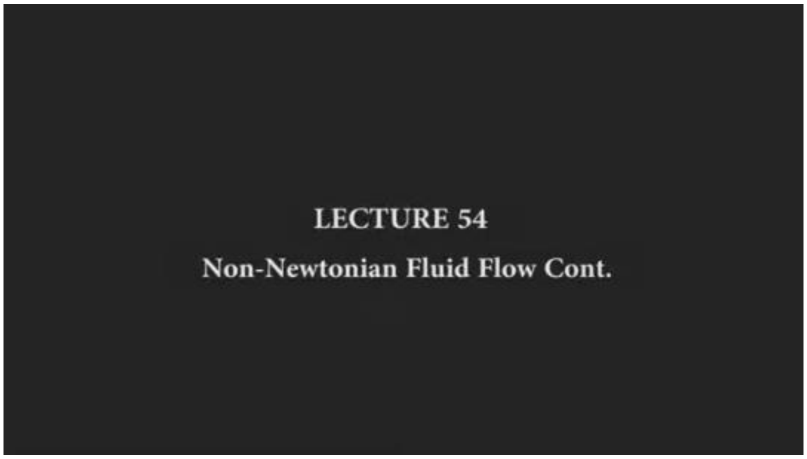
IMPACT OF FLOW OF FLUIDS IN FOOD PROCESSING AND PRESERVATION

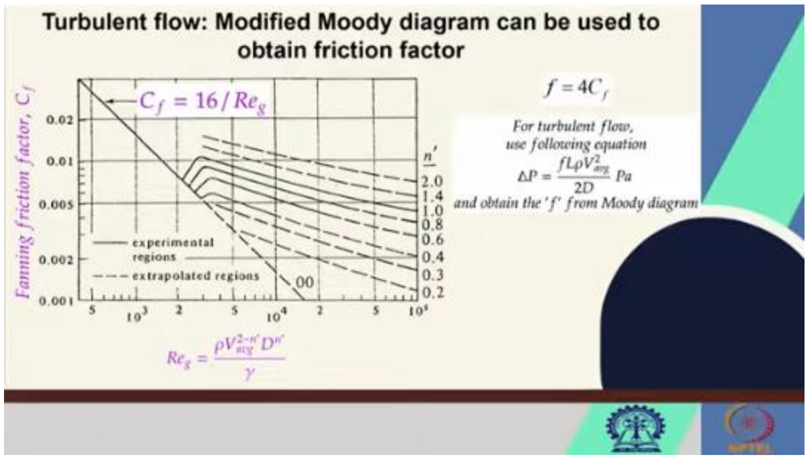
Lecture54

LECTURE 54 : Non-Newtonian Fluid Flow Cont.

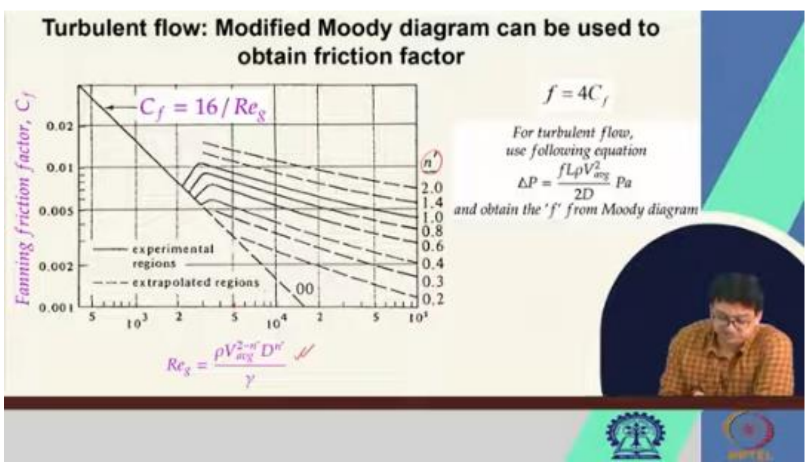
Welcome back. So, in the last class, what did we discuss? We discussed the pressure drop equation of the non-Newtonian fluid when it is in the laminar region, ok. So, we have derived the whole pressure drop equation and we have seen the final form of the equation, which one to use in order to calculate the pressure drop. Now, what if



there is a turbulent flow, how to do it. So, in this topic under the non-Newtonian fluid flow, when we will be calculating the pressure drop for the non-Newtonian fluid flow under turbulent region, we will use a modified Moody diagram or modified Moody chart, ok. So, if we had Newtonian fluid, we have seen perhaps we have used Moody diagram only, but in this case, we will be only using the modified Moody diagram because it has been modified for the turbulent flow and non-Newtonian fluid flow.



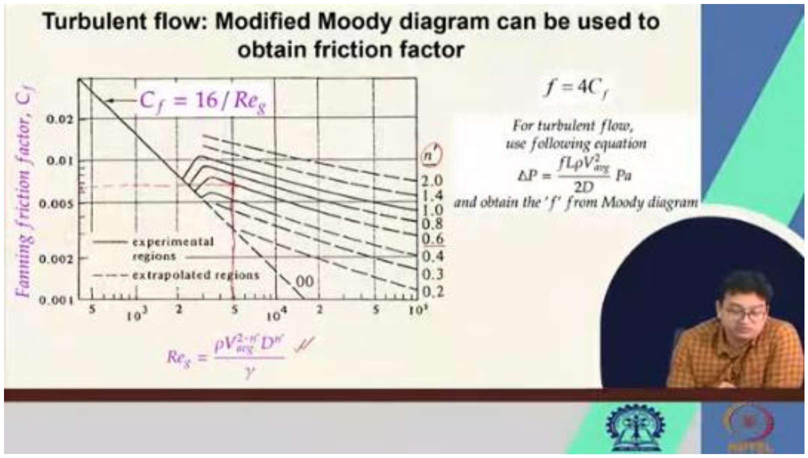
So, look over here: in the x-axis, we have a generalized Reynolds number in the x-axis, it is in the log scale, and in the y-axis, we have the friction factor, fanning friction factor, and those values are plotted at a different value of n prime, the flow behavior index. Now, let us say we have the calculated generalized Reynolds number of 5 into 10 to the power 3. So, how will we do it? And n prime is, let us say, 0.6.



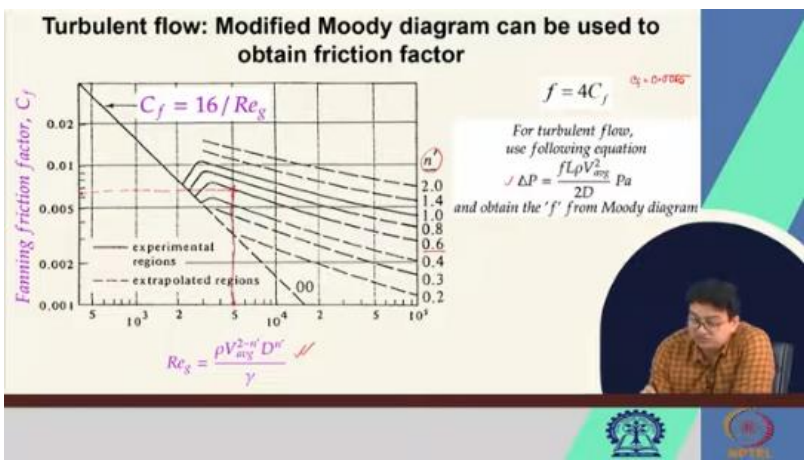
So we would follow this line like this. And this is the line over here. It will go up. 0.6 up to this. So this is the R intersection point.

