IMPACT OF FLOW OF FLUIDS IN FOOD PROCESSING AND PRESERVATION

Lecture07

LECTURE 7 : EQUATION OF CONTINUITY IN DIFFERENT FORMS

Good evening, my dear students and friends. We are in the development of the equation of continuity of fluids, right? We have defined what a fluid is, we have defined how the equation of continuity is developed, and on what principle, that is, the conservation of mass. And also, we said that we begin with Newton's second law, right. And from there, we are continuing to develop the equation of continuity, and it is used in the processing industries, right.

So, in that case, we say, that the equation of continuity can be expressed in Cartesian coordinates, that is, x, y, z, as ∂ρ/∂t. This is equal to -∂/∂x (ρv_x) + ∂/∂y (ρv_y) + ∂/∂z (ρv_z), right. It is one form of the equation of continuity, right. And if we explain the significance, what is the significance of this form of the equation of continuity?

$$\frac{\partial \rho}{\partial t} = -\left[\frac{\partial(\rho v_x)}{\partial x} + \frac{\partial(\rho v_y)}{\partial y} + \frac{\partial(\rho v_z)}{\partial z} \quad ...(1)\right]$$

We have said earlier that ∂ρ/∂t was with respect to x, ∂/∂x (ρv_x) with respect to y, ∂/∂y (ρv_y), and with respect to z, ∂/∂z (ρv_z). The summation of all these with a negative sign because this is ∂ρ/∂t, ∂/∂x (ρv_x) + ∂/∂y (ρv_y) + ∂/∂z (ρv_z) = 0. So, we have taken it to the other side and written it with a negative sign. The significance is that the rate of change of density, because our variable here is density, that rate of change of density at a fixed point, that is, x, y, z, results from the changes in the mass velocity.

Mass velocity is ρv_x or ρv_y or ρv_z. I repeat, the rate of change of density at a fixed point that results from the changes in the mass velocity vector →ρv, or in other words, that results from the changes in the mass flux in all directions. I hope you

$$\frac{\partial \rho}{\partial t} = -(\nabla . \rho v) \ldots (2)$$

know what flux is, right? Heat flux, mass flux, momentum flux, right. Flux, you see, we are saying ρv_x as mass flux.

Why? ρ is in kg/m³, and v_x is in m/s, right. So, mass is kg per meter cube, and in the numerator, ok. Let us go to that. That rho v_x, ok.

I think this color is better. It is equal to kg per meter cube into meter per second, right. So, meter cube goes out; it becomes meter square. So, kg Per meter square per second, right? That is the flux of mass, that is kg. The flux is nothing but any flux. The unit here is mass per unit time per unit area, right.

So, any flux that can be said To be equivalent to that parameter per unit time per unit area, right. So, mass flux we can say is kg per meter square per meter second, right. So, the The significance of this form of the equation of continuity is that it is the rate of change of density at a fixed point that results from the changes in the mass velocity vector rho v means all directions v_x, v_y, v_z, or in other words, it can also be said that it is

Resulting from the changes in mass flux in all directions. Mass flux, you have just said, is mass per unit area per unit time, right. So, any flux is that heat flux is also like that heat per unit area per unit time. Similarly, mass flux. Similarly, momentum flux. Any flux is that flux is of that Parameter per unit time per unit area, right.

So, then we go to another form of the equation of continuity in terms of vector, right. So, it is del rho del t that is equal to minus v→ vector into rho v. So, the vector rho v is the mass flux and its divergence. So, divergence v, which we have shown in the equation. The divergence has a simple significance: it is the net rate of mass flux, a flux

Per unit volume net rate of mass influx, right, per unit volume. It describes the fact that the rate of increase of density within a small volume element fixed in space is equal to the rate of mass influx to the element. divided by its volume, right. So, if we look at it, I repeat that it describes the fact that the rate of increase of density within a small volume element fixed in space is equal to the net rate of mass efflux, that is efflux, right. That is the net rate of mass influx to the element divided by its volume, right. So, with this, the second way of expressing the equation of continuity in terms of vector is expressed. Now, another way of expressing it is in the form that capital D rho capital D t, right? We said capital D operand that was one was the time derivative del rho del t, another was substantial rather total time derivative that is D rho D t. and the third one was the substantial time derivative that is capital D rho capital D t. So, that is equal to del rho del t, this we have shown also, plus del rho del t plus v_x del rho del x, v_y into del rho del y plus v_z into del rho del z, right. This we have derived and shown that the substantial time derivative capital D rho capital D t is nothing but del rho del t plus v_x del rho del x plus v_y del rho del y plus v_z del rho del z where obviously, D rho D t is called. substantial time derivative of density. Its significance is like this: that derivative for a path following the fluid motion. We said that canoe people are rowing with a canoe, right? They row in the direction of the flow of the stream; it is not the other way that the direction of the stream is this and you are moving this way with a canal.

$$\frac{D\rho}{Dt} = \frac{\partial \rho}{\partial t} + v_x \frac{\partial \rho}{\partial x} + v_y \frac{\partial \rho}{\partial y} + v_z \frac{\partial \rho}{\partial z} \quad \ldots (3)$$

Yes, you can move in this way, but with some other thing like you have to have any motorized vehicle, right? Otherwise, you cannot. So, if you want to move along the stream of flow then that is the significance of the substantial time derivative that is the time derivative for a path following a fluid motion or it can also be said in the form that the rate of change of density can be seen by an observer when floating along with the fluid. I told you that you are observing.

