not there.



So, you correct it this was kept unchanged because I wanted you to identify whether it is right or wrong or is there any. Thing wrong, ok. So, mass in minus mass out at the face x is ∂/∂x of ρv_x into Δx Δy Δz. This Δx is because from Δx/2 plus Δx/2. So, that has become Δx no more by 2, ok. Then we can say.

That this was in the x direction. Similarly, in the y and z directions also you can write that mass in minus mass out at the face y that is also here with a negative ∂/∂y ρv_y Δx Δy Δz again a negative because similarly it is only cut and paste and changing with the notations. And at the face mass in minus mass out at the face z is minus ∂/∂z of ρv_z Δx Δy Δz this minus is missing in both the cases you see now it is corrected. So, adding mass in minus mass out in all the three directions we get here it is corrected minus ∂/∂x of ρv_x. Plus ∂/∂y of ρv_y plus ∂/∂z of ρv_z right into Δx Δy Δz because this Δx Δy Δz was everywhere in all three directions.



Let me show you the previous slide. Here it was Δx, Δy, Δz. So, ∂/∂x of ρv_x, ∂/∂y of ρv_y, ∂/∂z of ρv_z, that is what is in the next. Here also, it was Δx, Δy, Δz, and as I said, the mistake with the minus is taken care of here. And things become ∂/∂x of ρv_x plus ∂/∂y of ρv_y plus ∂/∂z of ρv_z times Δx, Δy, Δz, right? And accumulation, that can be said equal to, with time, whatever has been accumulated in the volume element. So, what is the volume element? Δx, Δy, Δz. And with time, what has been accumulated.

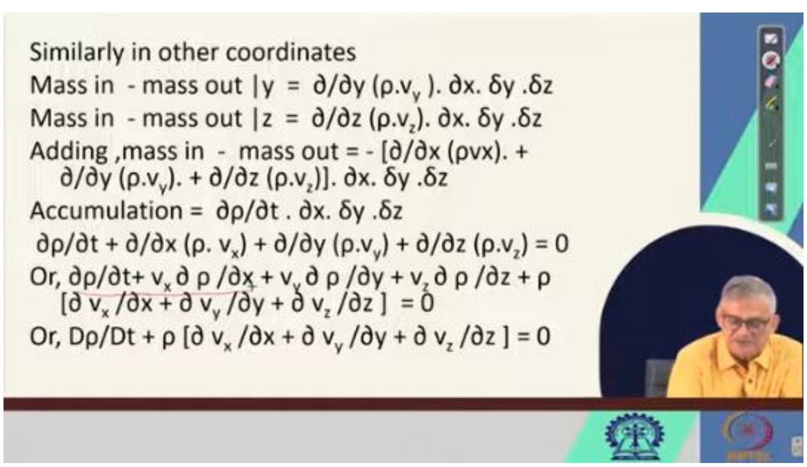
So, ∂/∂t of Δx, Δy, Δz, right? ∂ρ/∂t Δx, Δy, Δz. So, that is the volume, ρ is the variable. So, ∂ρ/∂t times Δx, Δy, Δz. So, accumulation and Adding all mass in and mass out, we get ∂ρ/∂t, right? Plus ∂/∂x of ρv_x plus ∂/∂y of ρv_y plus ∂/∂z of ρv_z equals 0 because this minus has now moved to the other side in the accumulation, which is why ∂ρ/∂t becomes plus. Okay, and all Δx, Δy, Δz, this volume got canceled out, okay. Now, it is canceled out, then we write ∂ρ/∂t plus ∂/∂x of ρv_x plus ∂/∂y of ρv_y plus ∂/∂z of ρv_z equals 0, right?

Now, $\partial\rho/\partial t$ earlier equation we have said that $\partial\rho/\partial t$ plus this can be written as v_x $\partial\rho/\partial x$ plus v_y $\partial\rho/\partial y$ plus v_z $\partial\rho/\partial z$ plus ρ into $\partial v_x/\partial x$ plus $\partial v_y/\partial y$. Plus $\partial v_z/\partial z$ is equal to 0. How did you do that? Any idea? I think you have read that u v method, right? Sorry.

So, if you have $\partial(uv)/\partial t$, then you can write u $\partial v/\partial t$ plus v $\partial u/\partial t$, isn't it? This is called the product u v method, right? Using this same method, we have done here that This $\partial\rho/\partial t$ or this cannot be shown; I do not know if it can be, but I have no idea how it is, yeah, okay.

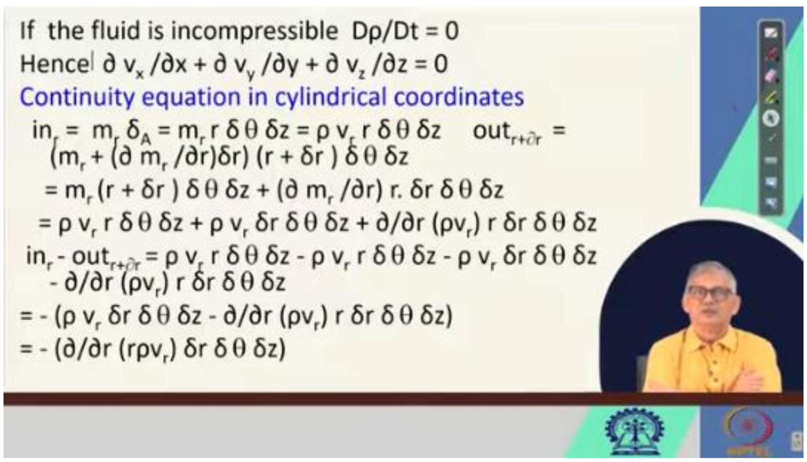
So, del rho del t says here you see del del x of rho v_x plus del del y of rho v_y plus del del z of rho v_z, if that be true. This is a product of rho v_z, this is a product of rho v_y, this is a product of rho v_x with the same derivative del x, del y, and del z. So, you can use that as the u v product, right? del del whatever x of u v means u del vx. Or u into rather del v del x plus v into del u del x, this is called the product method, right. So, you can do this and similarly, if you do that, then it becomes the product del rho del t plus v_x into del rho del x plus v_y into del rho del y plus v_z del rho del z. This is one, say u plus now the other one is constant. Earlier, v_x, v_y, v_z were constant, now rho is constant.

