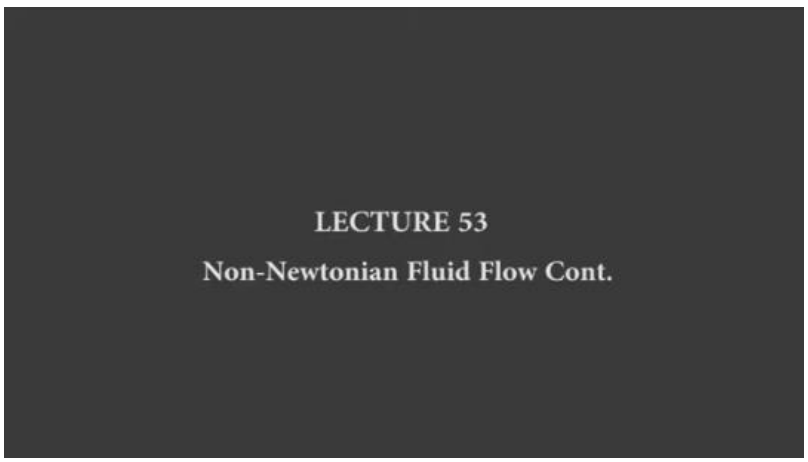
IMPACT OF FLOW OF FLUIDS IN FOOD PROCESSING AND PRESERVATION

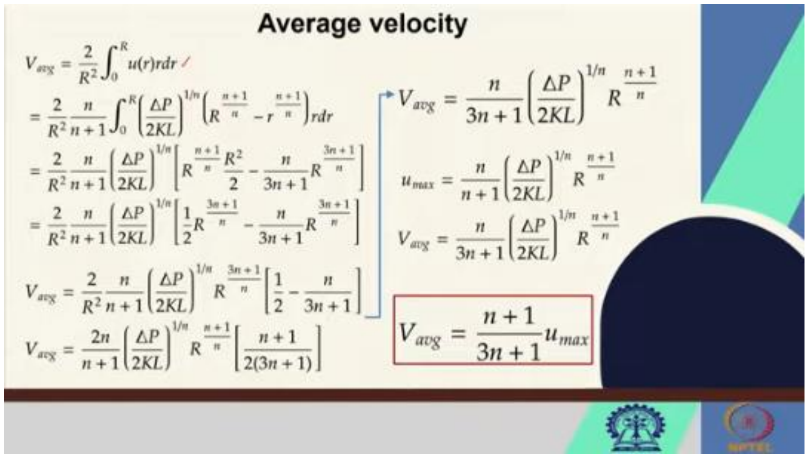
Lecture53

LECTURE 53 : Non-Newtonian Fluid Flow Cont.

Hello friends, welcome back. So, we have been dealing with the non-Newtonian fluid flow. In the previous classes, we have seen—we looked at how to derive the velocity profile of a non-Newtonian fluid flow through a pipe, OK? And we derived the velocity profile. Now, what we will be doing—a very important thing—we will be going through the average velocity, OK? Now, how to calculate the average velocity?



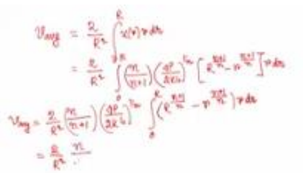
So, we have the velocity profile. From there, we will try to derive the average velocity of the non-Newtonian fluid flow through the pipe, OK? So, here is the thing. So, average velocity, if you look at it, can be calculated as it equals to 2 by R square. 2 by R square integral 0 to R u_r r dr, OK? So, this is the very basic equation. You can go back to it, OK?



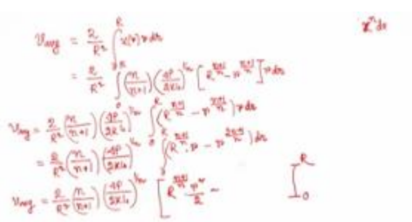
Now, we will put the velocity profile over here, OK? So, let us look at it, OK? So, $v_{average}$ equals to 2 by R square 0 to R u_r r dr. Now, what we will do here—we will put 2 by R square. We will put this velocity profile u_r smaller dr. What was that? n by n plus 1 multiplied by delta P by 2kL, delta P by 2kL to the power 1 by n multiplied by r to the power n plus 1 by n minus smaller n plus 1 by n r dr.

So, this one now we will have to work with. So, what can we do? We average. So, we can take all this out. So, 2 by R square n by n plus 1 delta P by 2kL to the power 1 by n. Now, 0 to R.

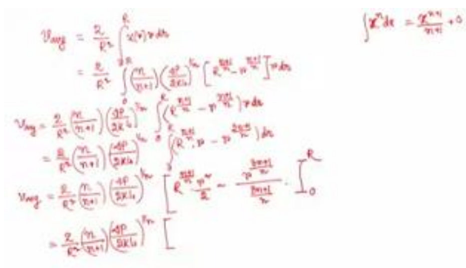
r to the power n plus 1 by n, so what we will be having is n plus 1 by n, so r dr. All right, r will be simply multiplied inside, so 2 by r square. I am not writing $v_{average}$ anymore, so just putting n by n plus 1, delta P by 2 kL, delta P by 2 kL to the power 1 by n. So, 0 to r, r to the power n plus 1 by n multiplied by r minus, minus what will we be having? r. So, plus 1, that is 2n plus 1 by n dr. So, 0 to r we have to do. So, 0 to R. So, if you put 0, everything will be 0, we will simply put capital R. So, $v_{average}$ is 2 by r square n by n plus 1 delta p by 2kl to the power 1 by n. Now, let us put it this way: r n plus 1 by n. So, I am going step by step.



