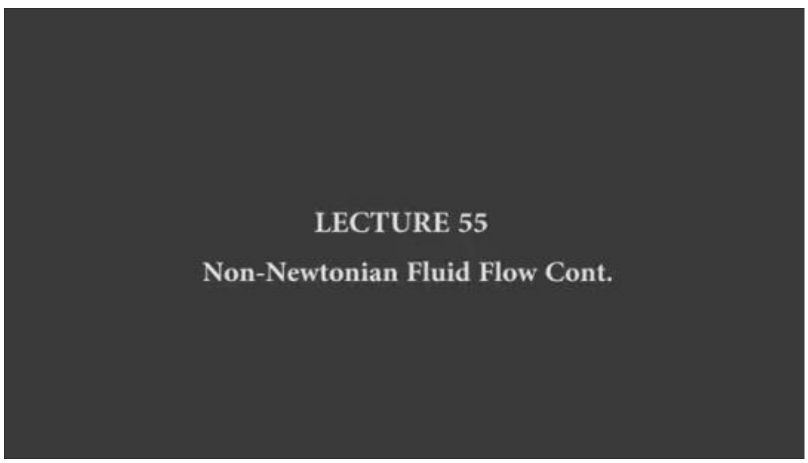
IMPACT OF FLOW OF FLUIDS IN FOOD PROCESSING AND PRESERVATION

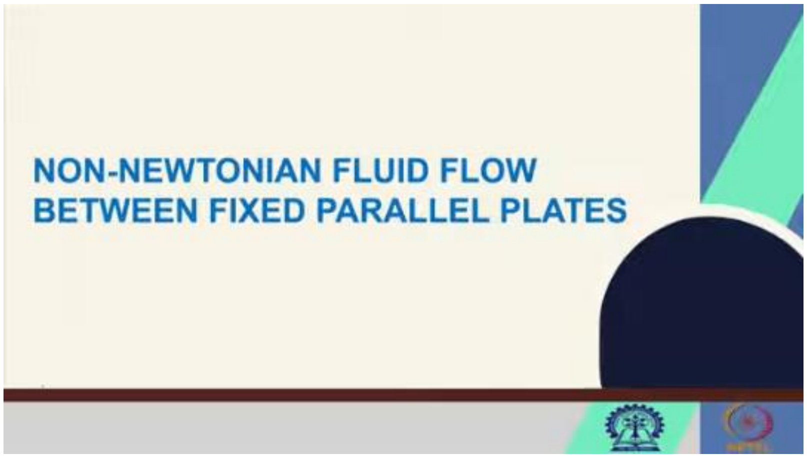
Lecture55

LECTURE 55 : Non-Newtonian Fluid Flow Cont.

Hello friends, welcome back to another class of week 11. So, we have been dealing with non-Newtonian fluid flow, which means the flow of fluids that are non-Newtonian in nature. So, in the previous classes, what we have actually looked at— if you recall, we have looked at the characteristics of non-Newtonian fluids and then we started discussing— how the flow behavior and pressure drop characteristics occur when a non-Newtonian fluid flows through a pipe—

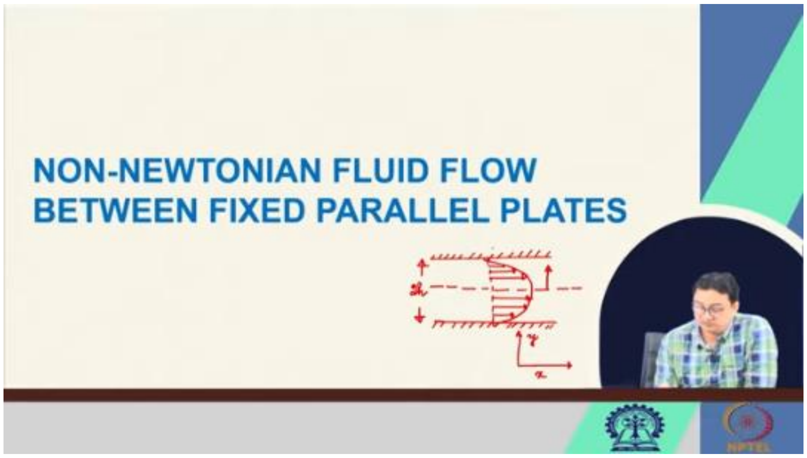


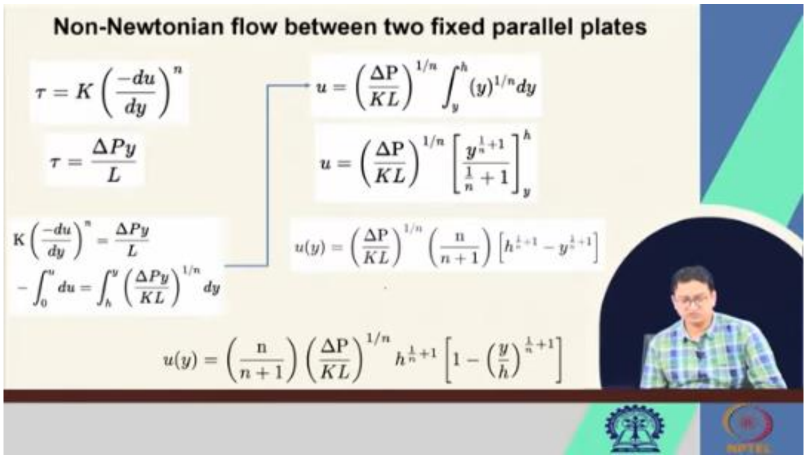
and between. So, in the last few classes, we have completed the non-Newtonian fluid flow through— a pipe, and we started discussing how the flow characteristics occur when it flows between two fixed parallel plates. So, we will continue from there in this class. So, we have been discussing non-Newtonian fluid flow between fixed parallel plates. So, if you recall, we had this kind of scenario where non-Newtonian fluid flows between two parallel plates.



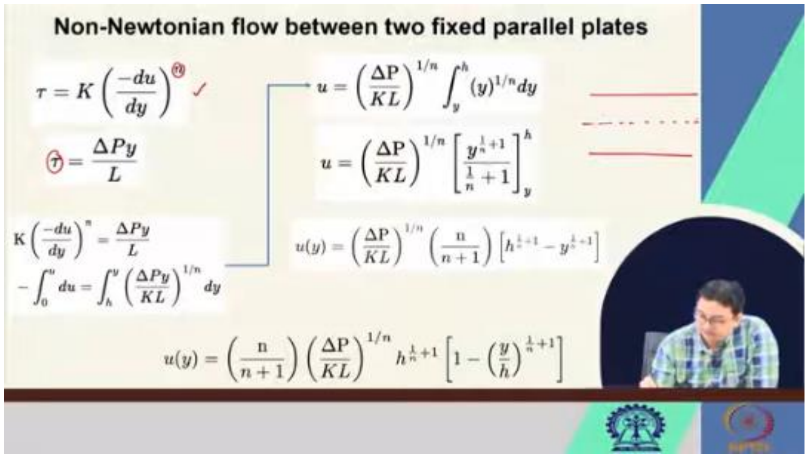
Many times, it is also called slits. So here, we are having fluid flow like this, and this is the center line. The gap between these two plates is considered 2h. So, this is along the x-direction. Here, in the y-direction, all right. So, the velocity profile develops along y as you move away— from the center line. We have a velocity profile, and the velocity decreases until it comes to a complete stop.

For the fluid that is actually adhered to the plates over here, ok. So, this one we can say plus h here, it is minus h here, we are having y equals to 0. So, this kind of situation we had when actually we tried to develop the flow behavior and the velocity profile for this kind of situation when the non-Newtonian fluid flows between the two plates. So, we will go back a little bit. So, what we had, we considered.