Now, you are observing from the canal flowing along with or moving along with the velocity of the stream, right. Obviously, the velocity of the stream will have its velocity component that is v_x, v_y, v_z and when it is coming in totality for finding out the substantial time derivative, then it is the partial time derivative del rho del t plus velocity component v_x into the density at that point x del rho del change of density at that point del rho del x plus v_y velocity component of the y Y direction rather times the density difference or change in density in that direction, that is del rho del y, plus the z, that is velocity component in the Z direction into del rho del z. It is the density component in the z direction, right.

So, both are explained, and the definition of the time derivative of substantial time derivative. That is d rho dt is also, as said many times, as substantial time derivative of density because density we have taken as the variable. Significance also we have stated and explained that the time derivative for a path following the fluid motion or the rate of change of density that can be seen. By an observer when floating along with the fluid, that is the substantial time derivative, okay. Next, the fourth one is in the again in terms of the vector component, that is the capital D rho capital D t is equals to minus rho.

The divergence of V times velocity that is the equation of continuity in this form describes the rate of change of density as seen by an observer floating along with the fluid, right. So, the equation of continuity in this form describes the rate of change of density as seen by an observer. Floating along with the fluid, right. So, these four forms are known as very much in the equation of continuity. Any form of this can be used afterwards or subsequently, and you can use them.

$$\frac{D\rho}{Dt} = -\rho(\nabla . v) \ldots (4)$$

And when as the form required in the way as it is explained. Now, another thing which is very, very important is that a typical idealized condition such as if the density is constant. That is d rho del t capital D rho capital D t is 0. If density is constant, obviously with time it has no change, is it not? Earlier also, in one of the examples, we had said that if the density is constant.

Here also we are saying if the density is constant. The moment density becomes constant; it cannot change with time. That is why substantial time derivative D rho D t is 0. Hope you understand this. If that is true, then you see.

If that is true, then not this form of equation, but this form of equation capital D rho D t right is equal to del rho del t. Since rho is constant, so both substantial time derivative and partial time derivative are 0, right. So, capital D rho capital D t is 0 that is equal to del rho del t that is also 0 plus v_x del rho del x plus v_y del rho del y plus v_z del rho del z equals to 0. That is the velocity components and its localized density effect this product. In all directions, r equals to 0 which we have arrived at here that del v_x del x del v_y del y del v_z del z is equal to 0. Let me show you, let me show you again this.

Oh, yes. So, density is said to be constant, right. Since density is said to be constant, then the velocity component in the x direction, velocity component in the y direction, and velocity component in the z direction, right. So, their summation is becoming equal to. So, it becomes then del v_x del x plus del v_y del y plus del v_z del z this is becoming equal to 0, which we have shown here.

So, under typical condition where density is constant, that is capital D rho capital D t. is equal to 0. And again, for compressible or rather incompressible fluid, you remember we had said earlier that for incompressible fluid, the density does not change appreciably. With change in pressure, right. Density can change with pressure, but if the density is not varying with pressure appreciably, that is the change in density is insignificant with the change in pressure.

So, under that situation. We can say del v_x del x del v_y del y plus del v_z del z equals to 0, right. We just want to do to and fro a little, right. Here you see this was our first one del rho del t is equal to. Obviously, that fourth one I am doing to and fro because here the first one density is constant then our D rho D t that becomes 0, then the left side is 0 right side remains that is one possibility.

Second, if it is incompressible then again density becomes. Constant again, then we can say this that again density being constant del rho del t is 0. And this equals to minus rho is off because that is 0; rho is constant and comes out, then it becomes 0. Then, del v_x del x del v_y del y del v_z del z equals to 0, right? This is another form of the equation of continuity under some imposed conditions, and that imposition of the condition is also very good.

That is, if it is an incompressible fluid, then only the density becomes constant, right? Then only del rho del t becomes 0. Then only that rho comes out as a constant from the derivative, and dividing it also goes out. That negative sign goes out because it is equal to 0. Then we get this unique form: del v_x del x plus del v_y del y plus del v_z del z equals to 0. Mind it, it is again another form which may be used in the future for some derivative or some derivation or application because we have done the equation of continuity, right?

We have not done the equation of motion, but again, the equation of continuity: I do not remember whether I have put this in Cartesian coordinates, right? In cylindrical coordinates, what is the equation of continuity? I do not remember whether I have included it in the course or not. Subsequently, we will see whether it is there or not. However, then we can wind up like this: for Cartesian coordinates, because we started with that x, y, z, we have defined some derivative terms. We have also defined the continuity term, and we have defined the other term, that is, rho v, which is your momentum. Also, you have defined flux, right?

Similarly, momentum flux will also be coming sometime. That is momentum per unit time per unit area, right? Mass flux is mass per unit time per unit area, right? So, whatever flux is there—heat flux is heat per unit time per unit area. So, any flux is per unit time per unit area, okay?

So, kg per meter square per second per meter square per second—that is the flux. And we have established in Cartesian coordinates all four forms of course, two in the normal analytical form and two in the Vector form, right, and its significance. And we have also established the typical condition that velocity is constant, then the substantial time derivative capital D rho capital

That became equal to 0, and we also said if the fluid is incompressible, then for an incompressible fluid, again, density becomes constant. And under that situation, this equation of continuity in form 1 is transformed del v_x del x del v_y del y del v_z del z or plus, of course, that 0 equal to del v_x del x plus del v_y del y plus del v_z del z, right. So, with this, we conclude this class here, and the equation of continuity in Cartesian Coordinates That is established, and we have made you understand, okay.

Thank you.