So we have 5 into 10 to the power 3 and n prime is 0.6. Then we will try to draw a line that is parallel to the x-axis. And this is our value of the Fanning friction factor, that is 0.005, maybe 0.0065, 0.789, and 0.01, OK. So, we will have, therefore, C_f equals to we may have

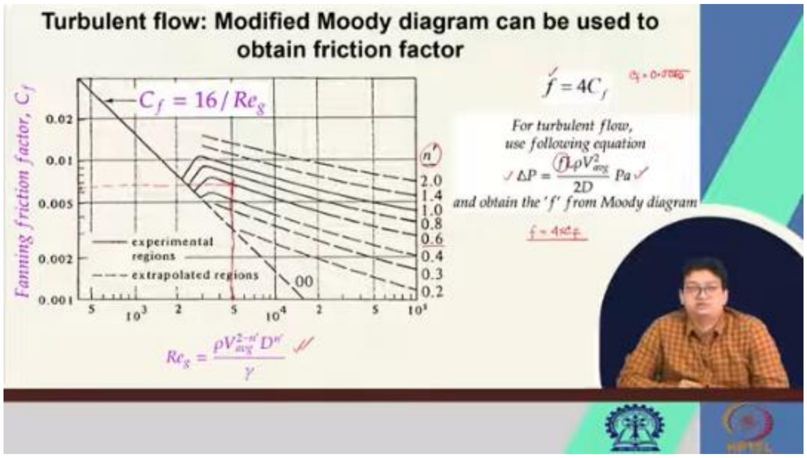
0.0065. Now, as I said, the mother equation is delta P equals to F L rho v square by 2 D, OK.



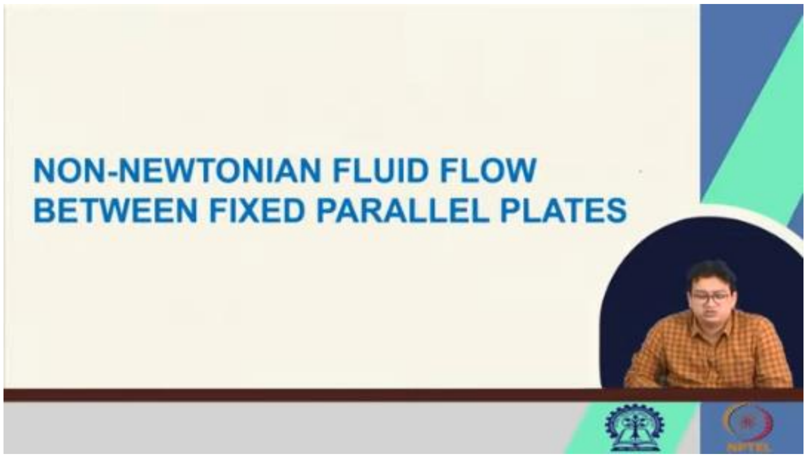
So, we have to have this small f as the only friction factor. The Fanning friction factor is C_f, that is 16 by Re_g, OK. Now, so 64 by Re_g is the f only. So, 4 into C_f is the friction factor. So, simply we have to do f equals to



4 into C_f, ok. So, once we have the C_f value, 4 into C_f multiplied by capital L rho $v_{average}$ square by 2 D, that will give the, you know, pressure drop in Pascal, alright. So, this one completes the non-Newtonian fluid flow through the pipe in the laminar region and in the turbulent region, ok. So, laminar flow and turbulent flow. Now, we will slowly move to another one: the non-Newtonian fluid flow between two fixed parallel plates, ok, between two fixed parallel plates.



So, now the thing is, where does it happen in the flow of fluid between the fixed parallel plates? So, in food processing, in food process operations in many industries, there is a heat exchanger, that is, a plate heat exchanger, you know, plate heat exchangers. There are several plates that are stacked. So, like this. So, through one, you will have the hot fluid going in, and another one, you will have cold fluid in, cold fluid and hot fluid. So, cold fluid is usually food, maybe milk or juices or sauces; hot fluid is usually hot water.



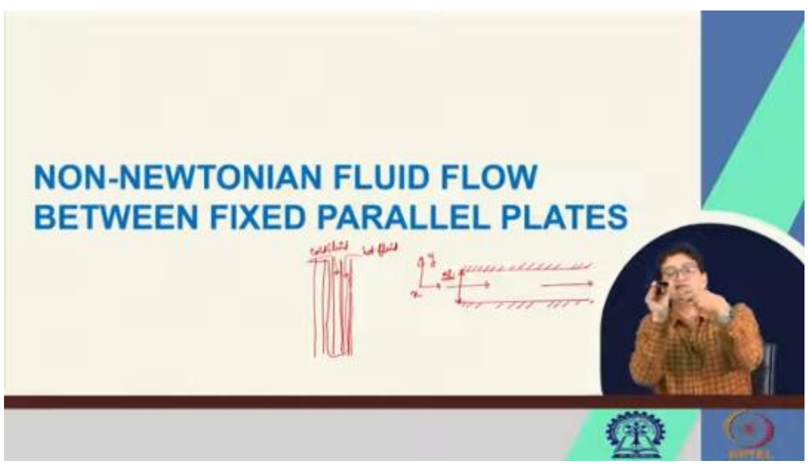
So, they will travel like this, and they do not mix because they are separated by the plates and the gaskets. So, while the hot fluid flows from one end to another end, and cold fluid flows counter-currently from one end to another end, the fluid gets heated, and the hot fluid loses some energy, and the temperature drops. You see, these are actually the kind of scenarios where the flow between the plates is happening, ok. Now, for those cases, how to calculate the pressure drop and how the velocity profile looks like, ok. Now, what we will do, for our sake of analysis and understanding, we will make it just, you know, like this, and we will ignore the effect of gravity, ok.

So, the flow of fluid is happening like this between the fixed parallel plates, all right? And these are what we will consider. So, this is in the y direction. And this is in the x direction. The gap between the two plates, we will say, is 2h. So, just like I have brought one thing.

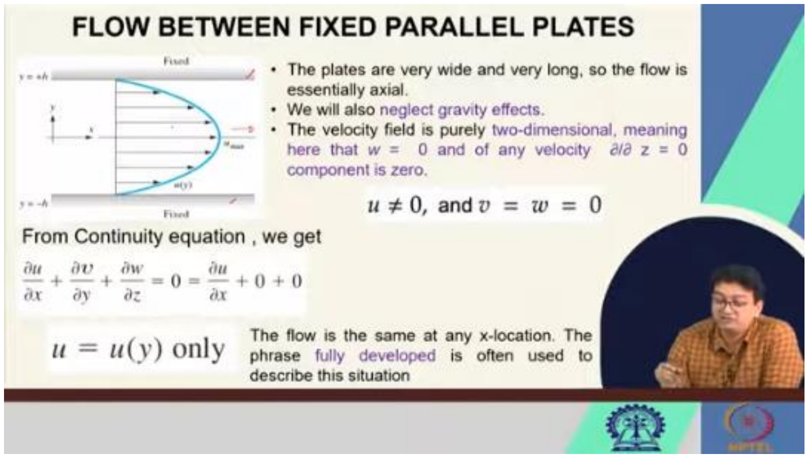
So, this is, you can see, this is the gap. So, this is the gap through which the fluid is flowing. So, this is the cross section. And this thickness, the gap between—so this is the top and this is the bottom plate. You can think, see, like this.