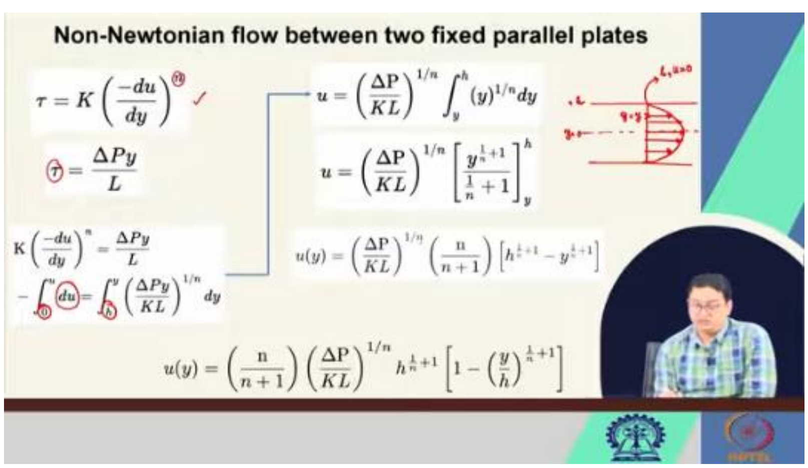


So, this is the relationship for the non-Newtonian fluid: tau equals to k multiplied by minus du dy to the power n, where k is the consistency index and n is the flow behavior index. And we also took help of this form of the tau: tau equals to delta P multiplied by y by L, ok. So, here we have, therefore, k. Last class we have already developed just to have a brief idea: k multiplied by minus d u / d y to the power n equals to delta p y by L. And if we really integrate from 0 to u and here h to y, that means, so over here.



So, here equals to y equals to 0 and h, ok. So, over here we are having some velocity. In this case, we are having a velocity profile velocity gradient develops. So, at h, that means, over here u equals to 0. So, that is at h equals to 0. At some point, if you take over here at any location, let us say it is y equals to y, and here we are having velocity equals to u equals to u. So, that is what we are putting as the integral limit. equals to, say, h integral h to y delta p y divided by k l to the power 1 by n multiplied by dy. Now, if you rearrange this one. So, what is our aim? Our aim is to get the u value, ok. So, u equals to delta p by k L to the power 1 by n integral of y to h y to the power 1 by n. Now, if you integrate this one, we will be

having y to the power n plus 1 by n, that means, 1 by n plus 1 divided by 1 by n plus 1. So, finally, we will have u_y equals to delta p by k L to the power 1 by multiplied by n by n plus 1. So, this part we can rearrange. So, the n at denominator will go to the top, and here at the denominator, we will be having n plus 1 multiplied by

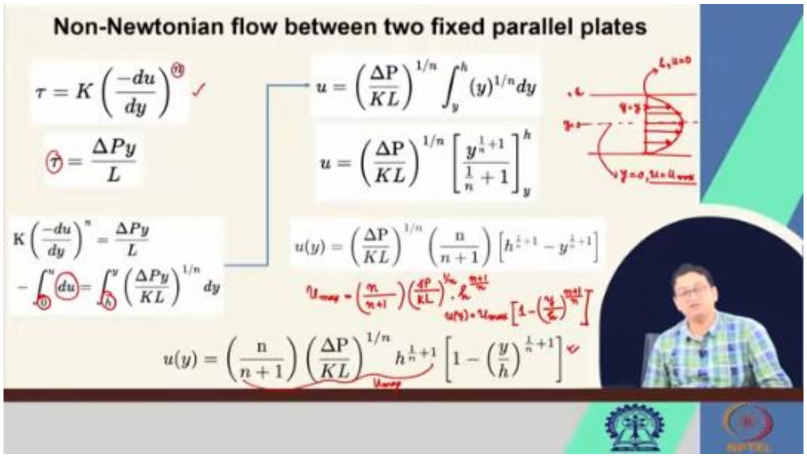


since we are having two limits, y and h. So, upper limit minus lower limit h to the power 1 by n plus 1 minus y to the power 1 by n plus 1. So, this is our velocity profile, ok. So, this is our final form, and if we take the h to the power 1 by n plus 1 out, we will be having 1 minus y by h to the power 1 by n plus 1, ok. So, this is our velocity profile.

Now, at the center line, we are having over here y equals 0. u equals u_{max}. So, here, the maximum velocity occurs at the central line. So, if you put y equals 0, then you see this part will be canceled; we will be left with multiplied by 1 only. So, then that means, u_{max} in this case we will be having n by n plus 1 delta p by

kl to the power 1 by n h to the power. So, I will just write n plus 1 by n. So, this one can be further written as. So, since this part is u $_{max}$, we can write this one u_{max} multiplied by 1 minus y by h to the power 1 plus n plus 1. So, that is uy. So, uy also can be written as if I write over here since this part is u_{max}, this part is what is this u_{max}, right?

1 minus y by h to the power n plus 1 by n. So, that is what we have now. Now, what is our aim? Our aim is to get the average velocity, ok. So, average velocity, how will you do it?



So, let us look at it. So, average velocity. So, see when we have a velocity profile like this. So, we can write the volumetric flow rate, which is often expressed as q or capital V. So, what can we write for this volumetric flow rate?

$v_{average}$ multiplied by the cross-sectional area. So, what is the cross-sectional area? This is 2 ok, and in the z-direction, we have the depth of b. So, I will just show you over here. So, let us say you imagine this is a flow this is actually the top part, the top plate, and this is the bottom plate. These two are actually fixed.

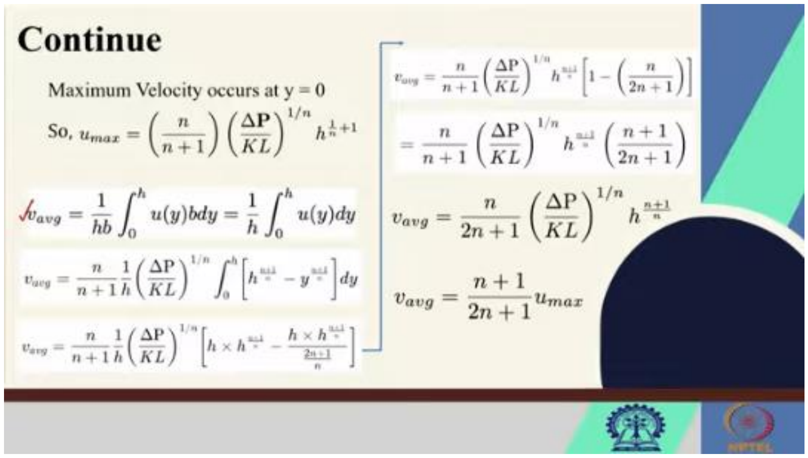
Flow is occurring like this, ok, through this cross-section, meaning this is the b, and this is 2h, this gap. So, the cross-sectional area is b multiplied by 2h, ok. Now, see, this can be $v_{average}$ multiplied by 2h multiplied by b, ok. Now, since we have a velocity profile, we have to integrate it, ok. So, we have to integrate.