This is capital R plus 1 by Okay, so what? Okay, so what we will do is wait a minute. There is one mistake here, okay? So over here, so we have to integrate, okay? So what will we be having? So here we have small r, that means r square by 2. So let us do it this way, n plus 1 by n small r square by 2 minus we have plus 1 divided by plus 1, x to the power n dx x to the power n plus 1 by n plus 1 plus C constant. So, here what we will do? So, r to the power 3n plus 1 by n plus 1 means it will give 3n plus 1. Here we will have 3n plus 1 by n in the denominator. So, comes 2 by r square n by n plus 1

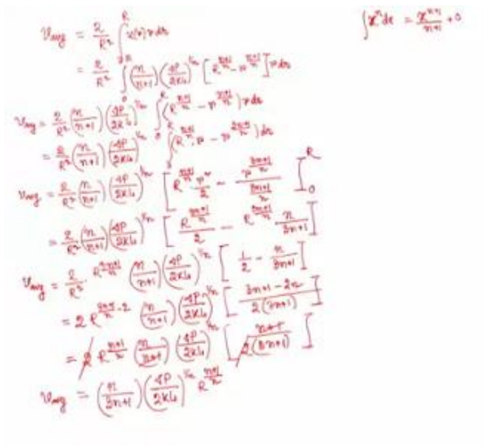


delta P by 2kL for 1 by n. So, 0 means everything is 0. We will put r. So, r square by 2 that means here multiplied means it will be simply added. 2n that means 3n plus 1 by n by 2 minus here r to the power 3n plus 1 by n multiplied by n by 3n plus 1. Now, $v_{average}$ equals to, we can take this thing out. So, 2 by r square.

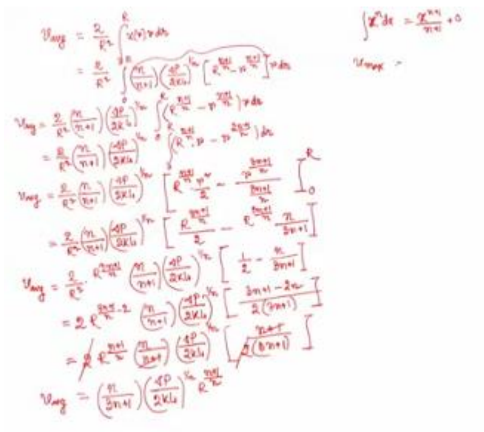


Now, I will put it over here so that we can have adjustment. Multiplied by n by n plus 1, delta p by 2kl, 1 by n. minus n by 3 n plus 1. So, 1 by 2 minus n by 3 n plus 1. So, if we do it 2 by r square, so that means, so, 2, so r to the power 3n plus 1 by n minus 2, right. So, n plus 1 delta p by 2kL 1 by n. So, here 2, 3n plus 1, 3n plus 1 minus 2n. That means 2 multiplied by r to the power n plus 1 over here by n, n multiplied by n plus 1 delta P by 2kL by 1 by n. Here comes n plus 1 by 3n plus 1 multiplied by 2. This 2, this 2 get cancelled. So, equals to n by n C, the n plus 1, n plus 1 get cancelled.

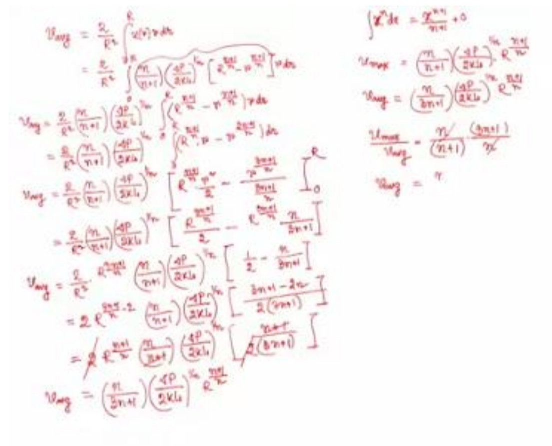
So, we can write n by 3n plus 1, right, delta P by 2kL to the power 1 by n. So, r to the power n plus 1 by n. So, this comes n by 3n plus 1 delta P by 2kL to the power 1 by n r to the power n plus 1 by n. So, this is the $v_{average}$. What was the u_{max}? We had u_{max}. So, this was our velocity profile.

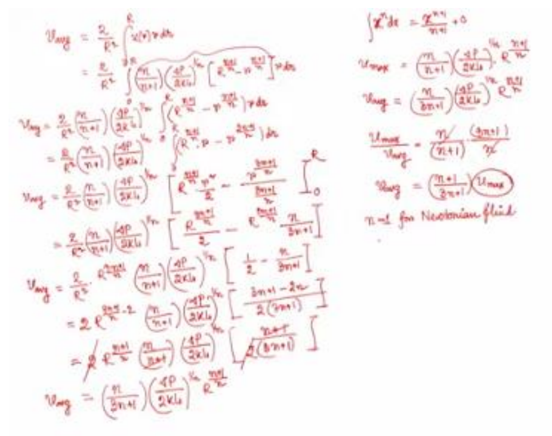


u_{max} means r equals to 0. So, we had n by n plus 1 delta P by 2kL to the power 1 by n. So, r to the power n plus 1 by n. It was u_{max}. So, that means, if you look at, I will just write n by 3n plus 1 delta P by 2kL 1 by n r to the power n plus 1 by n so if you do u_{max} by $v_{average}$ what will happen so n by n plus 1 okay so 3n plus 1 will go on top and divided when rest will be cancelled that means $v_{average}$ is