Top plate, bottom plate—they are both fixed. The fluid is flowing like this and coming out. So, the gap between these two plates is 2h. So, the flow is in the x direction. So, the velocity profile will be generated in the y direction.

So, that is, I think, what you can understand. And so, it is in the x-direction, y-direction, and this is in the z-direction. The z-direction, the depth is, let us say, it is a b. So, the depth of the plate is big in the heat exchanger. These gaps usually vary between 3 to 5 millimeters, sometimes even less. But the other plates are much bigger, like 200, you know, yeah, like 20 centimeters, something like that—15 centimeters, 20 centimeters, like that. Okay, so now we will go to the next part: how to proceed with the analysis. So, as I said, here we have both plates, and they are fixed. The flow is occurring in the x-direction, and the velocity profile is in the y-direction.

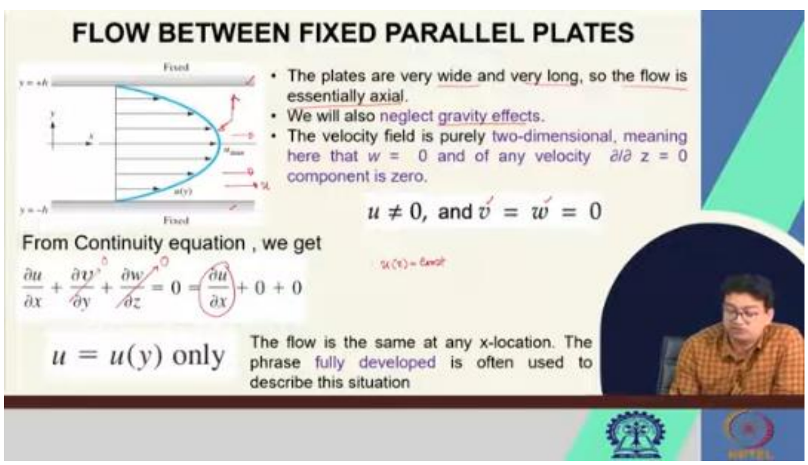


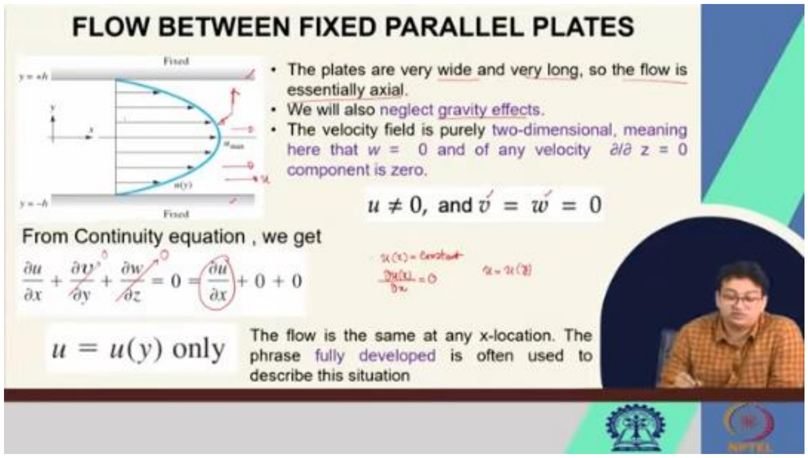
The plates are, as mentioned, very wide and very long. So, the flow is essentially axial. We will neglect the effect of gravity. And the velocity profile, or the velocity field rather, is two-dimensional. That means in the z-direction, there is no flow, okay? And in the y-direction—so overall, the flow is only in the x-direction, okay? So, there is no flow in this direction or the other direction either. So, that means if we look at it, we will essentially have the v component and W component—that is, in the y-direction and z-direction—the component of the velocity becomes essentially 0. u is not equal to 0 because u is in this direction.



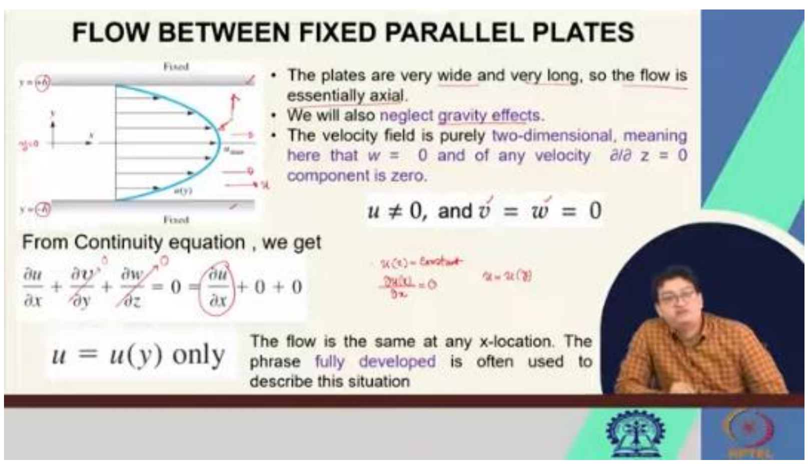
It is in the axial direction. Now, if you look at the continuity equation for the Cartesian coordinate, it looks like del u / del x plus del v / del y plus del w / del z equals 0. Since this is also 0, this one is also 0. So, we will be left with del u / del x, as you know, so that means, this is constant, right? This is constant.

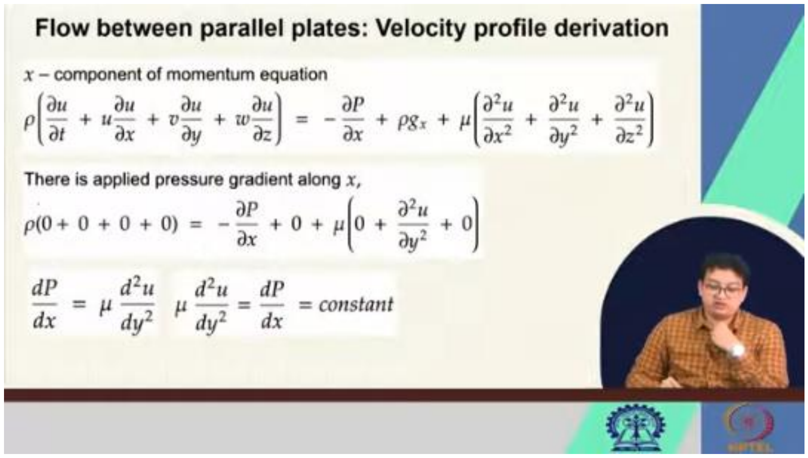
So, that is why if you do del u / del x, it becomes 0. That means, if you look at this figure, I have said earlier that u becomes function of u_y. That means, there is a velocity is changing along the y direction. So, in the center line, we have y equals to 0. Here we have plus h, here we have minus h. So, that is our