So, what we can write $v_{average}$ multiplied by A. So, this one can be written as so, we will be integrating uy multiplied by d A ok. So, over the so, we can consider very small very infinitesimal differential element, or small area and we will so, that is having the area of d A, very small infinitesimal and we are integrating the whole thing in order to get the total volumetric flow rate ok. So, u_y d A. So, $v_{average}$ we can write in that case 1 by A u_y d A. So, now what we can do this one is 1 by see

we can say. So, here we have this center line. So, this is axis ymmetric ok that means, the both the parts are mirror of of ah each other. So, we will only consider only one half ok only one half if you look at it. So, for the one half we can write, b multiplied by h because this is only h. See we can do

from minus h to plus h also for sake of our analysis for the simplification we can only consider the one of the half. Now, from here what we can write uy. So, what is therefore, d A, d A would be if you take very small element So, this small element will have the another direction b, but this one is the dy. So, in other direction is b ok. So, we can write b into dy.

So, this one we can write 0 to h, ok. So, 0 to h we can write. So, therefore, the b, b gets cancelled. So, I will just, you know, quickly show you it is over here, ok. So, 1 by b h. So, that is what we have done, 0 to h uy B dy, ok. Just I wanted to quickly show you, ok. Just let us derive it.

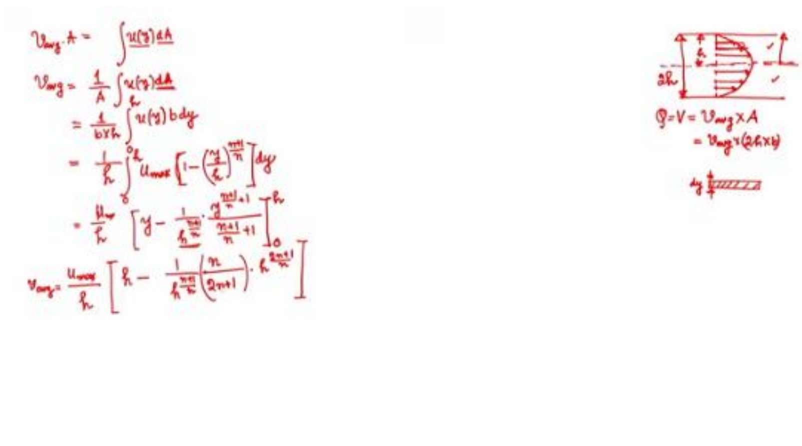


Now, what is this velocity profile? So, see b, b gets cancelled. We will have been 1 by h b root 2 h. So, this velocity profile we can directly write u_{max}. So, u_{max} 1 minus y by h. So, this part we can write like this, n plus 1 by n multiplied by dy, ok. So, then 1 by h 0 to. So, u_{max} we can also take out 0 to h. So, what we will be having.

So, we are integrating. So, dy this will give you y minus y by h to the power n plus 1 by n. This one will be, see, y to the power n plus 1 by n plus 1 divided by n plus 1 by n plus 1. So, that means, we will have n plus n plus 1 here. We will be having n plus n plus 1, ok. So, we will write 0 to h. Now, so what we will do now? So this equals to, so $v_{average}$ equals to $v_{average}$ equals to.

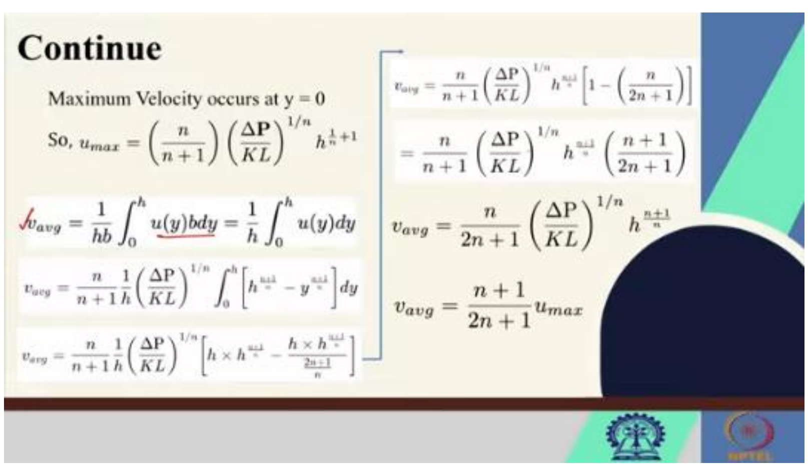
u_{max} by h, sorry, ah, what I will do is I will just remove this part then, okay, since we have already done it. Umax by h. So, this is h minus 1 by h to the power n plus 1 by n. Here we will be having n by 2n plus 1. Here we will be having h 2n plus 1 by. So, hopefully, I am correct over here. So, dy y. So, h will be here 0 if you put everything will be 0. This one will be fixed because already it has got h over here, 1 by h to the power n plus 1 by n multiplied by.

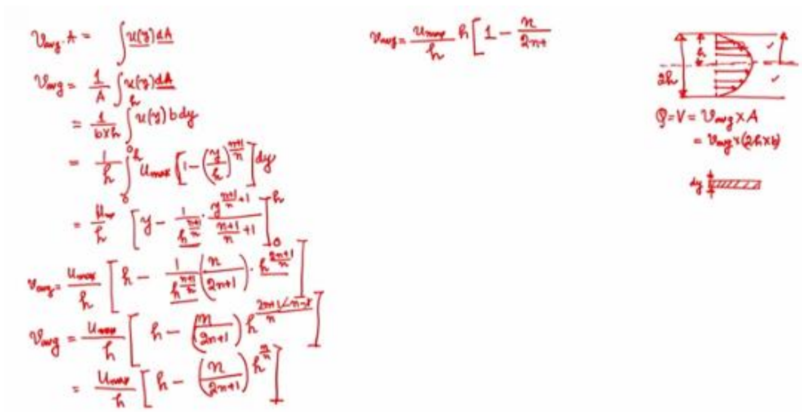
So, this n will go to the top. n by 2n plus 1 h to the power 2n plus 1. Now, this part we will adjust. So, $v_{average}$ equals to u_{max} by h. So, h minus n by 2n plus 1. So, h to the power 2n plus 1 by n minus n minus 1.



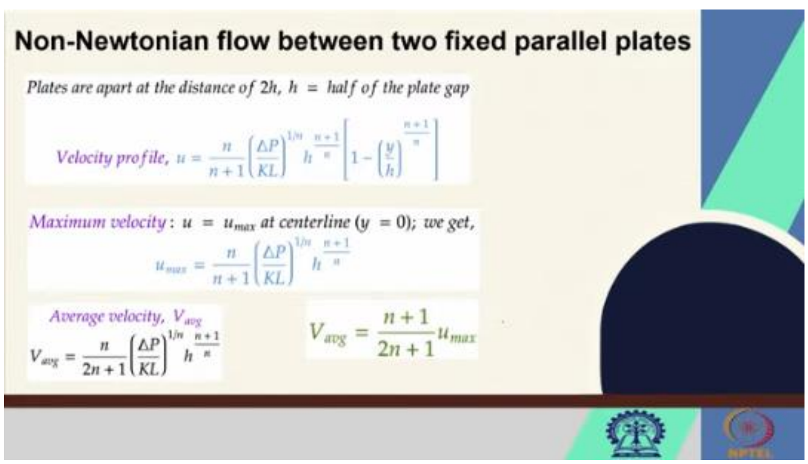
So, u_{max} by h. h minus n by 2n plus 1. So, what we have here? So, h to the power this one gets cancelled. So, h to the power n by n. So, h to the power n by n that means, only h. So, we will write over here $v_{average}$ equals to u_{max} by. h. So, we can take h out 1 minus n by.

