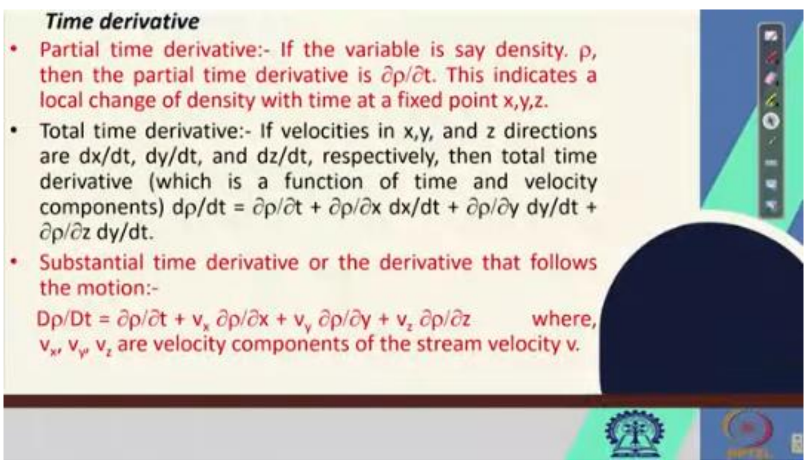
IMPACT OF FLOW OF FLUIDS IN FOOD PROCESSING AND PRESERVATION

Lecture06

LECTURE 6 : EQUATION OF CONTINUITY TO BE CONTINUED

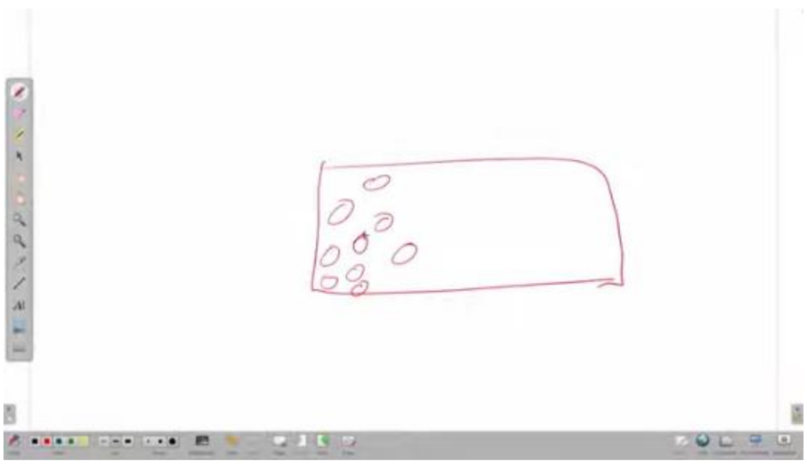
Good afternoon, my dear students and friends. Actually, we had started the equation of continuity in the flow of fluids. Now we are in our normal class; no preamble, it is a requirement for the course. So, we came up with the Definition of some of the time derivatives, right? And we said in the previous class that in the next class, we will try to explain them, right?

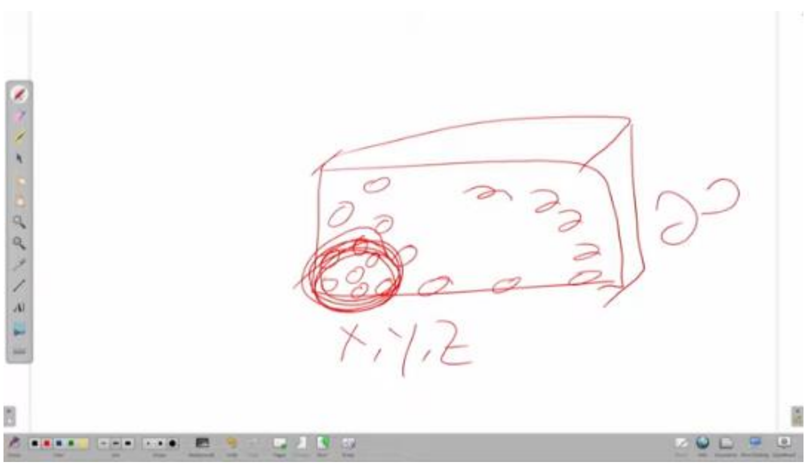
So, this is the sixth lecture on the equation of continuity and its use in processing; it is a continuous class. So, the time derivative, which we defined, is right. So, it is the derivative with respect to time, right, and that is called the partial time derivative. Now, if we take density as a variable, where you are taking density as a variable, density can be said in many different ways, in numerous cases, right.