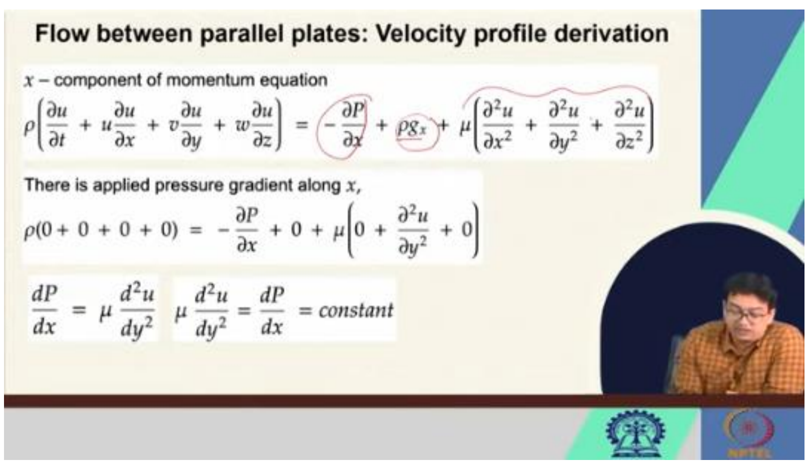


basic understanding. The flow is fully developed and we will look at the scenario of the laminar and the turbulent flow. Now, flow between the parallel plates how to derive the velocity profile. So, again we will take the help of the momentum equation. So, momentum equation in the x component of the momentum equation is rho del u / del t u del u / del x plus v del u / del y plus w del u / del x equals to minus del p / del x plus rho g_x plus this is the viscous term ok.



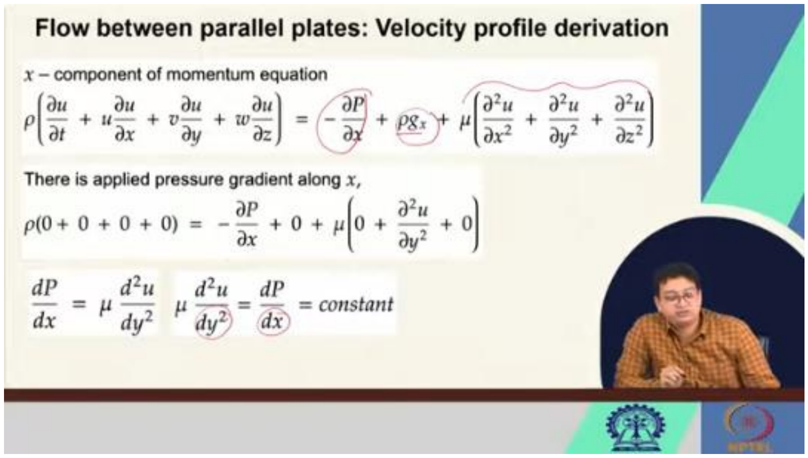


So, this is the term. Now, this is fine pressure gradient along the x, here steady state this becomes 0, this also becomes 0 del u del x it has come from your derivation earlier. del u / del y equals to 0, del u del z equals to 0 because no component in the y direction in the z direction becomes equals to minus del p / del x. Ignore the effect of the gravity. Here you will be left with only del square u by del y square u. So, del square u by del y square. Now, what comes finally?



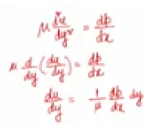
So, since this is the function, here you see we have the y, here we have the x, p is changing along the x. So, we can omit the partial form and write it as an ordinary differential form. So, we have dp dx equals mu d2y / dy square, which equals a constant. How do we solve this? This is the separation of variables. It is a standard method because here we have y, here we have x. So, both are different.

But this equals a constant means we have to use the separation of variables. But we will not follow that path. But we want to use this equation to develop the velocity profile. How do we do it? So, now we will look at the final form.



So, mu equals dP dx. This final form we will be using. Because this is the one, u by dy square equals dp / dx. So, now, we can write this as d dy of du / dy equals dp / dx.

So, du/dy equals to 1/mu dp/dx. So, then we will be having dy over here. Now, what do we have to do? We have to integrate it, right?



So, we will integrate it. Sorry, this one is, OK. So, du is equal to 1/mu dp / dx dy plus some constant C_1. So, u is the velocity profile; therefore, 1 / mu dp / dx y plus we will come from this one. A little mistake is there.

So, du / dy, d/dy to 1 / mu dp / dx. du / dy therefore, will be 1/mu dp / dx y plus C_1. So, I have missed this one.

Now, if you look at the u, the further you will do it. So, 1 by mu dp / dx y square by 2. We will have a y square by 2 plus C_1 y plus C_2. So, that is the one we have.

$$\mu \frac{d^2u}{dy^2} = \frac{dp}{dx}$$

$$\mu \frac{d}{dy}\left(\frac{du}{dy}\right) = \frac{dp}{dx}$$

$$\frac{d}{dy}\left(\frac{du}{dy}\right) = \frac{1}{\mu}\frac{dp}{dx}$$

$$\frac{du}{dy} = \frac{1}{\mu}\frac{dp}{dx}y + C_1$$

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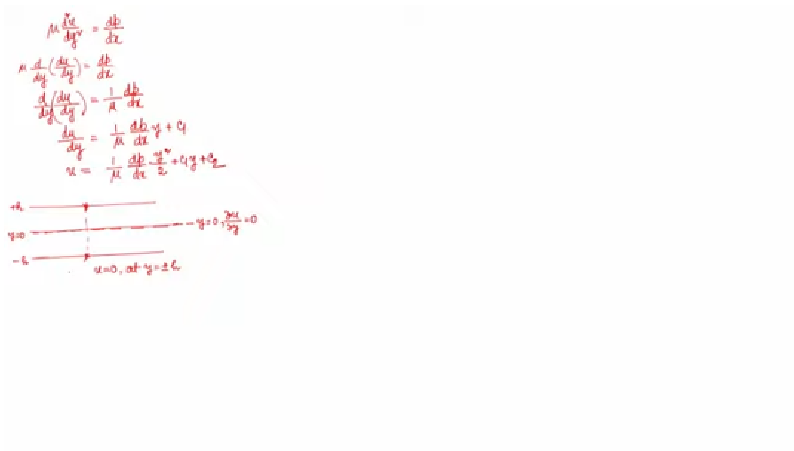
$$u = \frac{1}{\mu}\frac{dp}{dx}\frac{y^2}{2} + C_1 y + C_2$$

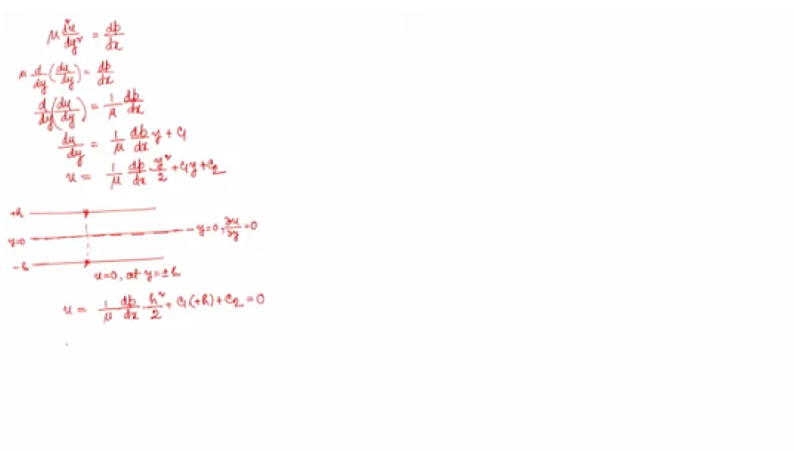
Again, we will go for the same boundary condition. What is the boundary condition? So, let us have flow. This is our line of symmetry, or rather you can say plane of symmetry. This is plus h minus h y equals to 0.