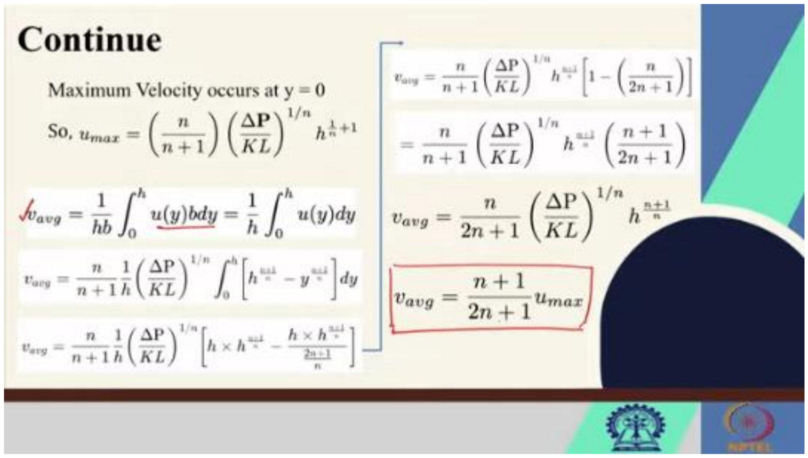
2 n plus 1. So, u_{max} equals to 2 n plus 1 minus n divided by 2 n plus 1. So, n plus 1 divided by 2 n plus 1 multiplied by u_{max}. So that means, average velocity of a non-Newtonian flow flowing between two fixed parallel place it is nothing, but n plus 1 divided by 2 n plus 1 times of u_{max} ok. So, if you look at it $v_{average}$ we have just derived $v_{average}$ equals to n plus 1 divided by 2 n plus 1 multiplied by u_{max} ok.





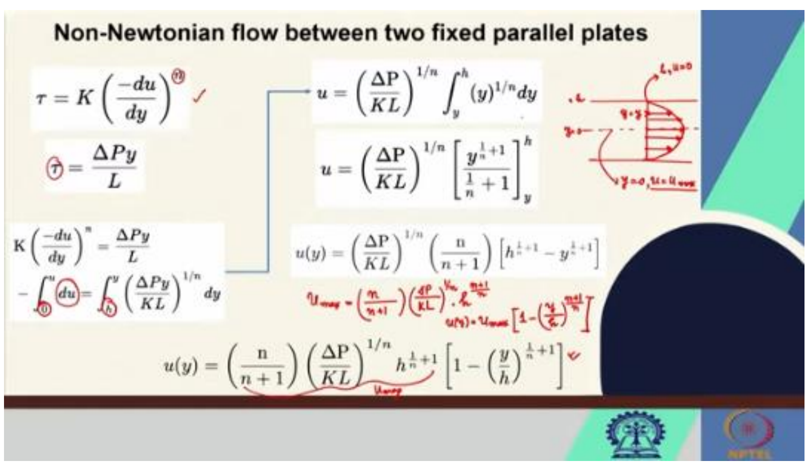
If you put n equals to 1 that means, Newtonian flow you will get as usual you if you recall the relationship is two-third of the u_{max}. The $v_{average}$ for the Newtonian flow between two fixed parallel plates is two-third of the u_{max}. So, that satisfies the equation. Now, so we will just quickly look at. So, what is the velocity profile therefore, u equals to n by n plus 1 delta p by k L to the power 1 by n h to the power n plus 1 by n multiplied by 1 minus y by h to the power n plus 1 by n. So, this is the velocity profile maximum velocity at y equals to 0.



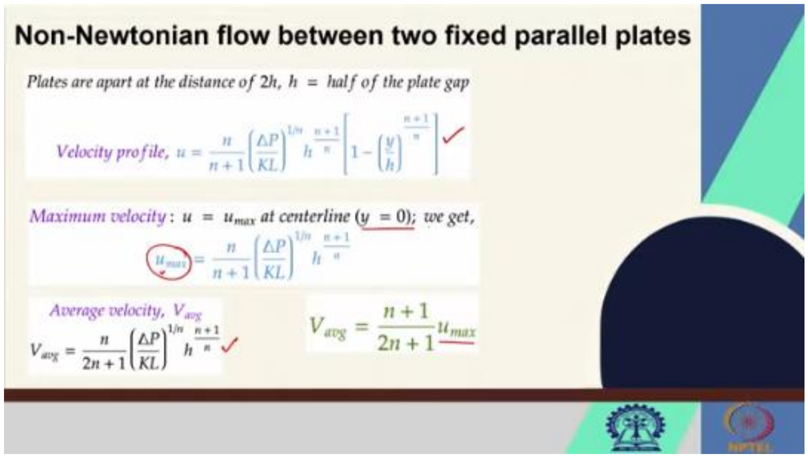


So, u_{max} equals to n plus n by n plus 1 delta p by k L to the power 1 by n multiplied by h to the power n plus 1 by n. So, this part also can be written as I said u $_{max}$ multiplied by this part. Average velocity we are having over here n plus 1 divided by 2 n plus 1 multiplied by u_{max}. Now, you see if you recall so, what we had I will just quickly go back over here.

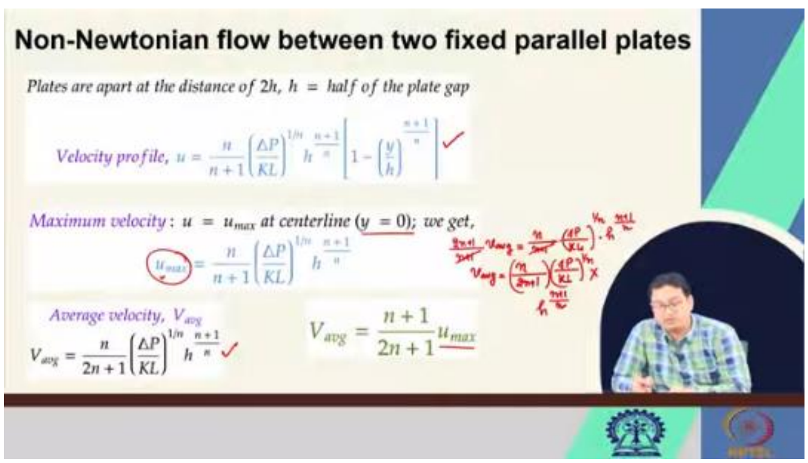
So, we had u profile like this one. u_{max} is sorry, u_{max} is over here n by n plus 1 delta p by kl to the power 1 by n. So, this part we can write. So, h to the power n plus 1 by n. So, u_{max} what is u_{max}? u_{max} is v_{averag}e multiplied by 2 n plus by n plus 1 from here we can also derive our this relationship.



So, if you look at it 2 n plus 1 divided by n plus 1 multiplied by $v_{average}$ equals to. So, that is u_{max} n by n plus 1 delta P by K L to the power 1 by n h to the power n plus 1 by So, this one will get cancelled. So, $v_{average}$ will be having a n by 2 n plus 1 delta P by K L to the power 1 by n multiplied by h to the power n plus 1 by n ok. So, this is our $v_{average}$ we get.



Now, what we have to do? We have to get our pressure drop. What is the expression for the pressure drop in terms of $v_{average}$ and finally, we will also be looking at in terms of Reynolds number. How to do it? So, let us consider that $v_{average}$ as I said equals to n by 2 n plus 1 n by 2 n plus 1 n by 2 n plus 1 delta P by



2 k L n by 2 delta p by 2 k L to the power 1 by n h to the power n plus 1 by n by n. So, what we have done for the non-Newtonian fluid flow through the pipe, we will do the same thing over here. So, delta p by 2 k L to the power 1 by n equals to 2 n plus 1 by n. So, 1 by h to the power n plus 1 by n multiplied by $v_{average}$. Now, we will start rearranging this one a little bit.

