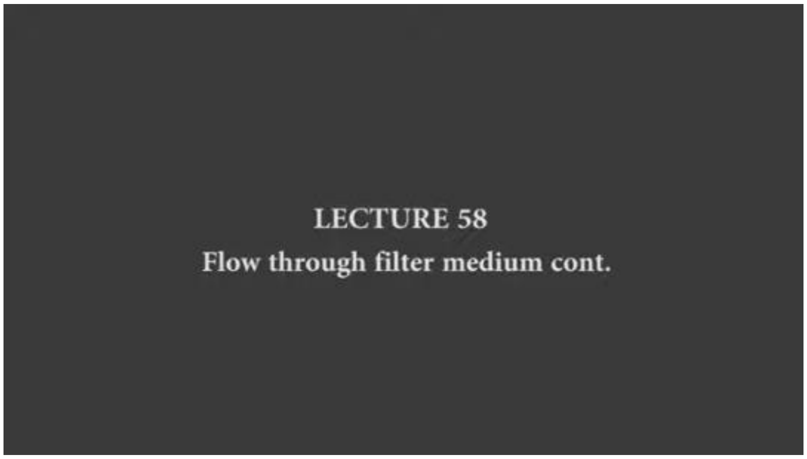
IMPACT OF FLOW OF FLUIDS IN FOOD PROCESSING AND PRESERVATION

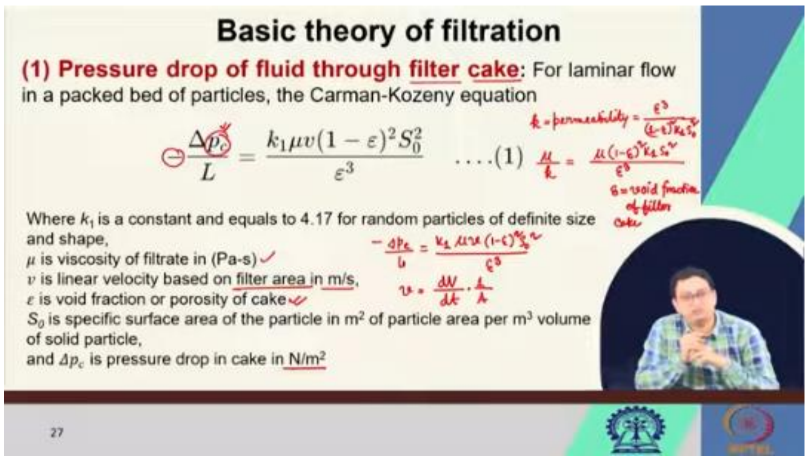
Lecture58

LECTURE 58 : Flow through filter medium cont.

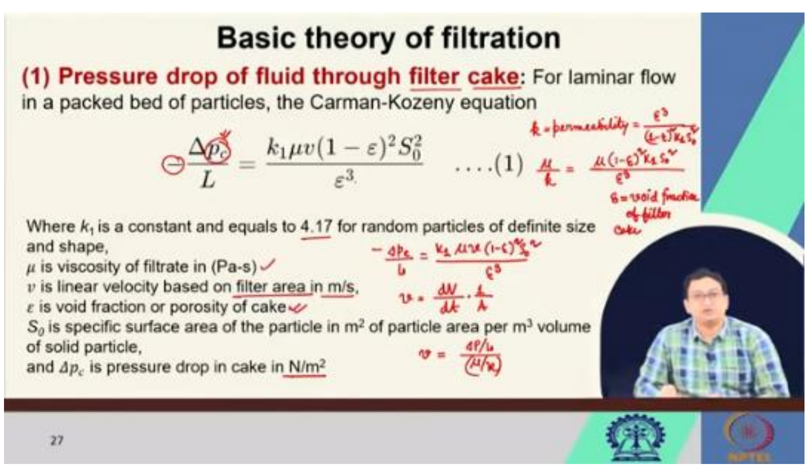
Welcome back, friends. So, in the week of the 12th, we have been discussing the flow through a filter medium, and what we have looked at so far is trying to develop the pressure drop equation or basic filtration equation for a cake filtration technique. That means the slurry is flowing across a filter medium, and we are getting the filtrate out. We are trying to look at what the filtration equations would be. So, what we have developed so far—if we can go back, we can recall—is that we have explained that