We only know that volume is by volume at a particular temperature and pressure, right. So, that, and mass. This is not the all-time density. In many cases, we say that population density, concentration density, and many other ways are used to express density. So, that is why if we take density as a variable, right, then the partial time derivative that is defined as rho del t, right, del rho del t why it is called time derivative because it is a function of time of the variable density, but we have to keep in mind that the position is fixed at x y z, right, at x, y, z. I do not know whether I can use it or not. Say, if we take such a volume element, oh my goodness—one minute after, a long time I have been using it. So, that is why I might be small; yes, if we take this as a volume element. right. And say, 'People are here, here, here; these are the heads,' okay.

Like many heads are there. I am not going to do all of it, okay. Now, if we want to say the density of people here at this location where x, this is the two-dimensional and this is the three-dimensional. Where x, y, and z are fixed because we have fixed the location. So, that ∂ρ/∂t, which we said was the partial time derivative, is fixed because we have set the location (x, y, z).

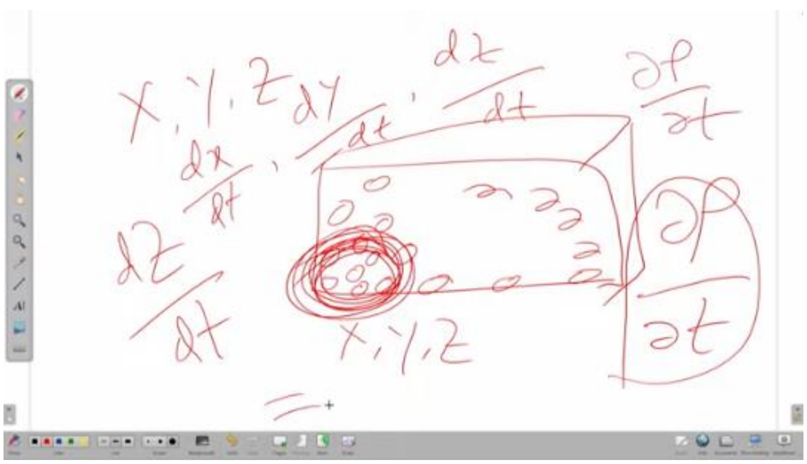




The time derivative, which is a function of time, the variable density as a function of time, is called the partial time derivative where we have specialized and specified the location x, y, z, right. Let us go back when we come to another. That is called the total time derivative, right. So, the total time derivative is that if the velocity is in the x, y, and z directions. So, we have a stream, right? You can imagine the Ganges flowing, right.

So, it will have. Say, for a particular fish or fishes, they will have velocity in the direction of x, in the direction of y, and in the direction of z. So, in all three directions, they will have velocity components, right, and these components are dx/dt, dy/dt. And dz/dt, these are the velocity components, right. So, we can say that the time derivative, which is a function of time and velocity, right, which is a function of time and velocity components, right. They can be defined as $d\rho/dt$.

Is equal to $\partial\rho/\partial t$ plus, rather $d\rho/dt$ is $\partial\rho/\partial t$ that is localized, and now with that localized condition, what are the effects of the x, y, z components? The velocity component is $\partial\rho/\partial x$ times dx/dt, $\partial\rho/\partial y$ times dy/dt, plus of course $\partial\rho/\partial z \times dz/dt$. There was a mistake; it should have been dz/dt. Sorry, it should have been dz/dt. I can use this right: dz/dt. So, we have x, y, z components, right. So, dx/dt is the x component, dy/dt is the y component, and dz/dt is the z component, and our time derivative was $\partial\rho/\partial t$, right. So, we are saying that this $d\rho$, we are saying that $d\rho/dt$ is equal to $\partial\rho/\partial t$ plus $\partial\rho/\partial x$ into dx/dt plus $\partial\rho/\partial y$ into dy/dt.