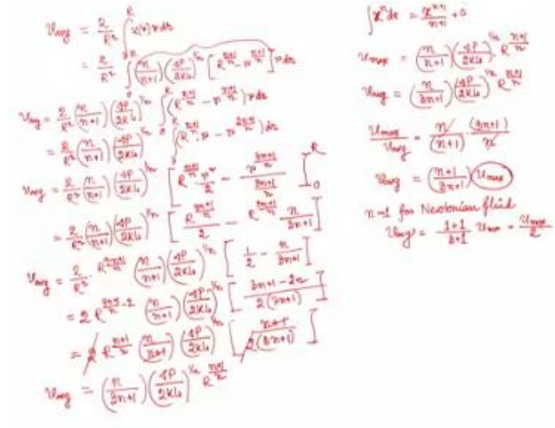


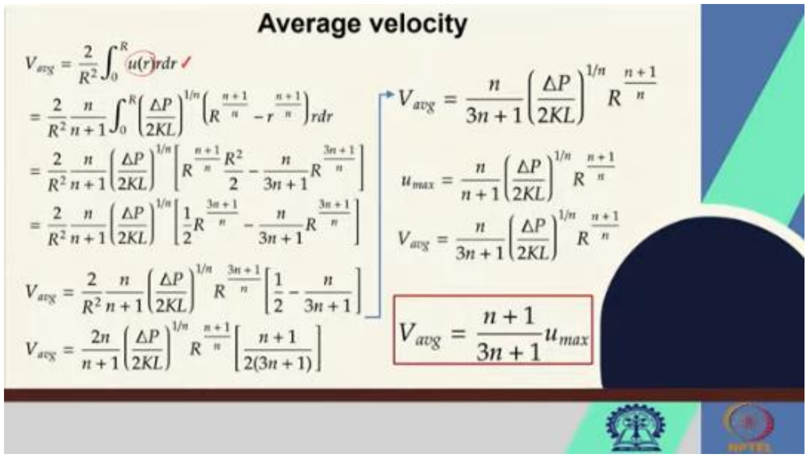
n plus 1 by 3n plus 1 of the u_{max}. So, we have maximum velocity, then n plus 1 divided by 3n plus 1 times of u_{max} gives us the $v_{average}$. Now, again, check if you have a Newtonian fluid, n equals to 1 for Newtonian fluid. Therefore, your $v_{average}$ nothing but will be 1 plus 1 divided by 3 plus 1 of u_{max} right that means 2 by 4 u_{max} by 2.



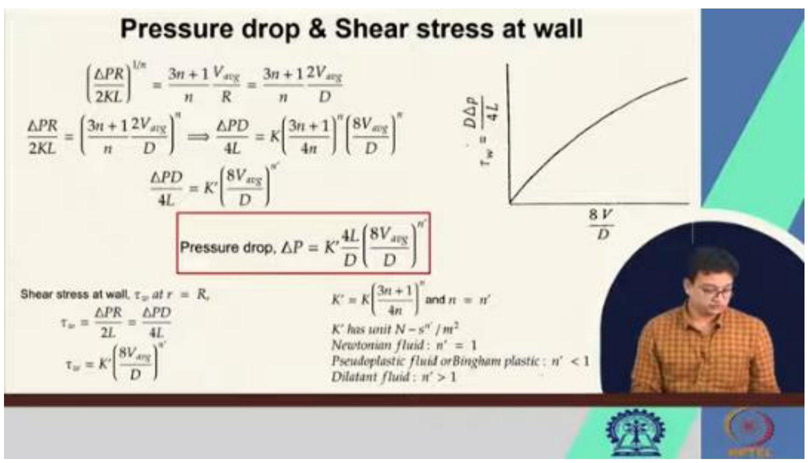


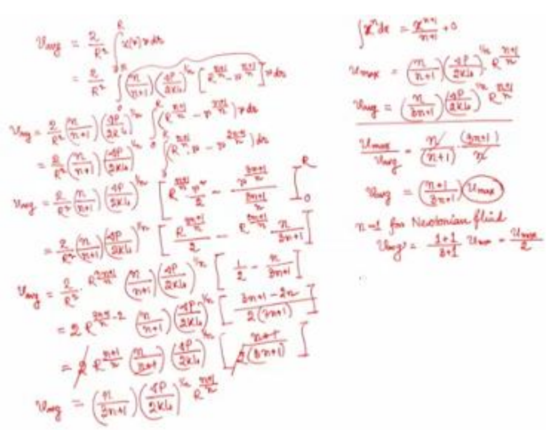
make sense ok. So, that is what we have also got that $v_{average}$ is or you can say u_{max} is the 2 times of the $v_{average}$ all right ok. So, coming back to over here. So, this is the one we just derived that $v_{average}$ equals to n plus 1 divided by 3 n plus 1 of u max ok.