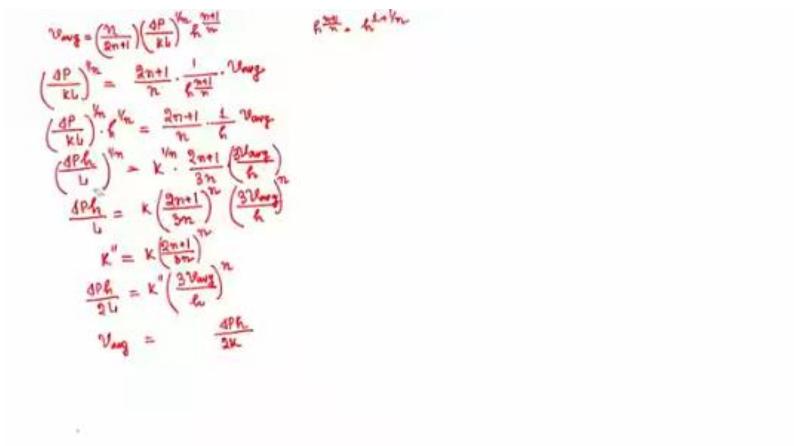
So, what we can do is delta P by 2 K L to the power 1 by n. So, here this one is h to the power n plus 1 by n. So, we can write h to the power. So, this one we can write 1 plus 1 by n. So, this 1 by n we will be taking over here. So, we will be left with at the right side 1 by h multiplied by V average. Now, what do we have to do?

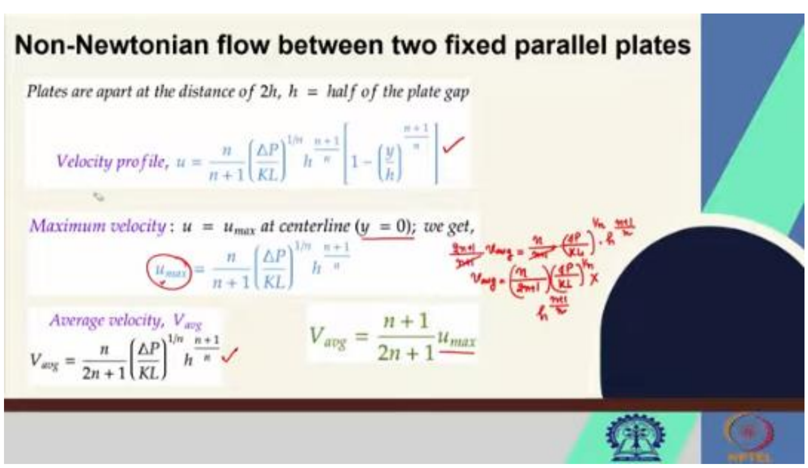
So, delta P h by 2 L. So, we will take k out to the power n, say k to the power 1 by n 2 n plus 1 by n $v_{average}$ by h. Now, what will we do? We will put 3 over 3 here. So, k to the power 1 by n 2 n plus 1 by 3 n $v_{average}$ by h delta p h by 2 L equals to. So, we will be having k 2 n plus 1 by 3 n to the power 1 by n 3 $v_{average}$ by h to the power 1 by n.

sorry not 1 by n. So, sorry such mistake to the power n and where because we are by taking this 1 by n out. So, you will be having to the power n to the power n over here. So, now, we will define k double prime equals to k multiplied by 3 n plus 1 sorry 2 n plus 1 by 3n to the power n. Now, what we can write?

So, delta P h by 2 L equals to K double prime 3 $v_{average}$ by h to the power n. So, from here we can obtain the v_{averag}e expression. So, what would be the $v_{average}$? The $v_{average}$ would be delta P h. So, by 2 So one thing quick, I think this is 2 is not here. So please bear this with me.

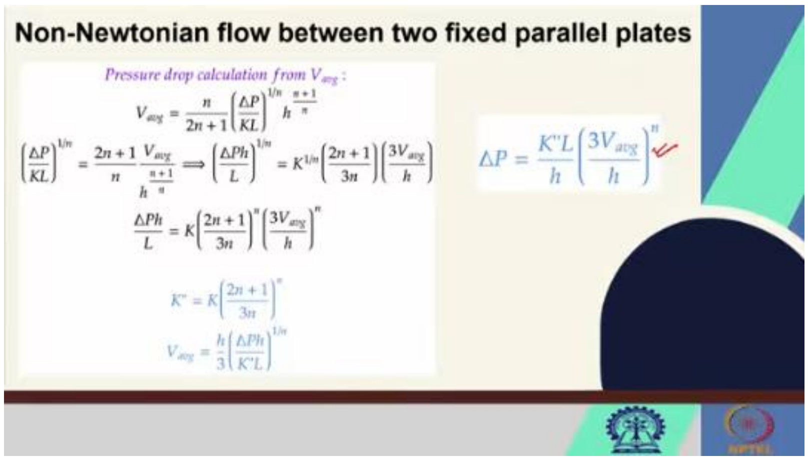
So this 2 is not here. So let us go back, look at it. Yeah, delta P by KL. Sorry for this mistake. So that 2 is not here.





So we will Remove these 2 and 2. I think everything is correct now. So, delta Ph by k double prime L to the power 1 by n and multiplied by h by 3. So, that is our v average.

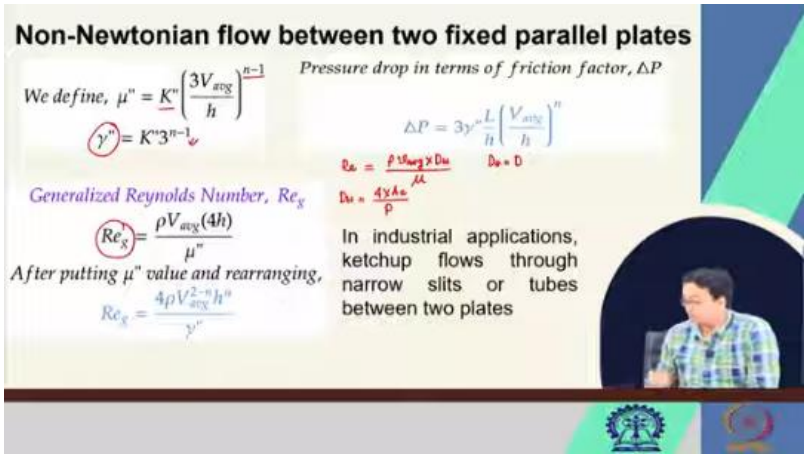
So, $v_{average}$, therefore, we can say h by 3 delta ph by k double prime L to the power 1 by n. So, please correct there should not be any 2 over here. So, it is a mistake; the rest of the thing is good. So, $v_{average}$ equals to h by 3 multiplied by delta p h divided by k double prime l to the power 1 by n. So, from here also you can derive the delta p. If you look at it over here, we can get this form also, not a problem. Now, what we have to do



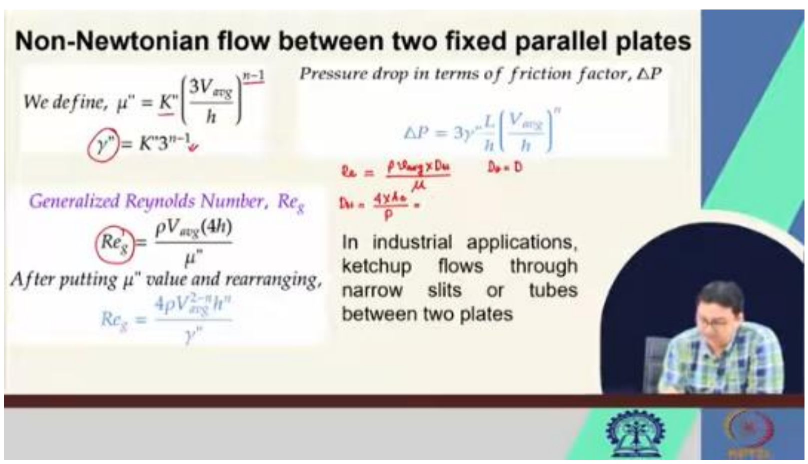
we will define that generalized viscosity gamma double prime equals to K double prime multiplied by 3 into 3 to the power n minus 1. So, mu double prime equals to K double prime multiplied by 3 $v_{average}$ by h to the power n minus 1. We take 3 out and we define this K double prime multiplied by 3 to the power n minus 1 equals to gamma double prime in this case, that is the generalized viscosity for the flow. Now, here generalized Reynolds number when we define.