flow through the porous medium obeys Darcy's law. The porous medium means it is, you know, a porous filter medium and that is actually, you know, explained by this basic equation. You have a basic equation where you have a filtration velocity that is driven by the pressure gradient divided by the resistance, OK? And this μ is the viscosity of the liquid or filtrate; K is the permeability of the porous medium. The permeability of the porous medium can be written in terms of the void fraction, which is ε^3 divided by $(1 - \varepsilon)^2$ multiplied by K_1 multiplied by $S_0^2\varepsilon$. So, K_1 can take the

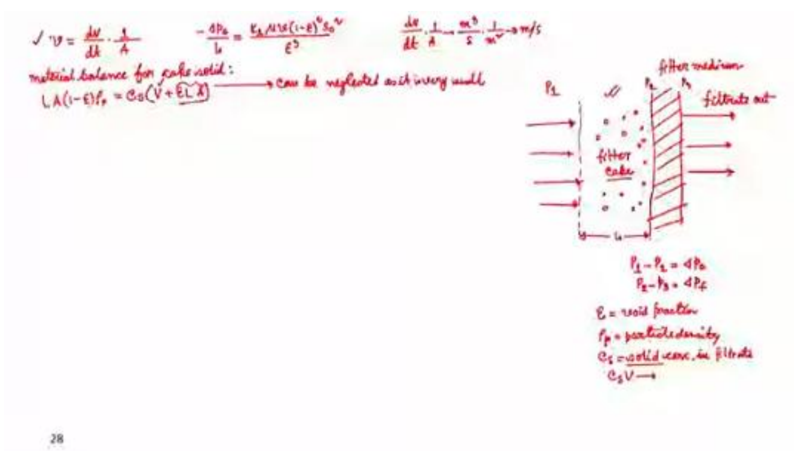


you know, values—as I said over here—4.17 for random particles of definite size and shapes. Sometimes it can also take the value of 5, and S_0 is the specific surface of the particle per unit volume of the solid particle. So, finally, if you look at the pressure drop across the filter cake, which is denoted by the suffix 'c,' then $\Delta P_c/L$ is the thickness of the cake medium, is $K_1\mu V(1 - \varepsilon)^2$ multiplied by S_0^2 divided by ε^3. So now, from here, we will try to develop the basic filtration equation because, in this figure, if you recall that



We have filter medium, we have filter cake, we have to consider the pressure drop across the filter cake and also the filter medium. Now, what we have written is that v equals the filtration velocity, which equals dv dt multiplied by 1 by v. That means, if you look at the unit of this one, dv / dt 1 by a. So, it is a volume of filtrate, meter cube per second divided by filtration area. So, filtration velocity gives meter per second, okay. Now, what are we doing?

We are performing a solid material balance for the filter cake, okay. Now, L A is the volume of the filter cake, which is very clear. multiplied by 1 minus epsilon. That means, epsilon is the void. So, 1 minus epsilon gives us the solid part multiplied by rho p, which is the particle density.