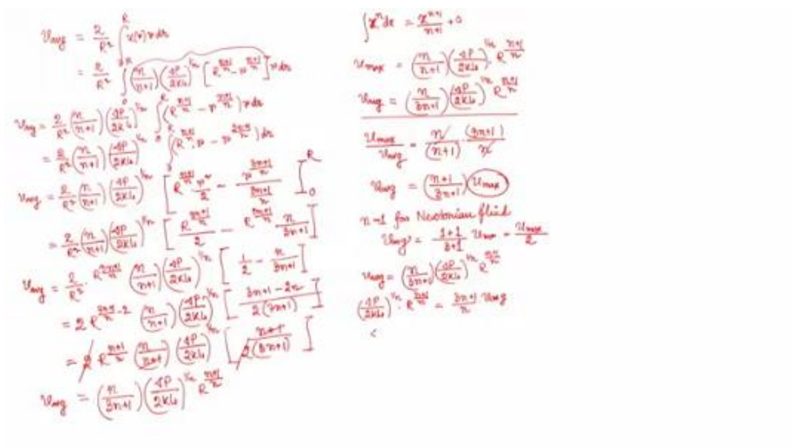
So, then now if you want to do that pressure drop want to look at the pressure drop and shear stress at the wall ok what we will do? So, we will come with this relationship. So, what we can write? So, ah if you look at this one ok vaverage We will take help of this one, vaverage.



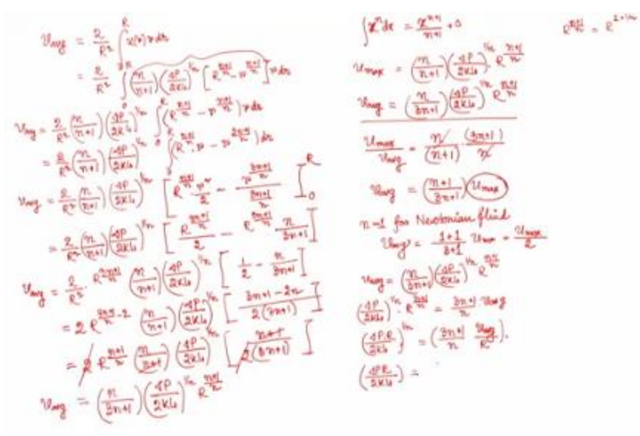


So, $v_{average}$, let us write it this way: $v_{average}$ equals n by 3n plus 1. Delta P by 2kL to the power 1 by n R to the power n plus 1 by n. So, now, this one we will rearrange because we want to calculate the pressure drop, delta P, OK. Now, how will you do it? So, delta P by 2kL to the power 1 by n to the power 1 by n. Multiplied by R n plus 1 by n equals 3n plus 1 divided by n multiplied by $v_{average}$.

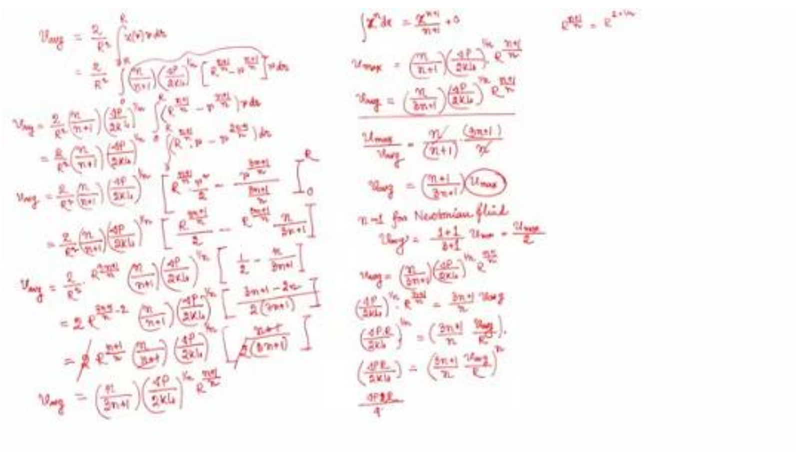
One of the R from here we will put inside this term. Delta P by 2kL R to the power 1 by n. That means we will be having over here, so R this term. So, if you take these things to the right-hand side, what will we be having? So, 3n plus 1 by n, $v_{average}$.



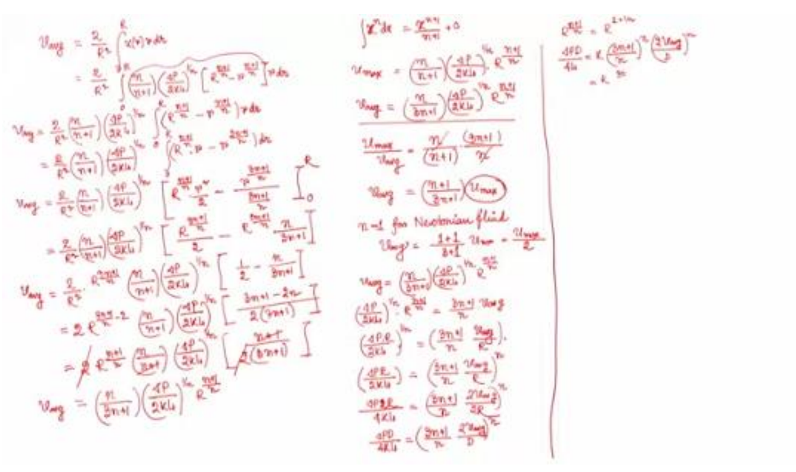
So, here what did we have? We had C R to the power n plus 1 by n. So, that means R to the power 1 plus 1 by n. So, 1 by n we have taken over here, we will have R over here. Delta P R by 2kL to the power 1 by, we want to take this thing, pin. So, 3n plus 1 divided by n, $v_{average}$ R. To the power n. So, if you want to remove that 1 by n, you will have the power n over here, OK.