So, what is actually the Reynolds number? In general, the Reynolds number is rho v average multiplied by hydraulic diameter divided by mu, okay. Now, for pipe flow, the hydraulic diameter is nothing but the diameter of the pipe. Now, how is the hydraulic diameter defined? dH equals 4 multiplied by the cross-sectional area through which the flow is occurring, divided by the wetted perimeter.

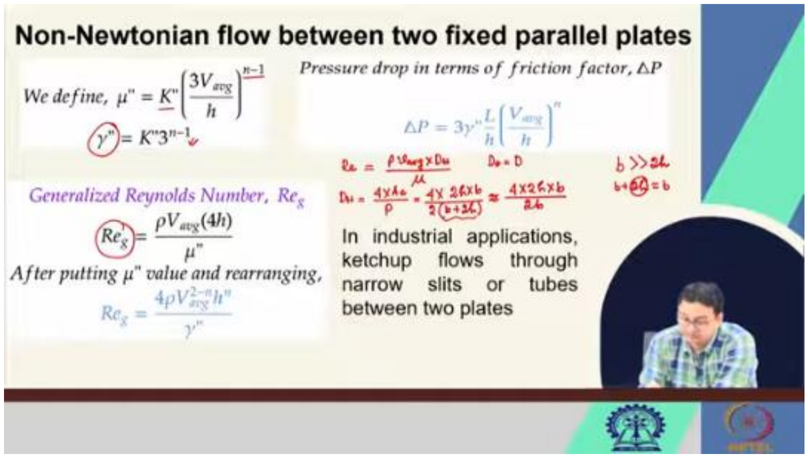
Now, you see in this case, what we have—let us say this is your top plate and bottom plate, both are fixed. What would be your cross-sectional area? b multiplied by 2H. And your wetted perimeter—what would be the wetted perimeter? The wetted perimeter has to be—see, one is b plus 2h, b plus 2h—that means 2 into b plus 2h, all right.



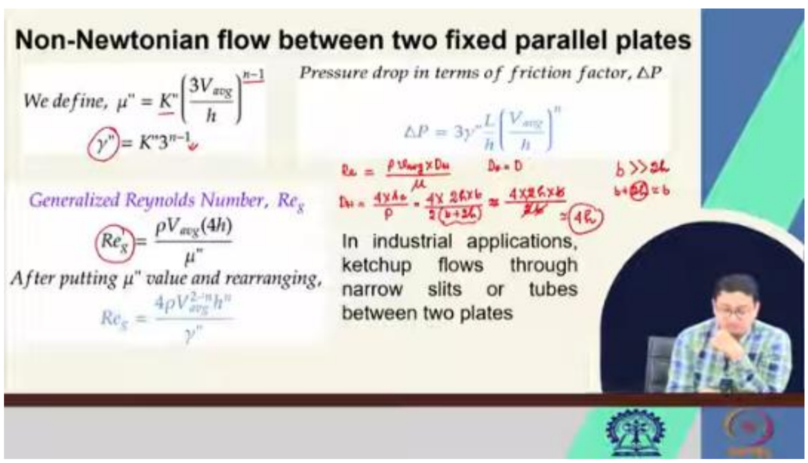
Now, we will consider one more thing. So, 4 multiplied by 2h multiplied by b, divided by 2 multiplied by b plus 2h, okay. Now, you see in practical cases, this dimension—the b dimension—is much, much larger than the 2h, which means the gap between the plates is very, very small compared to the b value. So, in the denominator part, we can ignore this b plus 2h part, okay? We can write it as b because the 2h value is so small, this one actually does not affect the whole sum. So, we

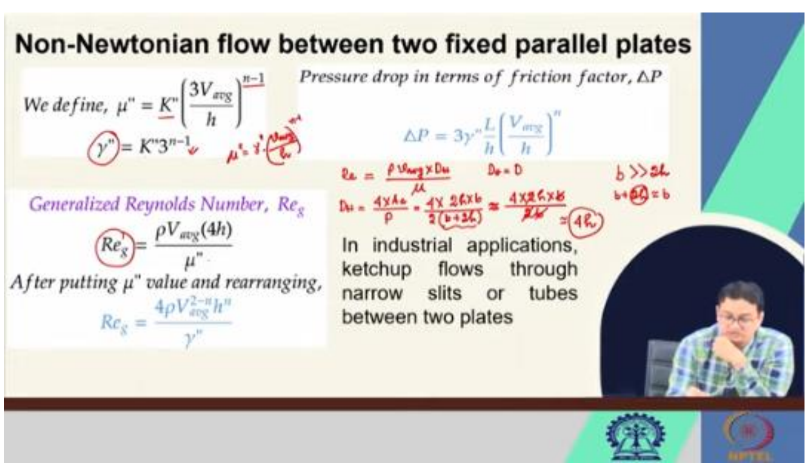


Can write over here 4 into 2h multiplied by b divided by 2b. So, we are equally writing like this. So, 2, 2, b, b gets cancelled. So, it is 4h. So, hydraulic diameter for this Newtonian flow between two pipes is equivalent to 4h, that is 2 times the gap between the two plates. So, we are having 4h, dh equals to. So, that is why Reynolds number we defined as rho vaverage multiplied by 4h divided by mu double prime, and this one we can rewrite as Re_g equals to 4, we can

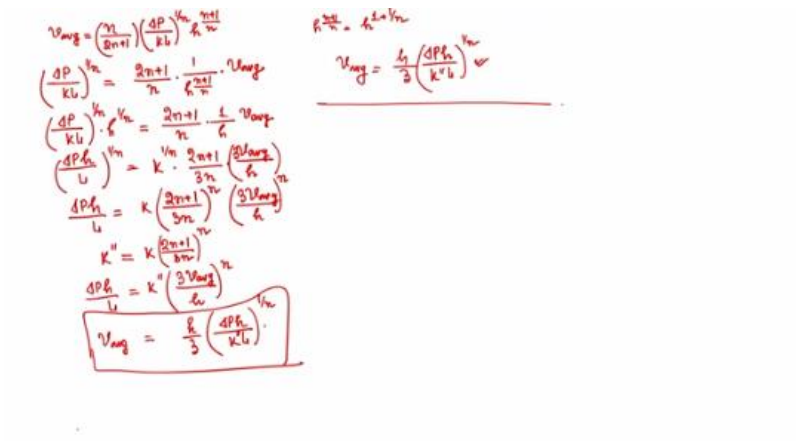


take over here rho multiplied by v_{average} to the power 2 minus n, h to the power n. gamma double prime. So, mu double prime simply what we are doing is writing it this way, ok? So, mu double prime is nothing, but gamma double prime multiplied by $v_{average}$ by h to the power n minus 1. So, if you put this value over here and rearrange, you will get R equals to 4h $v_{average}$ to the power 2 minus n multiplied by h to the power n divided by gamma