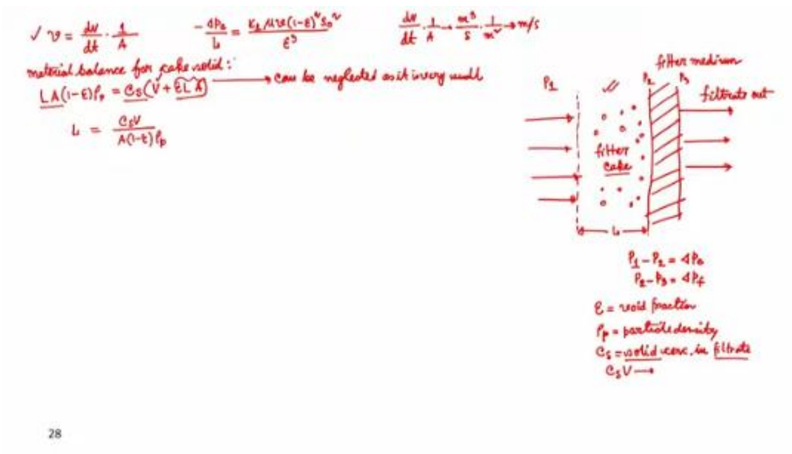


That means, we have kg per meter cube. So, we will have meter cube meter cube canceled, kg over here, and mass equals C_s. That means, what we are doing is the amount getting deposited as filter cake has to equal what is coming from the slurry, right? We have to make the balance. So, C_s is the solid concentration in the filtrate, okay. Now, C_s multiplied by V is the volume of filtrate.

So, kg per meter cube multiplied by meter cube plus we have added another term epsilon L A. So, epsilon L A, it actually considers that means the amount of solid that is retained in the with the liquid that is trapped inside the void fraction. Although it can be neglected as I mentioned before. So, let us neglect it and we will see that L equals to C_s V divided by A 1 minus epsilon rho P sorry rho P. the particle density.

So, we have so neglecting this one. So, C_s by v A epsilon rho p. Now, you remember what was our equation? So, this is our equation. So, let us do it dv / dt 1 by A dv / dt 1 by

A equals to minus P_c. So, let us do it mu by k So, minus P_c by L mu by so, this k value was so, let k value let us do it from since we already have let us do not require this one.



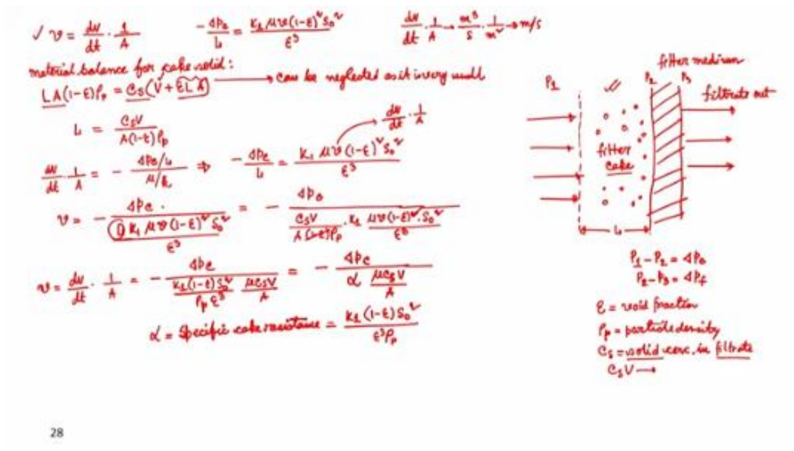
So, P_c by K L. So, it gives that minus P_c by L equals to K_1 mu v_1 minus epsilon square Snaught square epsilon cube. So, this v is the nothing but dv / dt 1 by A. So, let us do it. So, phi equals to what we can write. Let us rearrange little bit minus delta P_c by L k 1 mu phi 1 minus epsilon square is not square right.

Epsilon cube is missing, I guess. Then we put epsilon over here. delta P_c by L will all go over here. Now, since we have the L value, we will put this L value over here. So, minus P_c L is C $_S$ V.

A 1 minus epsilon rho p multiplied by k_1 mu v_1 minus epsilon square S_{naught} square divided by epsilon q. So, what does it give us? This one, this one gets cancelled. Now, what will we have? We will just rewrite this one, just do it on this side. So, pen V equals to dV / dt multiplied by 1 by A equals to minus P_c. We have to rearrange this one a little bit.