So, rho times del v_x del x plus del v_y del y plus del v_z del z, this equals to 0, right. Now, earlier when we had defined the substantial time derivative, if you remember, that substantial time derivative was del rho del t plus v_x del rho del x. plus v_y del rho del y plus v_z del rho del z equals to capital D rho capital D t, right. So, that is the substantial time derivative which we have seen earlier, right.



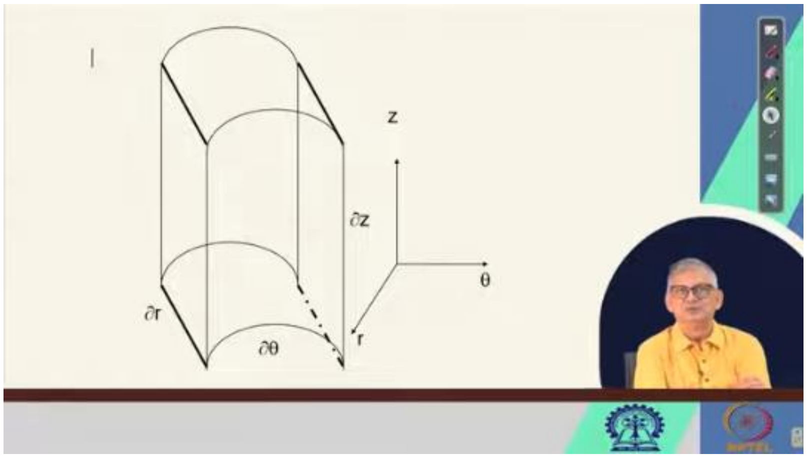
Now, this D rho D t plus rho del v_x del x plus del v_y del y plus del v_z del z, this becomes equals to 0, right. So, the substantial time derivative we can write, capital D rho capital D t plus rho del v_x del x plus del v_y del y plus del v_z del z equals to 0, right. Now, if the fluid is incompressible, then D rho D t is constant, which we have also seen earlier, that is

capital D rho capital D t is constant or 0, rather not constant, is 0, right. Hence, we can write from the previous equation, del v_x del x plus del v_y del y plus del v_z del z equals to 0. You remember in earlier class we had shown it under this identical situation that if the fluid is incompressible, then density is constant, then capital D rho capital D t is constant, also del rho del t is constant, right. So, this is what we have derived for Cartesian coordinates.



Similarly, we can also derive Because this is in Cartesian coordinates, meaning in x, y, z coordinates, right. Now, we have 5 more minutes in this class. I do not know whether we can finish it or not, but let us try. So, in cylindrical coordinates, that is r, theta, z In a similar fashion as we have done in the Cartesian coordinates x, y, z, we can write. Only here the areas will be different, right, for which I think I have taken, yeah, that area which we are supposed to take is like this, ok.

So, del r is the one variable Theta is the other direction, and del z is in the third direction, right. So, from there, keeping this pictorial view in mind, r, theta, z. So, if we take, that is, mass in at r is m dot or m_r del A, that is m_r r del theta del z, that is the area. r del theta del z is the area into m_r, that is m_r is rho v_r r del theta del z, that is in at the face r. Similarly, out at the face r plus delta r is equal to m_r plus del del r of m_r into del r. Earlier, in the Cartesian coordinates, we have taken the point at the center. Here, we are taking it at the end, right.



So, that is at r. That is why it is coming that out at r plus delta r is equal to m_r into A plus del del r of m_r into del r into the area r plus del in del theta del z, right. r plus del r is the other arc, right. This arc is r del theta, and that arc is r plus del r del theta del z, ok. So, now this is equal to m_r into r plus del r into del theta del z plus del m_r del r

into del r del theta del z. So, this can be written as rho v_r into r del theta del z if we expand it. Plus, instead of m_r, if we write rho v_r plus rho v_r del r del theta del z plus del del r of rho v_r r del r del theta del z. This is the in at r, right, this is in at r, ok. Now, out at r, r plus del r, yes. So, in we have already said that is m_r r del theta del z, that is rho v_r r del theta del z, that is in r. Out r is this one. So, in r minus out r is r in r minus out r plus del r is rho v_r r del theta del z minus rho v_r r del theta del z minus rho v_r

del r del theta del z minus del del r of rho v_r r del r del theta del z. Right. This on simplification can be written as minus rho v_r del theta del z minus del del r of rho v_r r del r del theta del z, right, which can be simplified as del del r of r rho v_r del r del theta del z. Earlier, if you remember, we had done del del x of u v we had written as v into del u del x plus v into del u del x sorry, rather u into del v del x right. The reverse ends right this is the expanded form.

In the contracted form, it is del x del u v right. So, the same thing can be written here. Same thing can be written here that in the expanded or contracted form that minus del del r of r rho v_r del r del theta del z, right. This is for the r coordinate, but I think our class time is over.

And I hope in the next class we will continue it for the conclusion of the r theta z coordinate, ok. So, we come to the conclusion. Thank you.