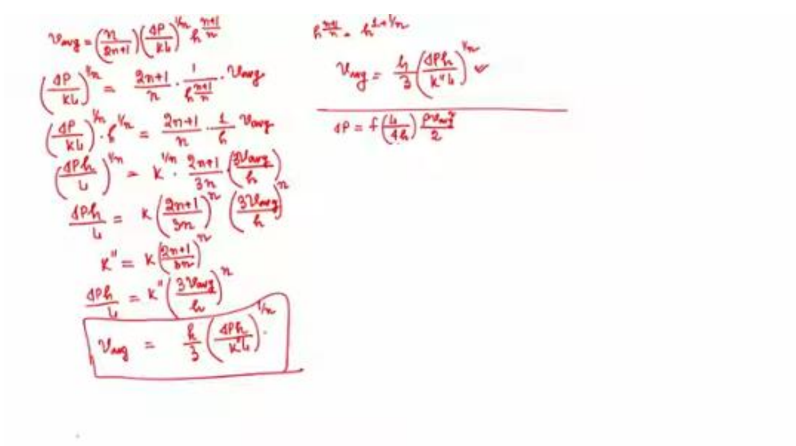




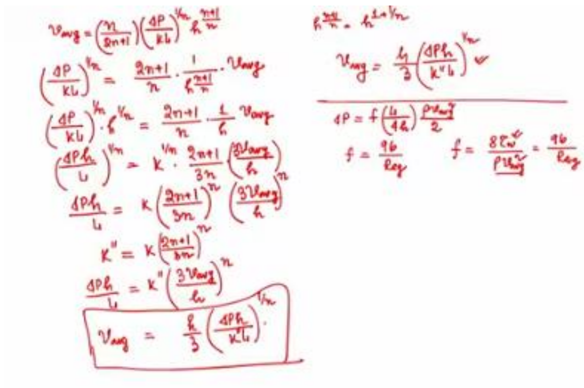
double prime, ok. Now, in terms of gamma double prime, we can get this expression: pressure drop equals to 3 multiplied by gamma double prime multiplied by L by h multiplied by $v_{average}$ by h to the power n. So, how can we get it? So, see, we will just do it over here. So, for flow between two fixed parallel plates. So, if you remember, what is our mother equation? It is delta P f L by 4h rho $v_{average}$ squared by 2.



Now, see for the pipe flow we had f L d rho vaverage square by 2. So, basically this part is nothing but the hydraulic diameter. So, for the pipe it becomes diameter, for this one, as I showed you a little while ago. So, d h becomes hydraulic diameter becomes 4 h, ok. Now, so f equals to, in this case, 96 by generalized Reynolds number, ok.

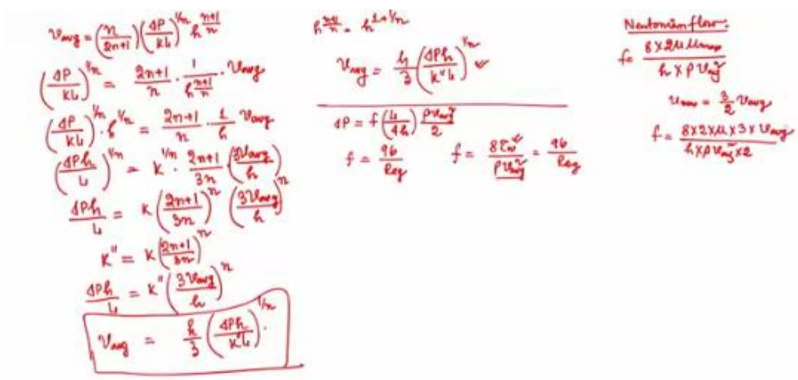


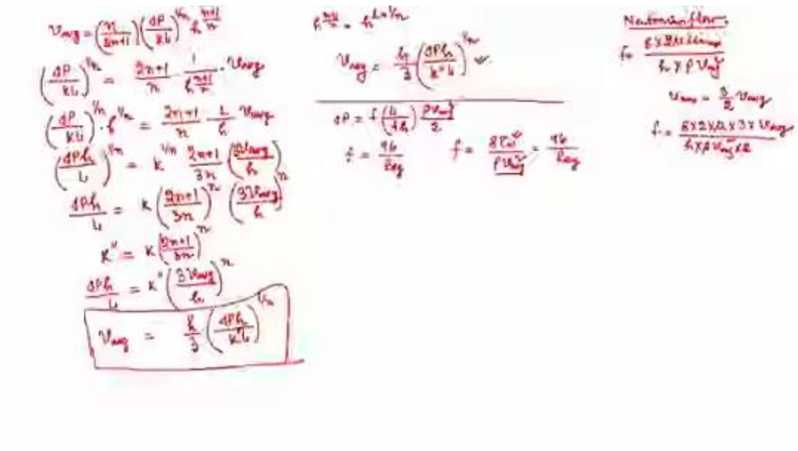
So, what is f? f is the Darcy's friction factor; it is actually defined by 8 τ_w by rho vaverage square. So, this part is viscous shear stress occurring at the wall, and this denominated part is nothing but the dynamic pressure. Now, this one becomes 96 by Reynolds number for the Newtonian, and for the non-Newtonian, we will put the Reg. So, how does it come?

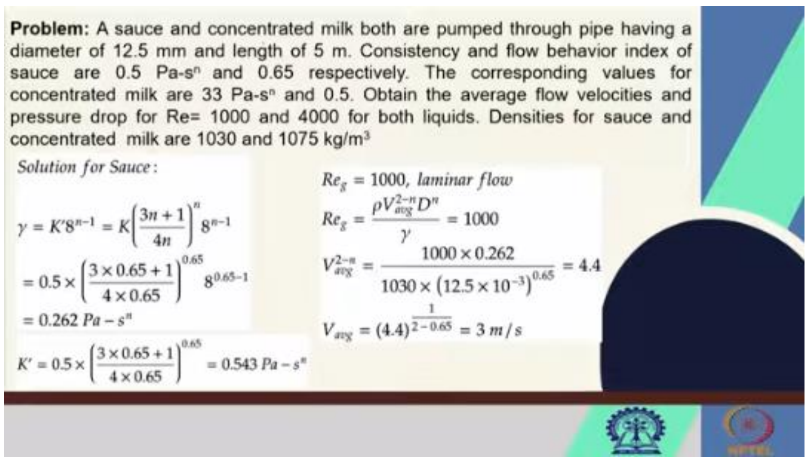


So, if you recall, the tau_w for the Newtonian flow, for the Newtonian flow, tauw becomes. So, 8 this tau_w mu u_{max} divided by h multiplied by rho $v_{average}$ square. Now, u_{max} is 3 by 2 times, and 1.5 times of $v_{average}$. So, here we will just put this one, the f value, f equals to 8 to mu 3 by 2 multiplied by $v_{average}$.

So, what is Reynolds number? Reynolds number is rho $v_{average}$ multiplied by 4 h by. So, if you put all those values over here, so you will rearrange this one, you will get delta p equals to 3 gamma to the power l by h 3 average by h, ok. Now, we will quickly solve a problem that it says. So, a sauce and concentrated milk both are pumped through a pipe having a diameter of 12.5 millimeter and length of 5 meter.