So, at y equals to 0, we have del So, we have one boundary condition, or maybe what we can do is we can also take another one. So, here also we can have del u / del y equals to 0, that also can do, or also another thing we can do is u equals to 0 at Another one we can do is u equals to 0 at y equals to plus minus h. That means over here and over here. That is at the boundary.

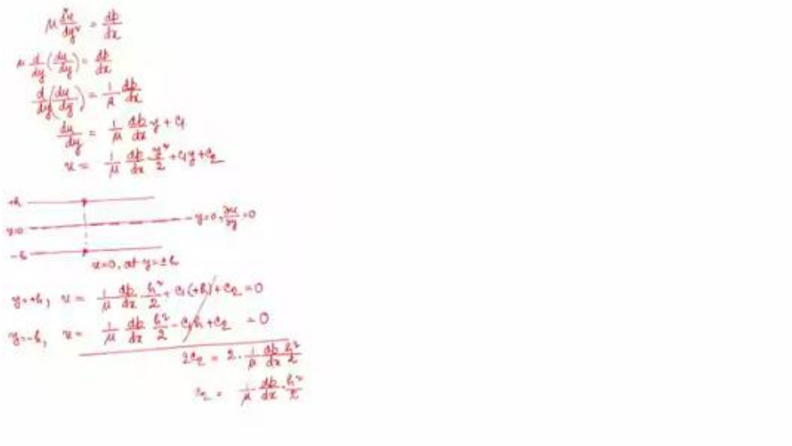
So, that is actually a no-slip boundary condition. So, let us do it. Therefore, let us apply this one boundary condition. So, u equals to 1 by mu dp / dx h square by 2. So, this is equals to plus C_1 plus h plus C_2.



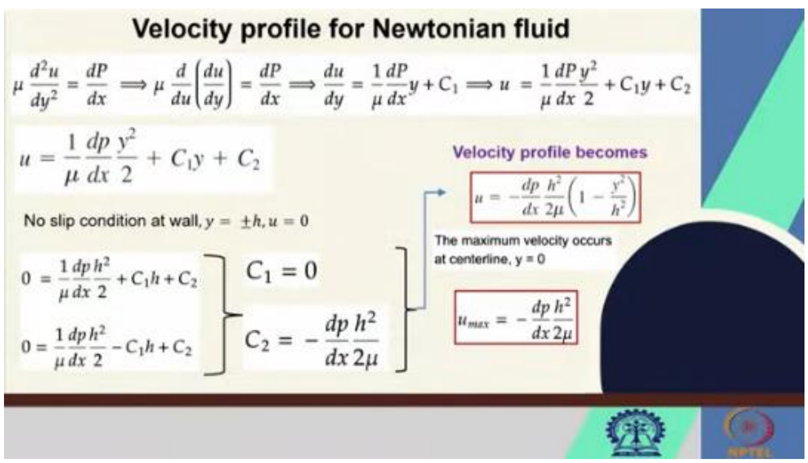
This is 0. Another one, u equals to 1 by mu dp / dx. This will be so when we are putting, actually, this is y equals to plus h, y equals to minus h, h square by 2. It will be squares, so minus C_1 h plus C_2, so that is also equals to 0. So, if you add them, what will happen? So, if you add them, then we will be left with this C_2 part. So, this is going to be 2 C_2 will be 2 times of 1 by mu dp / dx a square by 2.



So, C_2 is 1 by mu dp / dx a square by 2. So, both parts. Now, if you put, so let us look at this one. So, yeah.

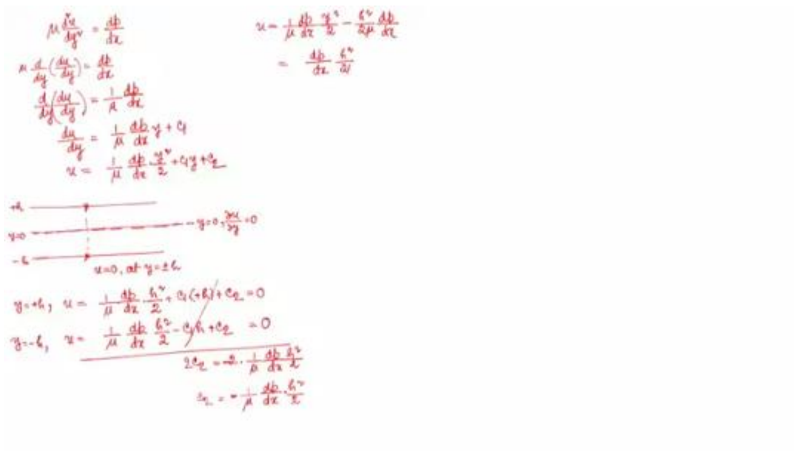


So, if you put this one, so C_1 becomes 0 and C_2 as I said 1 by a square by 2 mu. So, a square by 2 mu dp / dx over here, sorry, minus, yeah. minus over here because if you plus there you take it in the right side you will have a minus ok. So, that is one ah C_2 value we have C_2 equals to minus dp / dx a square by 2 mu. Now, that velocity profile therefore, if you put the C_2 value over here.