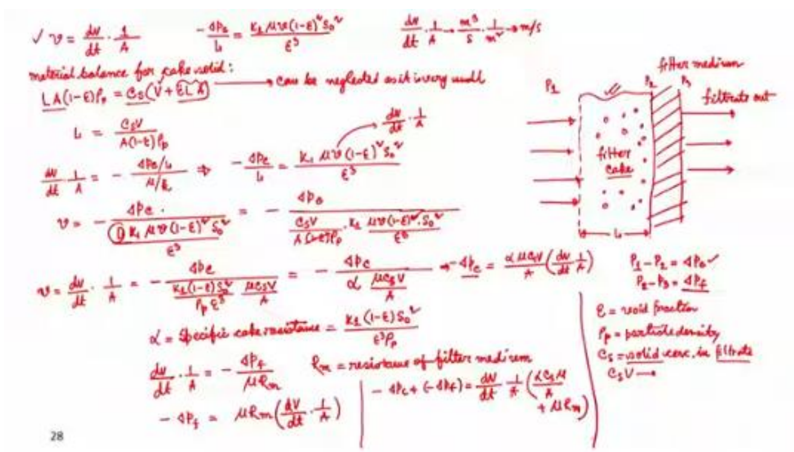
Let us do it. 1 minus epsilon is not square. Then we will have over here mu C_s v, right, sorry, v by A. This one we will have, rho p is there, another one is epsilon cube. So, this one, this part is called specific cake resistance, denoted by alpha mu C_s V by A, where alpha is known as specific cake resistance. Ok, and it is k_1 1 minus epsilon S_{naught} square epsilon cube rho p, ok.

So, that is known as specific cake resistance. So, this part is over. So, the delta P_c part is over. So, that means this can also be written as minus delta P_c equals to alpha mu Cs v by a dv / dt. Multiplied by 1 by a, this can be written, right? Now, see the delta P_c is say P_1 minus P_2. Now, coming to P_2 minus P_3, say delta P_f.



So, using the similar analogy, what we can write is dv / dt equals to minus delta P_f divided by mu R_m. So, where R_m is the resistance of the filter medium, resistance of the filter medium, ok. So, we have the filter medium, this one, and this one is over. So, we can write minus delta P_f equals to mu R_m dv / dt 1 by So, what can we write from here?

So, let us see. So, minus delta P_c plus delta P_f equals to dv / dt 1 by A alpha mu, sorry, C_s mu by A plus mu R_m, that can be written. Ok. That means we have added these two pressure drops to give the overall pressure drop, ok. So, we will come here.



So, minus delta P_c plus delta P_f, f means filter medium ok. We have dv / dt 1 by mu R_m plus alpha mu C_s v by A. So, this is minus delta P overall pressure drop equals to dv / dt 1 by A mu R_m plus mu C_s v by A. So, dv / dt 1 by A equals to minus P divided by mu, sorry, alpha, I have missed, alpha mu C_s V by A plus mu R_m. So, this is our final expression.

We can take delta P and you can take mu out. So, alpha C_s V by A Plus R_m. So, this is the cake resistance you can say this is actually you know filter medium resistance. So, we can write this alpha C_s V by A alpha, C_s V by A as a R_f filter cake resistance. So, that means, we can write minus P mu R_f plus sorry not r_f okay so let us denote in same way okay c means we have said let us say it is c okay otherwise it may create some confusion let us say c this is also c and R_m So, that means, sum of the resistance of 2, you have mu at the denominator and minus delta p. So, that is equals to dv / dt, multiplied by 1 by A. So, this is our final equation. Now, what else it can be in some book, if you follow, it can also be written as, just I want to give you the idea. We will take mu alpha C_s by A out mu alpha C_s by A out. So, v plus one is v_e.

Now, what is v_e? So, mu alpha C_s ve equals to mu R_m. So, this we are taking out we are putting. So, here you have A V. So, we are putting another one V that will actually representing for the filter medium. So, that means, what the V says V is the volume of filtrate volume of filtrate required

for building up a fixissus that means imaginary fixissus filter cake that has resistance equal R_m all right. That means, what it represents that let us say it is an imaginary volume of filters that is required to build up a as in you know fixation layer of the filter cake that has the resistance equal to R_m. So, that is the idea all right. Now, now what we will do we will so, this is our expression over here. Now what is the idea?