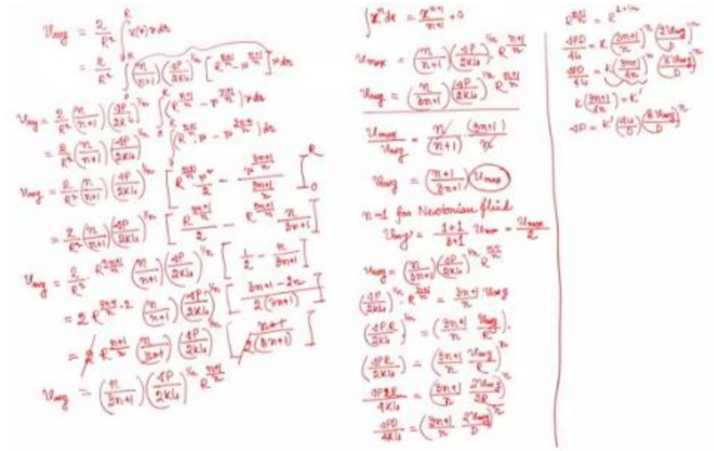
Now, you want to have this one and want to convert to the diameter. So, what will we have? Delta P, if you put 2 over here, 2 r means here will be 4, right? 2 r will give the—let us write it, ok. 4 k L 3 n plus 1 by n 2 vaverage by 2 R to the power n. So, delta P D by 4 k L equals to 3 n plus 1 by n 2 $v_{average}$ divided by D to the power n, ok. So, now coming to this side.



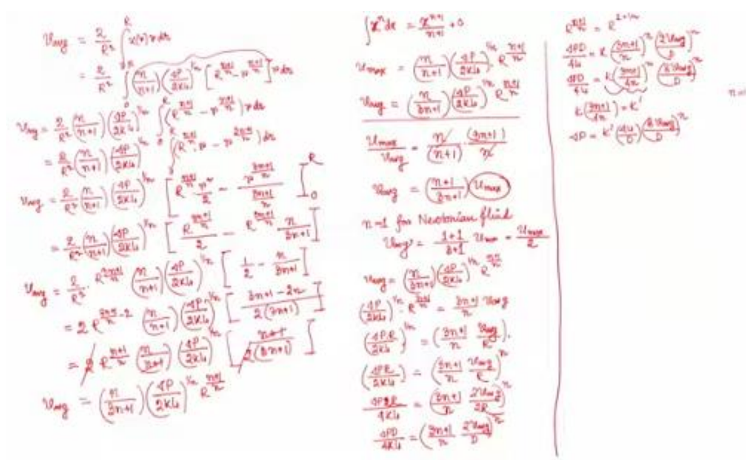
Delta P D by 4L equals to k multiplied by 3n plus 1 by n to the power n, then 2 $v_{average}$ by D to the power n. Now, what we will do is we will put 4 over here, ok. 3n plus 1 by 4n to the power n, 8 $v_{average}$ by D to the power n, ok. Now, this one delta P D by 4L, this one k multiplied by 3n plus 1 by 4n, we are denoting this one as k prime. Now, what would be the pressure drop? Delta P equals to K prime 4 L by D 8 $v_{average}$ divided by D to the power n.

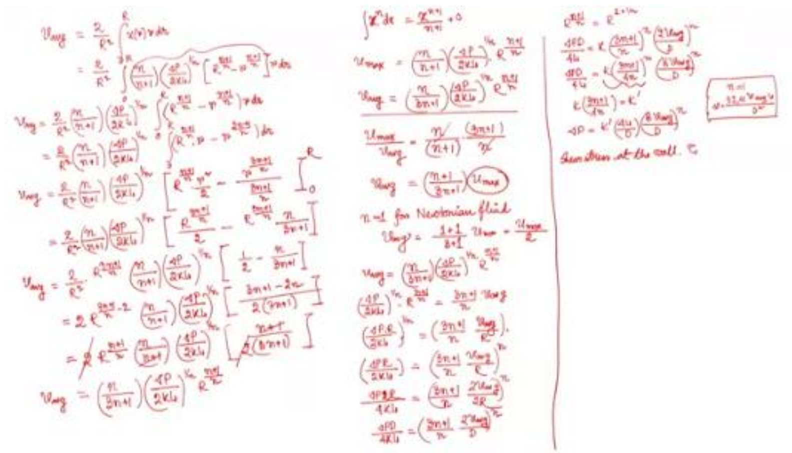


Now, you see why we have put 4 over here and then adjusted it over here. See now, if n becomes 1, ok? n becomes 1 for the Newtonian fluid. So, what does it give us? k prime will be nothing, but n becomes 1. So, it will be 4 here; it will be 4. So, k prime will be k equals to viscosity.