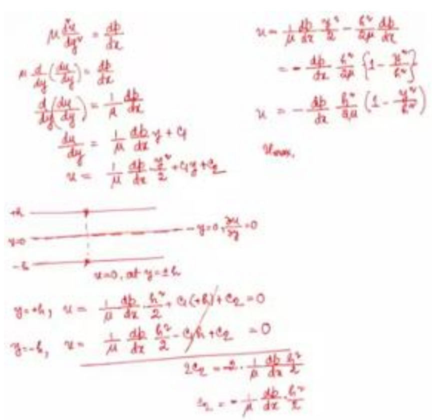


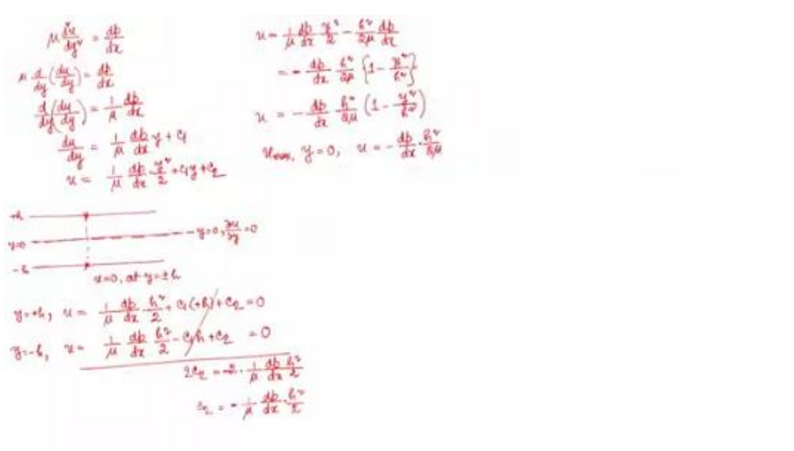
C_2 value over here u equals to where is it 1 by mu dp / dx y square by 2 so C_2 is minus a square by 2 mu dp / dx. So, it is a square by 2 mu minus dp / dx. So, that means what we can do we can take dp / dx we can take it out and we can also take a square by 2 mu.

Let us see how does it look like. Let us say minus. So, a square by 2 mu. So, 1 minus we will be having y square by h square. So, that means u equals to minus dp / dx h square by 2 mu 1 minus y square by h square.

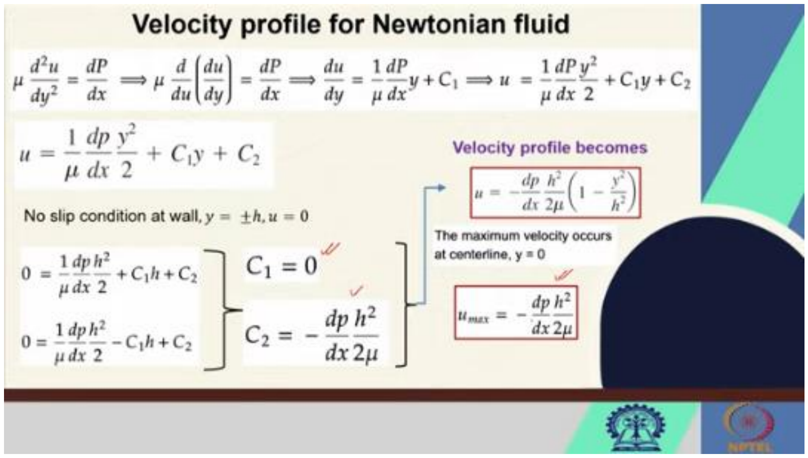


Now, what is u_{max}? u_{max} again at y equals 0. So, u has to be minus dp / dx divided by a square by 2 mu. So, this is our u_{max}. That means, u_{max} at the center of the

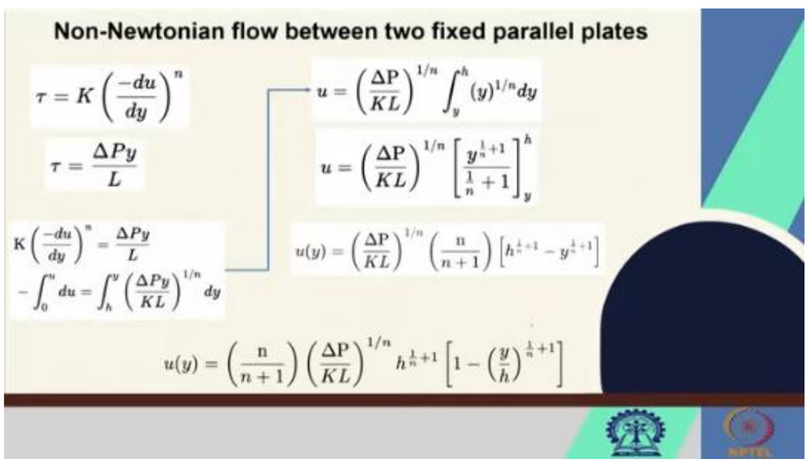




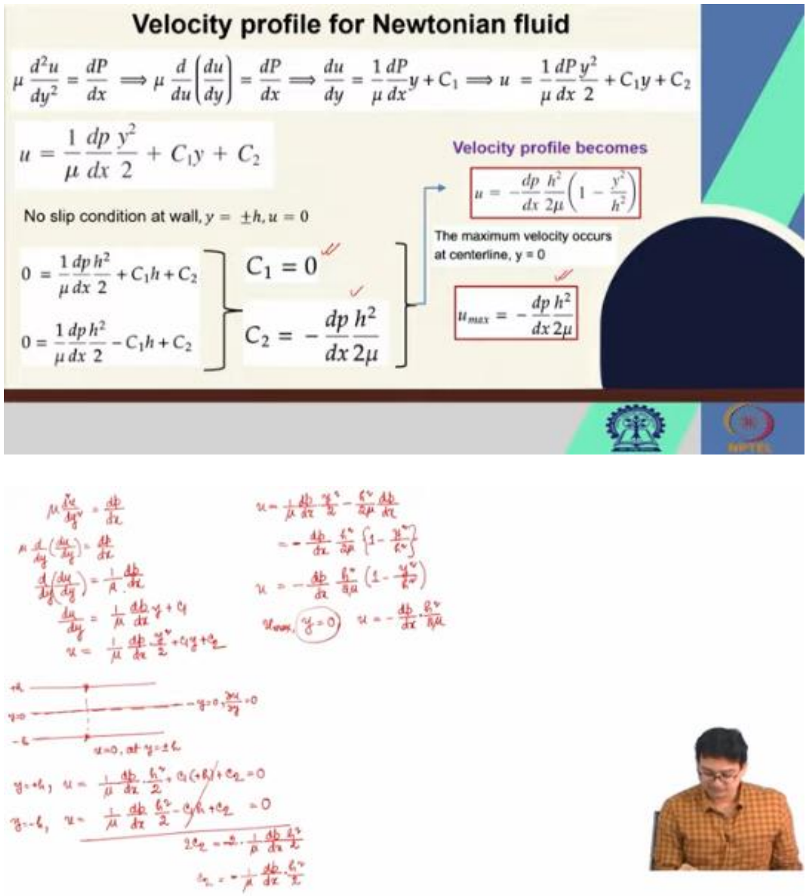
system. So, that is what we have: u max equals minus dp dx multiplied by h square by 2 mu, right? Now, what do we have to do? We have to look at the average velocity. So, how will you do it?



Let us look at it, ok? Again, what will we do? We will follow the same path, ok? So, so far we have this u max value: minus dp dash of a square by 2 mu, that is for our Newtonian fluid.

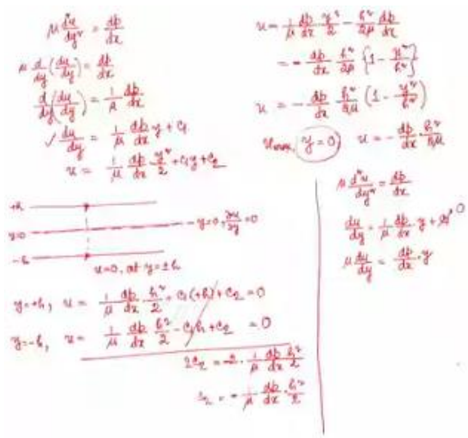


Now, the point as I said before we again take help of this final form over here ok, this one. So, that we can derive the non-Newtonian one ok and we will see how it actually differs from velocity profile for the Newtonian and for with the non-Newtonian one. How you will do it? In the same way, we will do again. I will go over here.