Idea is the how much time is required. Also we are interested in the pressure drop equation. Now at the very beginning I mentioned so there are actually two type. One is the constant pressure and one is the constant rate. So in the constant pressure means pressure remain constant.

$$-\Delta p_c + (-\Delta p_f) = \frac{dV}{dt}\cdot\frac{1}{A}\left(\mu R_m + \alpha\frac{\mu C_s V}{A}\right)$$

$$-\Delta p = \frac{dV}{dt}\cdot\frac{1}{A}\left(\mu R_m + \alpha\frac{\mu C_s V}{A}\right)$$

$$\frac{dV}{dt}\cdot\frac{1}{A} = -\frac{\Delta p}{\left(\frac{\alpha \mu C_s V}{A} + \mu R_m\right)} = -\frac{\Delta p}{\mu\left\{\frac{\alpha C_s V}{A} + R_m\right\}} = -\frac{\Delta p}{\mu(R_c + R_m)}$$

$$\frac{dV}{dt}\cdot\frac{1}{A} = -\frac{\Delta p}{\frac{\mu\alpha C_s}{A}(V + V_e)}$$

$\frac{\alpha C_s V}{A} = R_c$ = filter cake resistance

V_e = volume of filtrate required for building up a fictitious filter cake that has resistance equal to R_m

$$\frac{\mu\alpha C_s V_e}{A} = \mu R_m$$

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So, pressure remains constant. So, there will be a volume of filtrate accumulated, and during this period, there will also be a deposition of cake. The filter, OK. So, over the period, the volume of filtrate will go down, OK. So, that is the constant-pressure filtration.

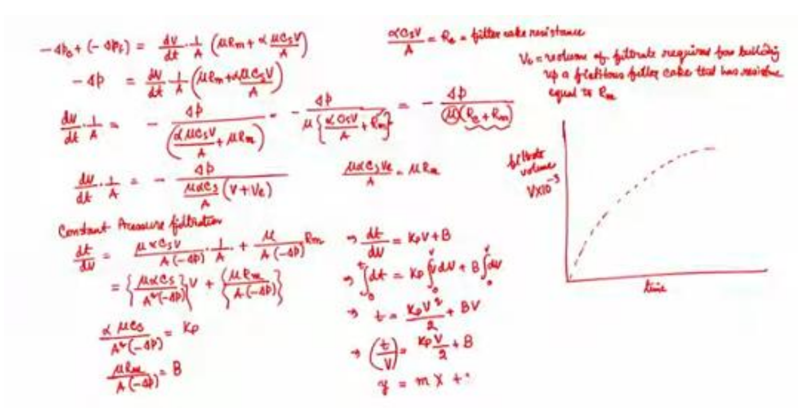
So, if you really consider the constant-pressure filtration—sorry, constant-pressure filtration—that means what we will be having in the constant-rate filtration, the pressure is constant—sorry, constant-pressure filtration is where pressure is constant. So, we are interested in the timing part. So, let us do it. So, dT/dV, we will just make a reciprocal of it, dT/dV over here.

So, if you do it, See, A will be on the top. So, if you do it, so mu alpha C_s v by A, right, multiplied by—no, it is, it will be in the denominator. Yes, sir. So, pen. So, multiplied by 1 by A multiplied—yeah, dT/dV, and V is over here, that is fine, plus we have the second term mu A. So, you

will be having mu A this will come over here in the denominator multiplied by R_m all right. I am correct. So, dT by dV. So, therefore, we can write this one mu alpha C_s by S square minus delta P V plus mu R_m A minus delta P. Now, these two terms, this one and this one, they are represented as a constant term.