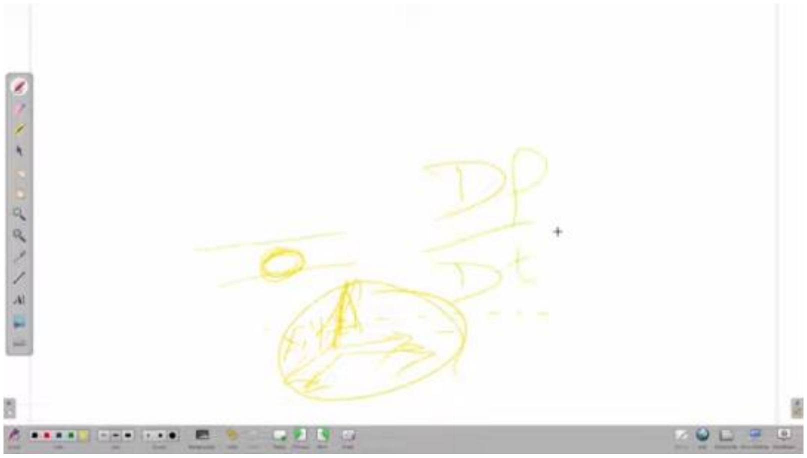


$\partial\rho / \partial z$ into dz/dt, that is the product of the individual velocity component. Some of the individual velocity components and that local x, y, z location, right? That location is what $\partial\rho / \partial t$ is. So, this is the total time derivative. Another one, which we have defined as the substantial time derivative, right. This substantial time derivative, or the derivative that follows the motion, which can be said as $D\rho / Dt$, is an operand; D is an operand. Like "small d" is an operand, "∂" is an operand.

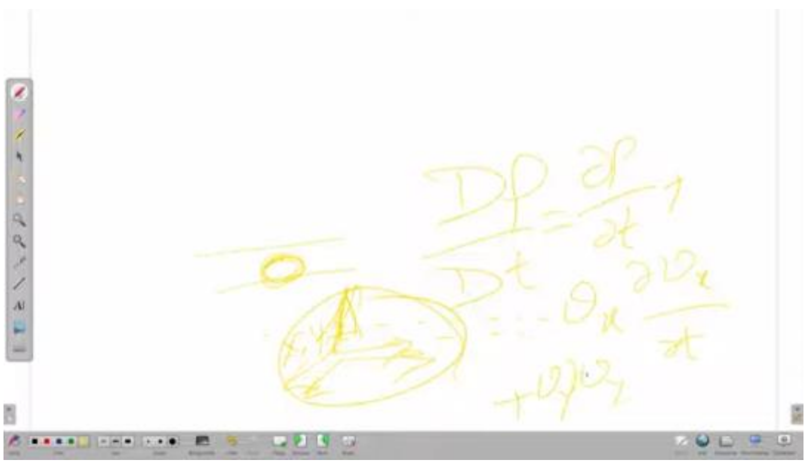
So, capital D is an operand, like small d, and ∂ are operands, mathematical operators. So, $D\rho / Dt$ is the substantial time derivative that is equal to $\partial\rho / \partial t$, that is the time derivative, which we said is the partial time derivative, plus the individual velocity component effect: v_x in the x-direction, v_x into $\partial\rho / \partial x$, plus v_y into $\partial\rho / \partial y$, plus v_z into $\partial\rho / \partial z$. Obviously, v_x, v_y, v_z are the velocity components of the stream velocity v, right. Now, if we explain this like this, we make a change; we make a change, perhaps.

So, what we say is that if you are standing on the bridge, this is a bridge, right, and you are standing here. Down below, in the river, fish are moving, right. So, fish are moving in all directions; this is the x-direction, this is the y-direction, and this is the z-direction in three-dimensional space. You are asked to stand; you are standing. You allow one canoe, you know, canoe people use to row, right.