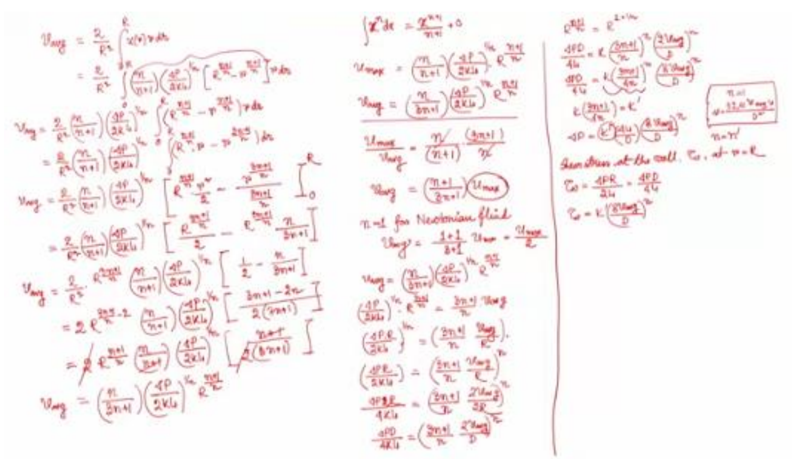


So, that means, you will be end up with 32 over here 32 mu vaverage L by D square that is very standard pressure drop equation for the Newtonian fluid ok. So, that is why we have done this adjustment ok. Now, shear stress at the wall shear stress at the wall how to calculate at a wall means tauw at small r equals to capital R. So, tau_w therefore, will be see we have learnt this one we have already learnt and you can write as in the diameter form also D by 4 L. So, tau_w therefore, if you look at from this expression you will be having k prime 8 $v_{average}$

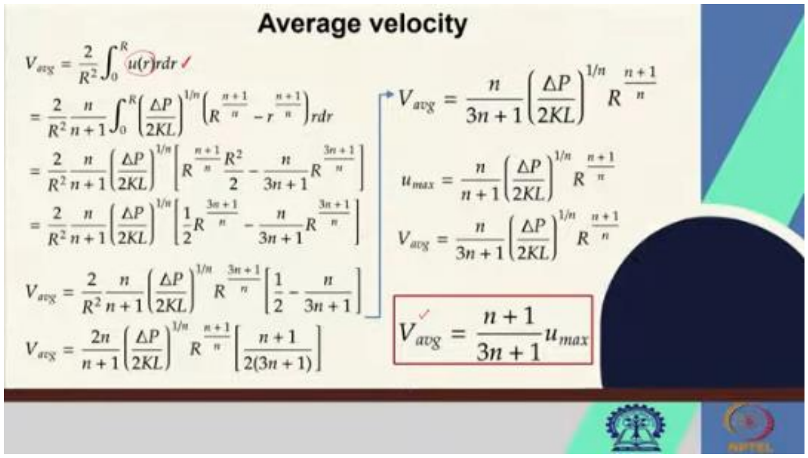


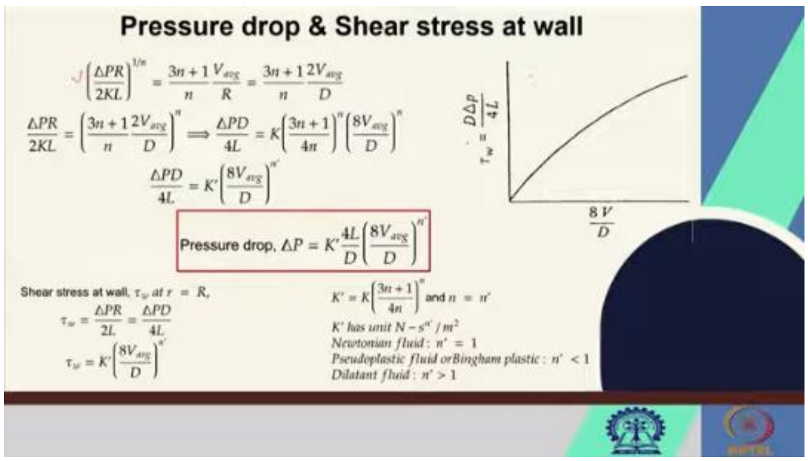


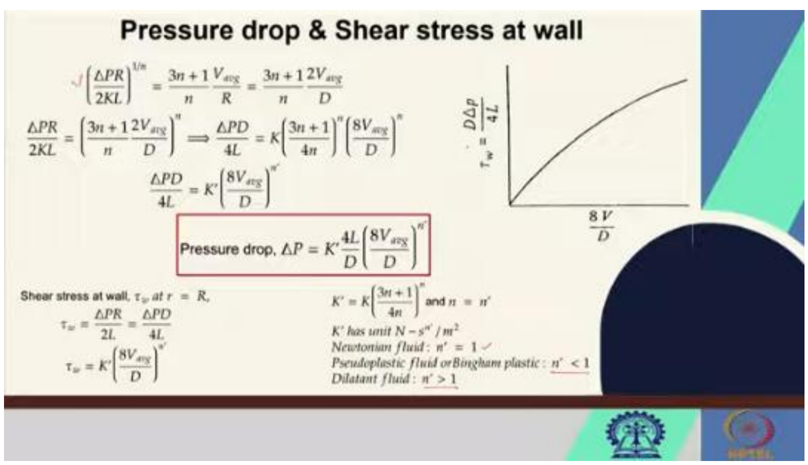
ok by D to the power n ok. Sometimes when k prime is used n also written as n prime ok. So, this is our pressure drop equation. So, once you can calculate the k prime, k prime is k multiplied by 3 n plus 1 by 4 n ok. We can calculate the pressure drop and also this one can be calculated ok.



So, we will go to the next ok. So, here if you look at it ah yeah K prime has the unit of ah again n s to the power n Pascal per meter square means Pascal s into the s to the power n for the Newtonian fluid as I said n prime will be 1 for this one and dilatant a to c a thickening n prime greater than 1 ok. Now, if you plot it so, with the a to v by D ok. with the tauw. So, it you will give it will give this kind of you know profile. So, how the you know stress at the wall is changing with the velocity diameter ratio.