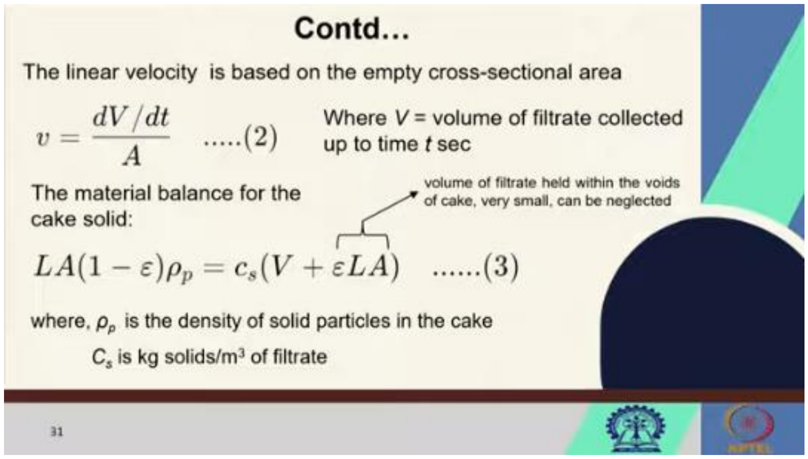
So, mu, let us say alpha mu C_s, alpha mu C_s divided by S square minus delta P, this is denoted as K_p and then R_m, mu R_m divided by A minus delta P that is actually B. So, this is written as therefore, dT / dV mu alpha C_s A v will be there right Kp v plus b. Now, d t will be K_p v d v plus b d v 0 to t. So, here let us say it is 0 to v_o to v d t equals to K_p v square by 2 plus b v. So, T by v k_p v by 2 plus b. Now, if you really plot the volume of filtrate let v let us say into d power minus 3 filtrate volume filtrate volume with time. So, it will be like this.

Now, if you take this form, then we will have A, say, what is this? So, this is only time and volume. Now, what we are doing? Here, it has t by v. So, y equals to mx plus c form. So, if you plot it, you will get a slope and intercept is b and slope is kp

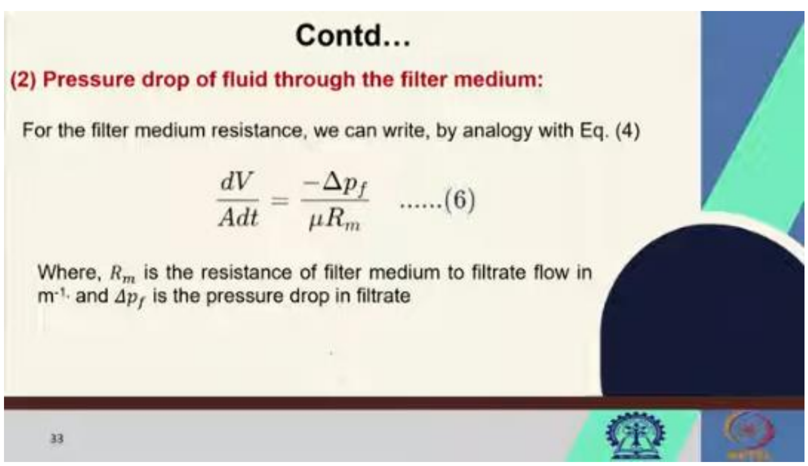


by 2, ok. So, over x is your v and your t by v. All right. So, now, what we will do? We will quickly go to the slide.

So, this part we have discussed already. So, solid balance. I will just quickly go through it. dv by A dt. So, that is minus delta P_c divided by alpha mu C_s by A. And finally, alpha is the specific egg resistance and it has the unit of meter per kg.



and it is equals to k_1 1 minus epsilon Snaught square divided by rho p multiplied by epsilon cube. Now, coming to the pressure drop across the filter medium ok. So, d v by A d t equals to minus delta P_f divided by mu R_m, R_m is the resistance of the filter medium ok. It is in per meter delta p_f is the pressure drop in filtrate ok. Now, if you combine those two equations, you will be having dV by A dT equals to minus delta P, delta P equals to delta P_c plus delta P_f divided by mu alpha C_s V by A Plus R_m.