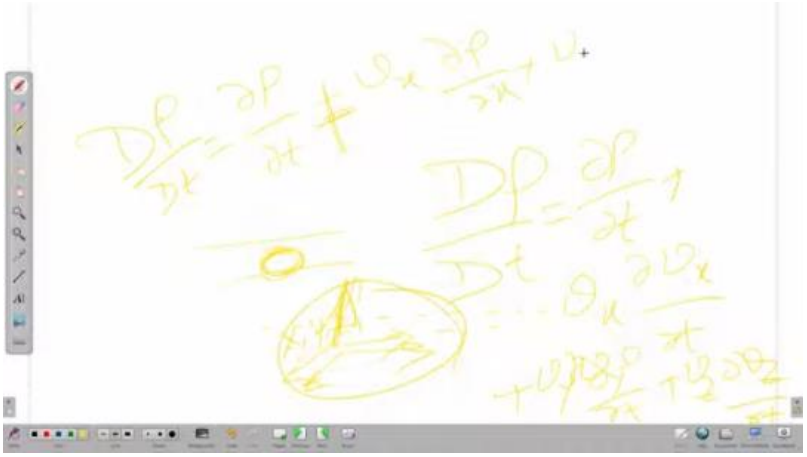
So, and that moves along the velocity stream only; it does not move all around, right. So, if it has a velocity that the stream has a velocity in this component v_x. It will also have a velocity component v_y, and it will also have a velocity component v_z, right. What is the impact when you are observing from here at the location (x, y, z) What is the impact of the density of the population of, say, fish? That is the total or rather substantial time derivative, that is $D\rho / Dt$, right.



So, it takes care of the partial time derivative, that is $d\rho/dt$, plus it takes care of the velocity component v_x $\partial v_x/\partial t$, right, plus v_y $\partial v_y/\partial t$. ∂t + v_z $\partial v_z/\partial t$. This is the substantial time derivative. ∂v_y $\partial\rho/\partial z$, that is what we said. That is what we said, v_y $\partial\rho/\partial y$. So, I write it clearly: $\partial\rho/\partial t$ is equal to $\partial\rho/\partial t$, or $d\rho/\partial t$ is equal to capital T. Sorry, this is plus, not equal to. This is

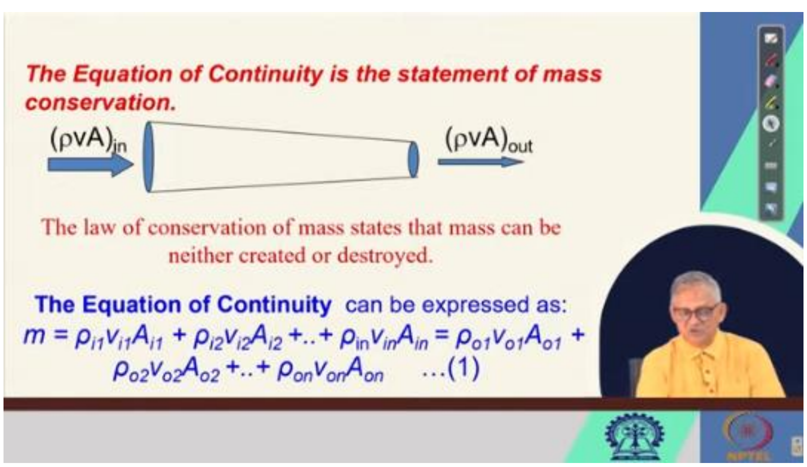


v x del rho del x, what is the velocity, what is the effect of velocity on this density right plus v y del rho del y plus v z del rho del z right. So, this is what. We have shown that the substantial time derivative is okay. So, what you are seeing is localizing your point (x, y, z), right? But as if you are moving along the flow of this stream—whatever it may be—it cannot be in all three directions; otherwise, it becomes turbulent. There will be human cries. In the water, or in the pond, or in the river.