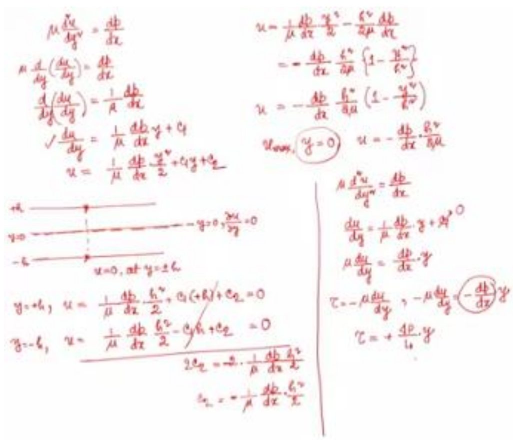


What is our main form of the equation? So, we had, if you remember, so mu d2y by dy square equals to dp / dx. So, this is the one we had. Now, if you look at this one, so du dy equals to 1 by mu dp / dx multiplied by y plus C_1.

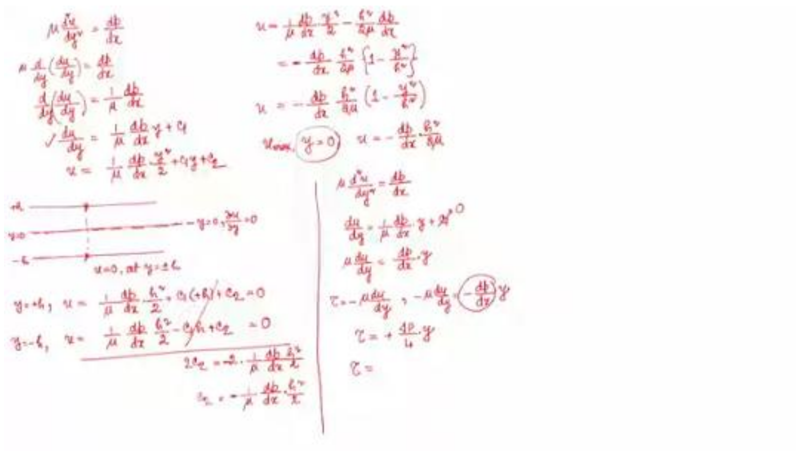
Now, what we have seen that C_1 is already 0. So, we will put this one equals to 0. Now, mu du / dy equals to dp / dx multiplied by y. Now, what is tau? So, tau you remember we had so tau is minus mu du / dy.



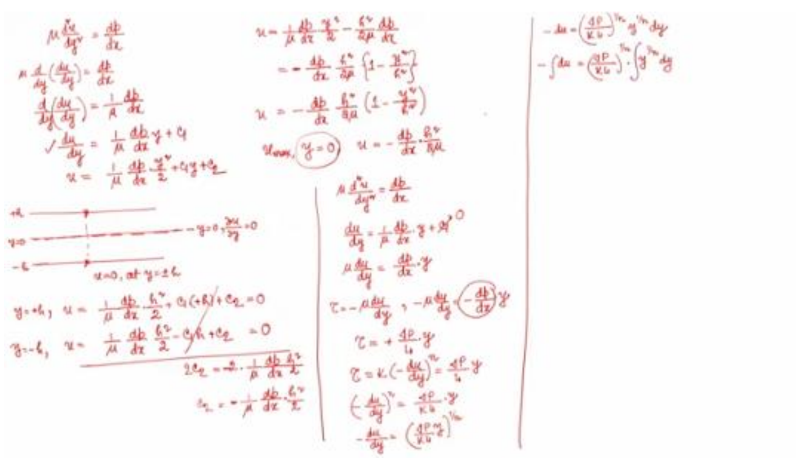
So, if we do this, that means minus mu du / dy is equal to minus dp / dx multiplied by y. Now, we put this one tau minus. So, this is actually minus dp / dx. Now, if you have a form, you can write it in the form of delta P by L if you integrate it multiplied by pi. So, that means tau is delta P y by L for the non-Newtonian, as we already know.



So, if you remember for the non-Newtonian flow through the pipe, it was different, OK. So, we had a delta P r by 12, OK. Now, here we will be putting again equals to k minus du/dy to the power m, that is also equal to delta P by L multiplied by y. So, what I said, our aim is to get the u. So, we have to rearrange a little bit. So, how will you rearrange?

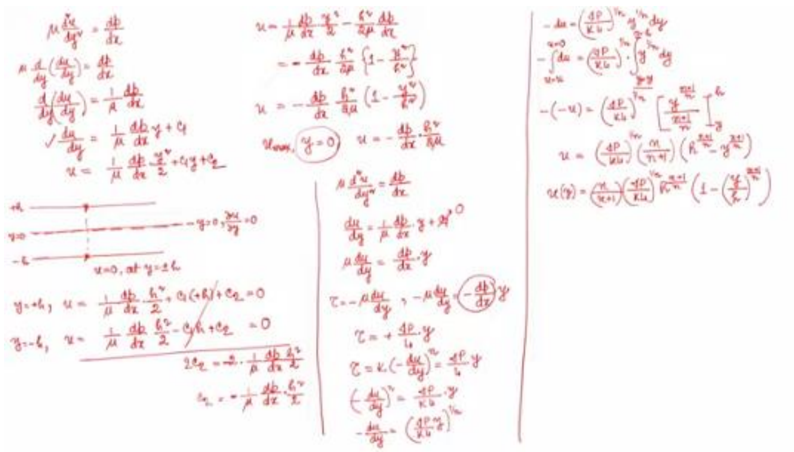


So, minus du/dy to the power n equals delta P by kL multiplied by y, du / dy minus equals delta P by kL y to the power 1 by n. Now, I will come to this side. Minus du equals delta P by kL to the power 1 by n y to the power 1 by n dy. We can write now we will integrate du equals delta P by kL to the power 1 by n dy. So, what will we do? So, if you integrate from y equals to y to y equals to h, what will be the values?



So, over here, that means you will have at y equals y, you have some velocity of u equals u, and at y equals h, u equals 0. So that means minus of du minus minus u, so you will have the positive delta p by. For n y to the power, this one will be n plus 1 by n divided by n plus 1 by n. So, we will have y to h. So, u will be delta p by kL, delta p by kL to the power 1 by n, and then n by n plus. h to the power n plus 1 by n minus y to the power n plus 1 by n. So, you take, so u is a function of y, n by n plus 1. Delta p by kL to the power 1 by n, h to the power, we will take this one out, n plus n, we have 1 y by h.