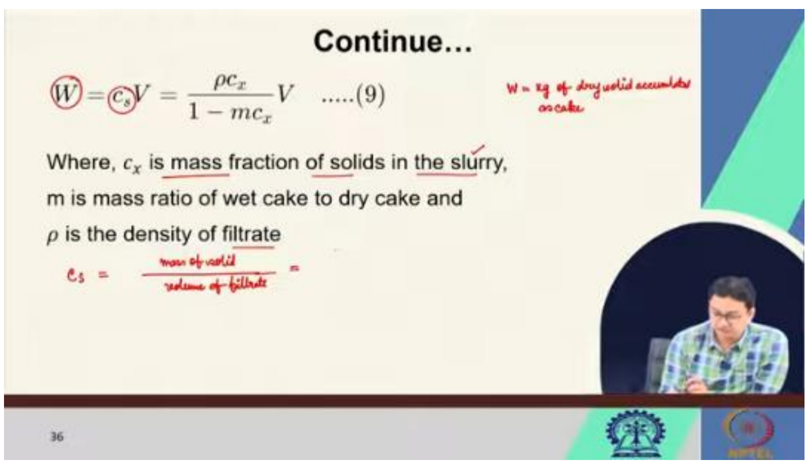
So, as I said, this R_m plus R_c is for the cake resistance, and this is for the filter medium resistance, ok. So, this part we have also talked about: dv by A dt equals minus delta p divided by mu alpha C_s by A v plus v_e. Now, this part is one thing that let us try to understand. Here, we are actually trying to think about w. So, what is w? w is, I will write over here, the kg of dry solid accumulated—kg of dry solid accumulated in or as cake as

as cake. Now, C_x is the mass fraction of solids in the slurry. This is actually the mass fraction, not the solid concentration. Mass fraction of solids in the slurry. C_s—what is Cs, if you remember? C_s is the solid concentration in the filtrate.

Ideally, what should happen? The kg of dry solid that is accumulated as k_x, ok. We are talking about dry solid, ok. That has to be equal to C_s multiplied by V because that is actually accumulated, ok. But now, how can it be written?

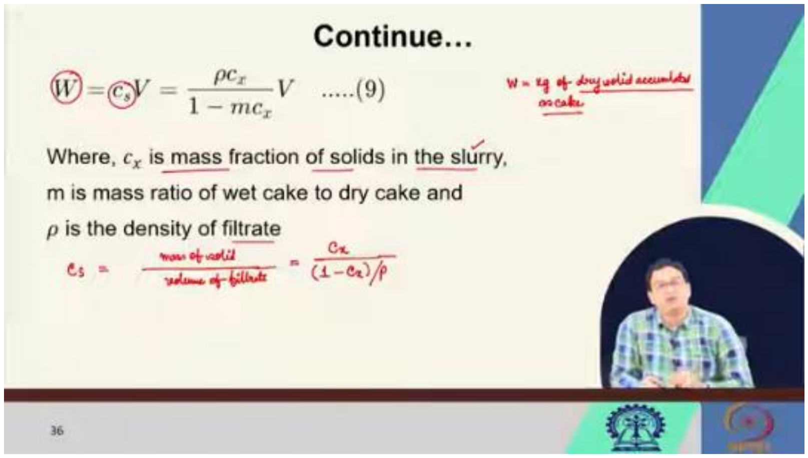
So, in terms of C_x and rho, which is the density of the filtrate. So, can we write this as C_s equals to? So, what we can write is A. So, what is that? That C_s is the mass of solid by, sorry, over here, the volume of filtrate. Just wait.

So, this is the C_x. Now, in terms of C_x, what can we do? So, we can write. So, the mass of solid is the mass fraction of solid in the slurry. It is coming from there, divided by, ideally, if we write here, we have the volume of the filtrate.



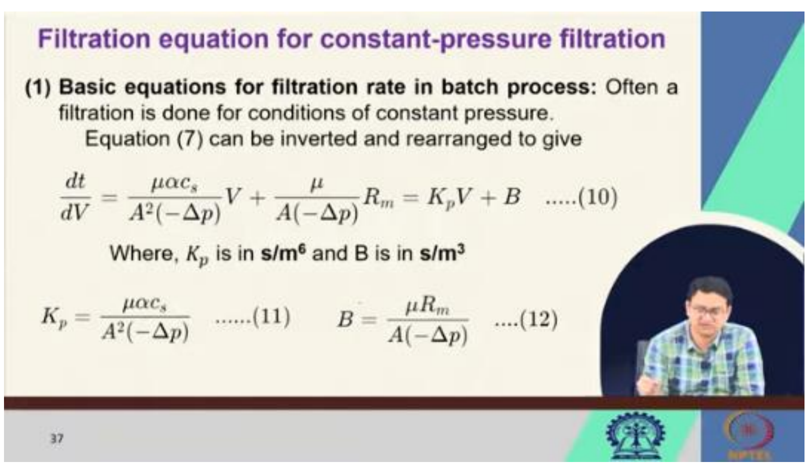
That means, if you do 1 minus C_x and divide it by rho. So, it will give you the volume of the filtrate. All right. Now, what happens? So, this is your ideal coincidence.