So, it is that you are moving like a canoe is moving along the flow of the fluid, right. So, in that case, all other components of the velocity and concentration ($\partial\rho/\partial x$), ($\partial\rho/\partial y$), and ($\partial\rho/\partial z$) at that point. Plus that local time derivative $\partial\rho/\partial t$, this is equal to capital $D\rho$/capital Dt, okay. Then we come to the equation of continuity, which is the statement of conservation of mass. This is a statement of mass conservation or conservation of mass.

So, we have taken one, we have taken another, yeah, I hope you can see that movement, okay. Conical, conically shaped, one unit where inlet is ρvA inlet. And outlet is ρvA outlet, right. And the law of conservation of mass is a test that mass cannot be created nor can it be destroyed; right, neither can it be created nor can it be destroyed; the mass is conserved and remains constant, right. So, the equation of mass or the equation of continuity, based on the conservation of mass, can also be expressed as, if we take m as the total, that is equal to $\rho_{i1}v_{i1}$, that is inlet at point 1, into area A at point 1, A_{i1}.



Plus, rho_{i2} v_{i2} A_{i2} that is at point 2, like that plus dot dot dot dot if we go up to rho_{in} nth term, v_{in} nth term, A_{in} nth term. So, rho_{in} v_{in} A_{in} Right. This is equal to rho outlet rho_{o1} v_{o1} A_{o1} plus rho_{o2} v_{o2} A_{o2} plus similarly we can go up to nth term that is rho_{on} v_{on} A_{on}: This is equal where there has been no creation or destruction of mass.

So, that mass total m is whatever mass got in that mass has come out we have not seen what has happened inside. So, conservation of mass does not say whatever has been done inside; it states that matter cannot be created or destroyed in a closed system. Somehow, it is getting a size problem. However, it is visible that m is the mass flow. rate that is in kg per meter second kg per second rather rho is the density, that is kg per meter cube, and v is the speed that is in meter per second, A is the area in meter square.

Now, with uniform density, if we take the density to be uniform for all points, we have i1, i2, in, o1, o2, on. So, if all the points density is uniform then rho comes out right. Then we can write that the uniform density this can be modified to if instead of m if we write it to be q is equal to the same it is mass flow rate right q that is mass flow rate equal to v_{i1} A_{i1}, v is in meter per second a is in meter square that is meter cube per second. So, now, from mass flow rate we are coming to volumetric flow rate that is Q, right.

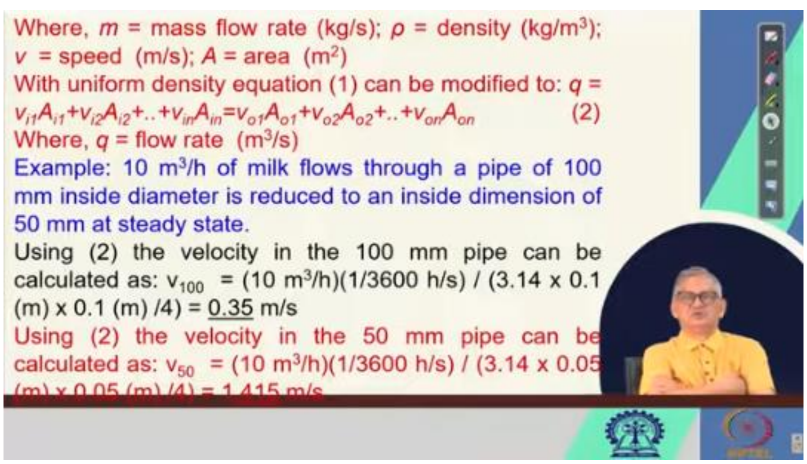
So, v_{i1} A_{i1} plus v_{i2} A_{i2} plus dot dot dot up to v_{in} A_{in} right. This should be equal to outlet in terms of volume per unit time right meter cube per second that is v_{o1} A_{o1} v_{o2} A_{o2} plus v dot dot dot v_{on} A_{on} right. Obviously, Q is the volumetric flow rate, right.