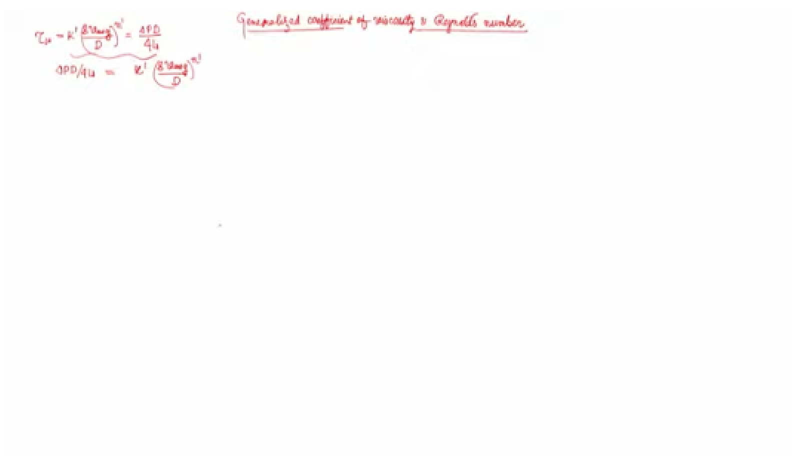




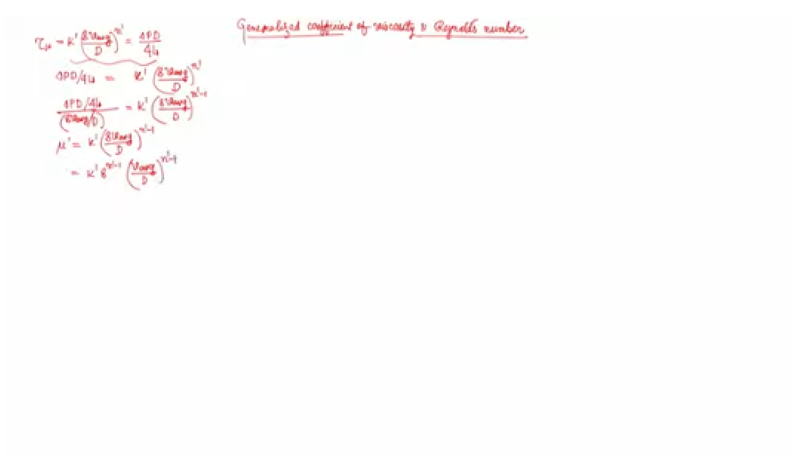
ok all right now coming to the generalized coefficient of viscosity ok how to work with this part generalized coefficient of viscosity and Reynolds number all right so I will just do it for here then come to the slide so what will come from here tauw ok so tau w equals to k prime it was k prime 8 $v_{average}$ divided by D to the power n prime that is also delta p d by

4 L because that is very known. So, we are trying to get as I said generalized coefficient of viscosity and also Reynolds number. Now, we have to rearrange this one little bit.

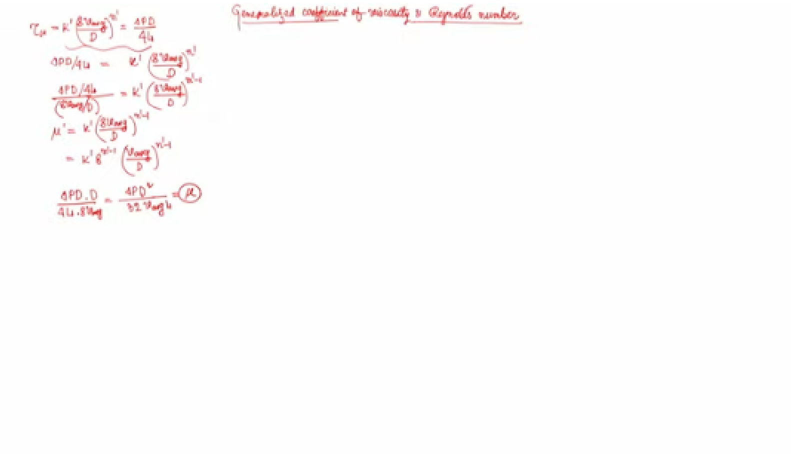
Now, we will work with these two parts. So, delta P, let us say d by 4 L equals to 8 K prime 8 $v_{average}$ by D to the power n prime. Now, what we can do? delta P D by 4 L. So, 8 $v_{average}$ by D over here we can take ok. Then what we will be having over here minus 1 k prime 8 $v_{average}$ by D I will come in a moment why we are doing this why this adjustment we are doing ok.



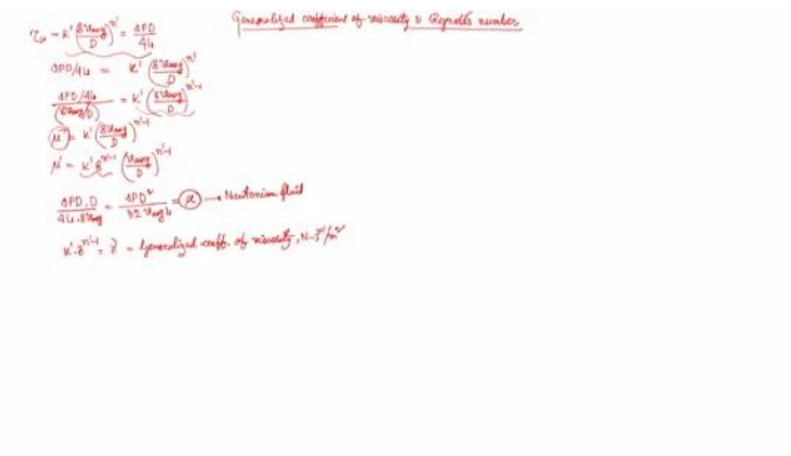
So, now we will define the mu prime equals to k prime multiplied by 8 $v_{average}$ by D to the power n prime minus 1. ok. So, we can write this one as a K prime 8 to the power if you take the 8 out n prime minus 1 $v_{average}$ by D n prime minus 1 ok. So, we have this ok. Now, what we will do