Now, the thing is, we are talking about the dry cake over here. Dry, solid accumulated cake over here. Now, you remember I mentioned that in the cake, there is also trapped liquid inside. And it also has some solid fraction, which means it has got the moisture, not the dry cake. That means, if we do not consider the solid over here, the moisture part over here in this, you know, filter cake, it will simply



underestimate the value of W, the dry solid cake. So, that is why we are actually introducing another one, this is actually M, it is just a moisture ratio of cake, usually called the moisture ratio of cake, that is equal to mass of wet cake to mass of dry cake. And that is actually this one is multiplied by the C_x in order to consider this one, the whole scenario. Then we will have W equals to C_x divided by 1 minus m C_x multiplied by—sorry, rho is there—rho is multiplied by v. Now, when this moisture ratio is 1, then it

gives the ideal condition. So, 1 means 1 minus C_x divided by rho, and rho will go up, rho C_x by 1 minus C_x, the moisture ratio is 1. Now, this part we have already started talking about in the constant rate—sorry, constant pressure filtration. So, what is the basic equation for the constant pressure filtration? So, this is for the batch process. We have learned that dT by dV equals to can be written as K_p multiplied by V.

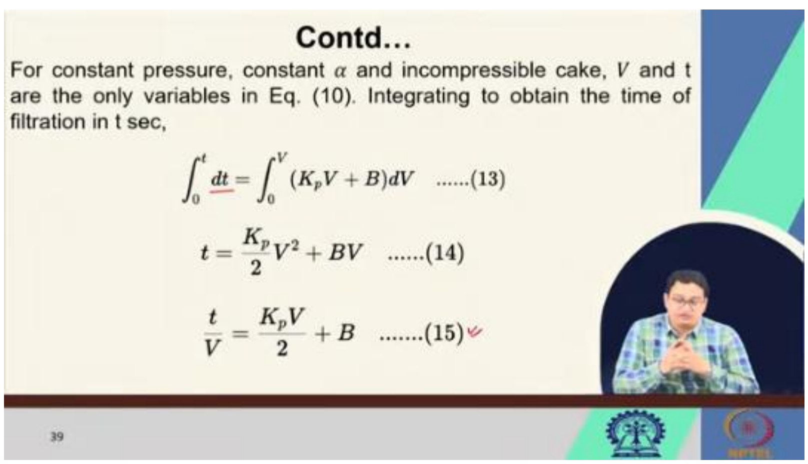


plus b, where K_p has the form of mu alpha C_s divided by a square multiplied by minus delta p. b is the mu R_m divided by a multiplied by minus delta p, and K_p has the unit of s per meter to the power 6. b has the unit of s per meter cubed. So, if you derive it, you can easily get it. So, you will have a Pascal second s squared. Now, here you will have the Pascal, and R_m is per meter.

If you put all those things, it gets cancelled: s per meter cubed. You can easily derive the unit of K_p and V from here. Now, this part also we have A already done that. The time required equals to K_p by 2 multiplied by v squared plus b v, and if you do t by v equals to K_p v by 2 plus v, ok. So, this is our final expression, all right.

So far, what we have actually learned is that we started from the basic filtration equation, where we took care of the two resistances that are actually in series: one is for the filter cake, and another resistance is coming from the filter medium. We combined the pressure drop across these two media and finally came up with the basic filtration equation. Now, when we talk about constant-pressure filtration, in constant-pressure filtration, the pressure is constant. So, over the period, there will be a deposition of solid or K on the filter medium. Your accumulated volumetric flow.

The rate of the filtered will slowly go down. So, then we are interested in the time requirement. So, we have this form finally, okay. So, we will stop here now and continue from here in the next class.



Thank you.