So, that is in meters cubed per second. So, this is what we know as the equation of continuity, which is based on the conservation of mass, right. We can do problem-solving also; like we have 5-6 more minutes, OK. So, it can be done that 10 cubic meters per hour of milk flow through a pipe with an inside diameter of 100 millimeters.

is reduced to an inside diameter of 50 millimeters per piece at a steady state, right. I repeat: 10 cubic meters per hour, that is the volumetric flow rate of milk flowing through a pipe with an inside diameter of 100 millimeters, and this pipe is reduced to an inside dimension of 50 millimeters at steady state, right. So, use volumetric flow rate instead of mass flow rate at the inlet and outlet. So, using the previous volumetric flow rate equation, we can say that the diameter is 100 millimeters, velocity in the 100 millimeter pipe, that is equal to v_{100} and that is equal to 10 meter cube per hour, divided by π by 4, right. π by 4, yeah, you see it is 10 meter cube per hour divided by meter cube to second 1 by 3600, hour to second divided by π that is 3.14 into 0.1, 0.1, is what it is given diameter right 0.1. So, it is in meter 100 millimeter, it was right, into 0.1 divided by 4 that is π d square by 4, right. And this amounts to 0.35 meters per second; that is, the velocity at 100 millimeters is 0.35. meters per second.

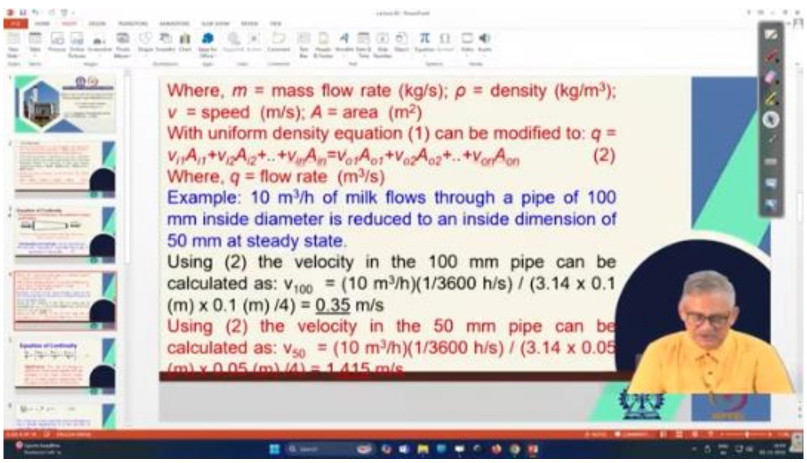
Now, if we use the same equation for the 50 millimeter pipe, right, we can write at 50 millimeters. This is equal to a flow rate of 10 cubic meters per hour, which is converted into seconds by multiplying 1 by 3600 (hour to second) divided by the area, which is 3.14 × $(0.05\ m)^2$, over 4. So, that is equal to 1.415 meters per second. From point B (100), where the velocity was 0.35 meters per second, very low velocity.

We have come to the point where it is 1.415, a very high velocity compared to 0.35, but why? Because you have reduced the diameter of the pipe, right. Like, I am putting my words with a huge opening, and if I put my words in a small opening, you will see the difference, right? It is more so if you blow air with your mouth. With a bigger mouth and with a smaller mouth, you will find the difference.



In the former case, you will not be able to sense any or minimal airflow; and in the second case, you will see a blast of air through your mouth, which you are able to make because you have reduced the diameter of your mouth opening, right. So, it is also true. So, this is how, from the equation of continuity, we can determine the velocity, provided that the density is uniform, and you know the diameter of the pipe at the inlet and outlet. Points where you want. So, if you know the diameters at those points, you can find the velocities very easily.

This is a very trivial matter rather than a problem, which anybody can do anywhere, right. Our time is also going up for this typical class; the equation of continuity is different from this one. We will try to develop it in the next class, and at least, we have established that based on the conservation of mass; the equation of continuity can be derived. Only the thing is that the volume element which you are considering is low, then it does not matter whether you are coming from or what you are.



So, with this, we complete this class. Thank you.