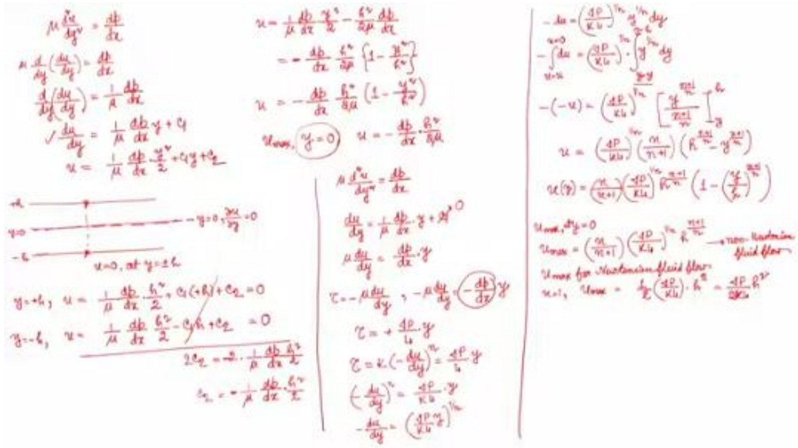
So, this one we will have n plus 1 by n. So, this is the velocity profile we have derived for the non-Newtonian fluid flow between the. So, we have come from the basic fundamental equation using the momentum and continuity equation, and initially, we have used the form in between to derive the non-Newtonian flow, ok. Now, what would be all right? So, now let us say again if n equals 1.



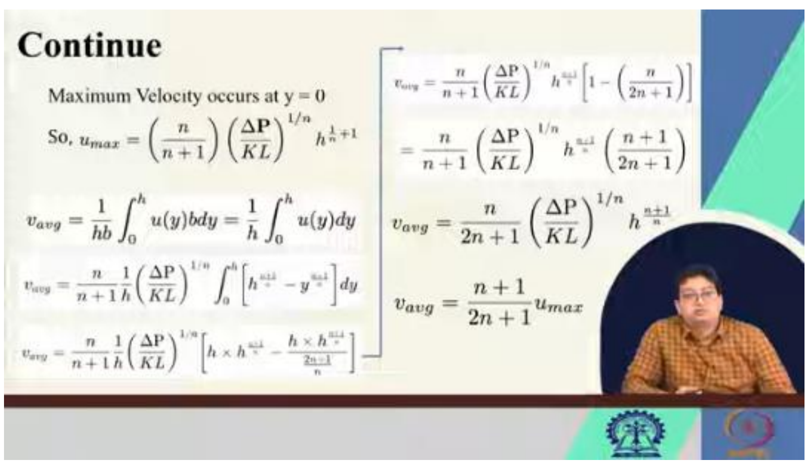
So, it will give us a form of the Newtonian flow. So, how about u max? u_{max} at y equals 0. So, u, sorry. So, u_{max} equals n by n plus 1, delta p by kL to the power 1 by n. So, here 0, that means we will be left with this one.

we will be left with this one all right. Now, if we put u max, so this is for non-Newtonian weight flow ok. Now, u_{max} for Newtonian fluid flow how it looks like then that means we will put n equals to 1 right so 1 by 2 delta p by kL so 1 by L equals to n so h to the power 2 by L so f is delta p by 2 kL h square So, delta p square by 2 k L ok.

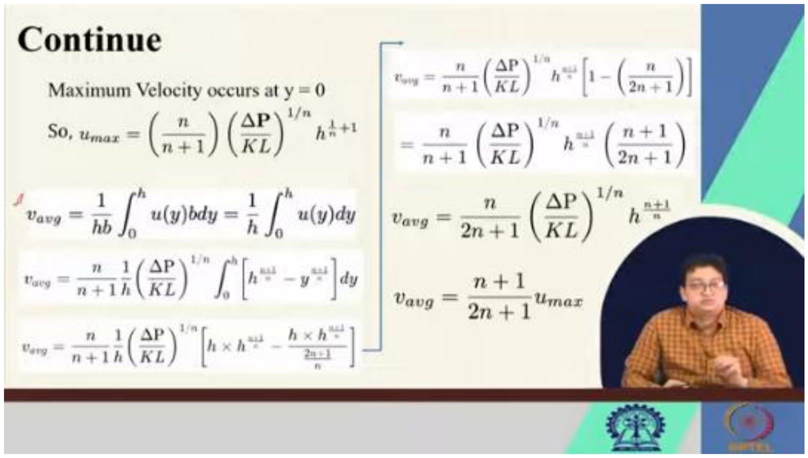
So, that is u_{max} for the Newtonian fluid flow and for the non-Newtonian we have more generic form n by n plus 1 multiplied by delta p by k L to the power 1 by n multiplied by h to the power n plus 1 by n ok. So, yeah so, the I have already derived it you can go through it some step maybe missing, but you can still since I have already derived it you can go through it and so, u_y equals to n by n plus 1 delta p by k L to the power 1 by n and here is the final form. ok and maximum velocity occurs at y equals to 0. So, u_{max} will be n by n plus 1 delta p by k L to the power 1 by n h to the power 1 by n plus 1 ok. what we will do our next interest is to calculate the v_{averag}e ok.

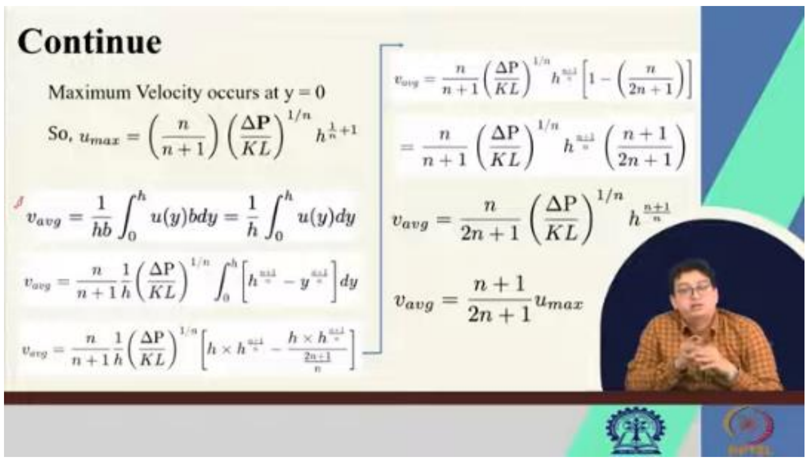


So, before I go for the calculating the $v_{average}$, $v_{average}$ velocity over here, we will just have a recap what we have done so far. So, what we have done that we have two parallel plates that are both are fixed and the fluid is flowing between the plates and we are trying to achieve or get the velocity profile and the pressure drop equation ok. So, again we have taken help of the momentum conservation equation and the continuity equation in order to derive that one. And what we have seen that by ah the basic assumptions where we have neglected the gravity



And there is no flow in the y or z direction; it is the flow in the x direction, and u is essentially a function of y because the velocity gradient develops in the y direction. If you move from the center line, you will be able to see the velocity profile. Now, using all this information and applying the boundary condition, we are able to calculate the velocity, the velocity profile, and velocity in a more generic nature. We are also able to calculate u_max, which is the maximum velocity that occurs at y = 0, meaning at the center line or central plane of the flow between the parallel plates.





So, for calculating the average velocity, we will do it next time for now. We will stop here. Thank you.