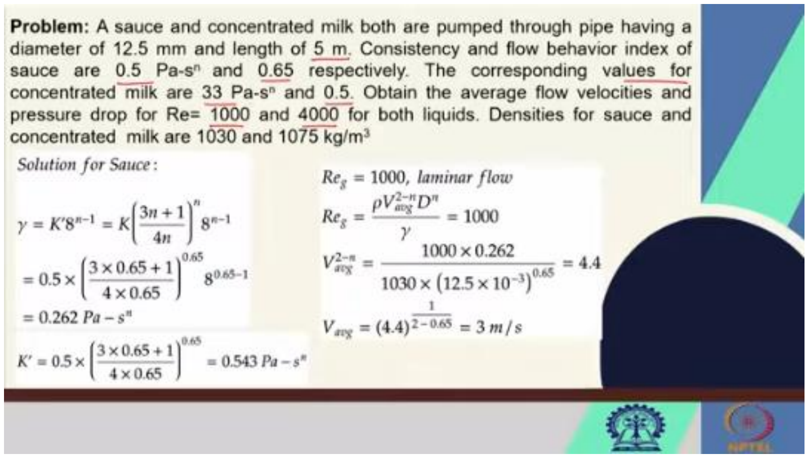






Consistency and flow behavior index of sauce are 0.5 Pascal s to the power n where n value is 0.65. The corresponding values for concentrated milk are 32 Pascal s to the power n and 0.5. Obtain the average flow velocities for pressure drop Re 1000 and Re equals to 4000, and the densities of sauce and concentrated milks are given. So, the thumb rule is if the Reynolds numbers are not given, always check for the Reynolds number at first whether the flow is laminar or turbulent, ok. Now, here since we will just, you know, take example of 1, ok, then we will just look at it how to do it.

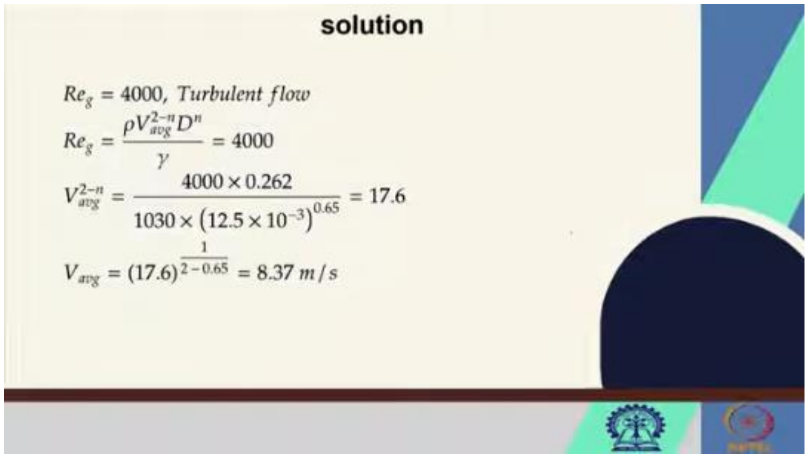
So, Reynolds number over here is given 1000, it is that means laminar flow. Now, gamma equals to k prime to the power 8 to the power n minus 1. So, n values are given, k values are given also. So, if we put all those values, we will get gamma equals to 0.262 Pascal s to the power n. We can get k prime is nothing but only this part, so 0.5 multiplied by 3 into 0.65 plus 1 divided by 4 into 0.65, it will give us 0.543, so these values will be handy

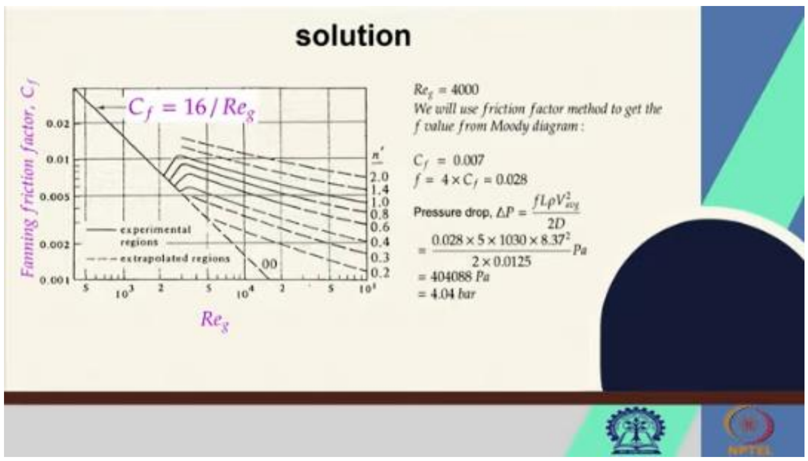


Because gamma is required for the calculation of the Reynolds number, now you see average flow velocities are being asked over here. That means we will get the velocity from the Reynolds number 1. So, we have 1000, gamma value we have already calculated, $v_{average}$ value is unknown, d is already known. So, 12.5 millimeter diameter. So, 12.5 multiplied by 10 to the power minus 3 gives us meter. So, 1000 is the Reynolds number multiplied by 0.262, this is the gamma, and finally, if you calculate.

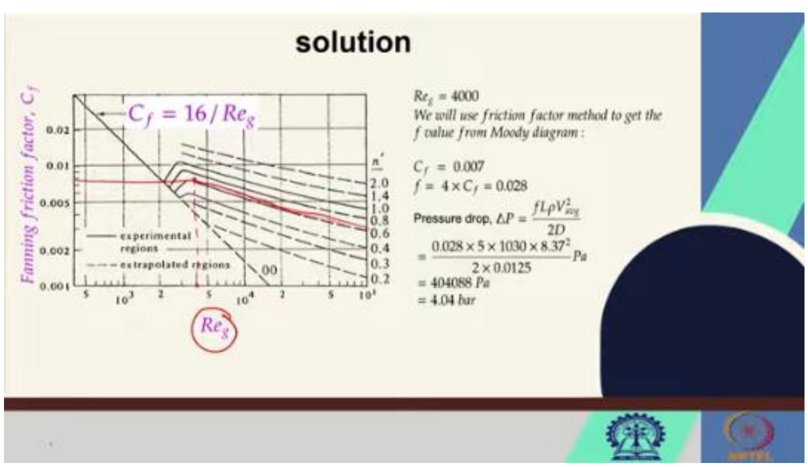
So, $v_{average}$ is coming to 3 meters per second. Now, we can calculate the pressure drop using this equation or you can directly put f L $v_{average}$ square by 2 G D where f value is, you know, your 64 by Re. Now, delta P equals to K prime 4 L by D 8 $v_{average}$ by D to the power n. n values are known already; we have calculated vaverage over here. If you put all these values, K prime, you will be able to get 1.18 bar. Or if you want to use the different form, this also gives you 1.18 bar, as in the friction factor method.

Now, if you have 4000 as the turbulent flow. So, the same way, you will calculate the $v_{average}$ velocity is 16 points to the power 2 minus n is to 17.6, and $v_{average}$ you will get 8.37 meters per second. So, remember, we have turbulent flow; we have to take help of the modified Moody's diagram for the non-Newtonian fluid flow. Therefore, we will get the Reynolds number over here.





So, 4000. And we are having n prime. What is n prime? We are having 0.4, right? Ah, 0.65. Okay, n prime we have 0.65. So, if you look at it, 4000 over here and 0.65 over here, to draw it like this, okay? We will end up with getting. Over here. So, let us say it is C_f equals to 0.007.



So, f equals to 4 into C_f, 0.028. And finally, you can put this f value, and you will be able to get the pressure drop, all right. So, this is our whole discussion about the non-Newtonian fluid flow between fixed parallel plates. So, we will, you know, complete this part over here, and we will stop here.

Thanks.