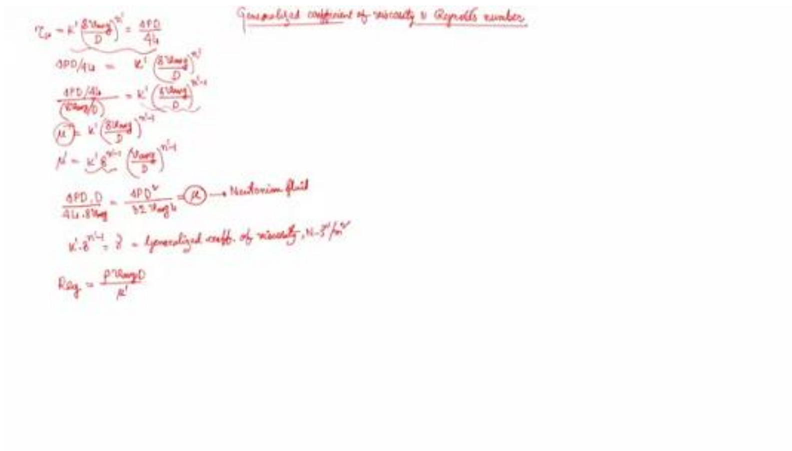
See, if you look at this part, what does it give us? This part ΔP D by 4L, right? 8 $v_{average}$ D, so what is it? ΔP D squared by 32 $v_{average}$ multiplied by L, which is nothing but—I mean, if you look at the Newtonian fluid, okay? The flow of Newtonian fluid—this is nothing but the viscosity for the Newtonian fluid. So, in order to get a similar structure over here, we have done this.

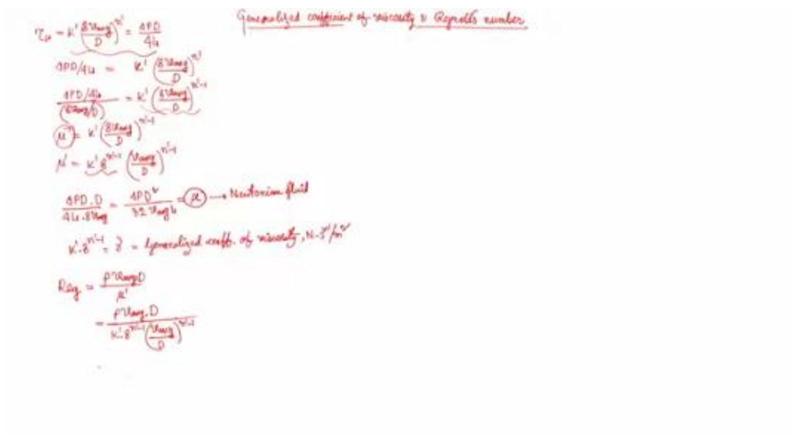


That means we can write this as μ′ for the non-Newtonian fluid, okay? So, for Newtonian fluid—therefore, for the non-Newtonian fluid—we can write this as μ′. So, μ′ this one, and this part, k′ multiplied by n to the power minus 1, is written as a small γ. It is known as the generalized coefficient of viscosity. So, it has got the unit of n, of course, h to the power n′ meter squared, okay? Now, also, the Reynolds number is expressed as the generalized Reynolds number, okay?

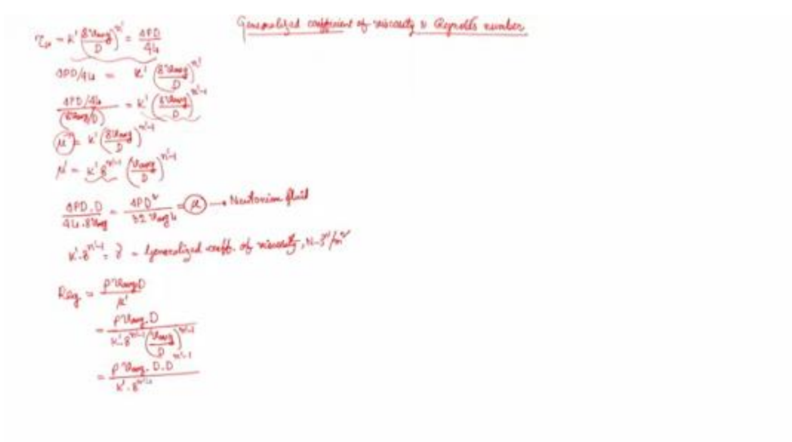


So, we denote it as Re_g, Re_g ρ $v_{average}$ multiplied by D divided by μ′. μ′. So, we will put those values: 2 $v_{average}$ multiplied by D, μ′ is k′ $v_{average}$ by D to the power n′ minus 1. Now, we will rearrange this. So, ρ $v_{average}$ D multiplied by D to the power n′ minus 1.

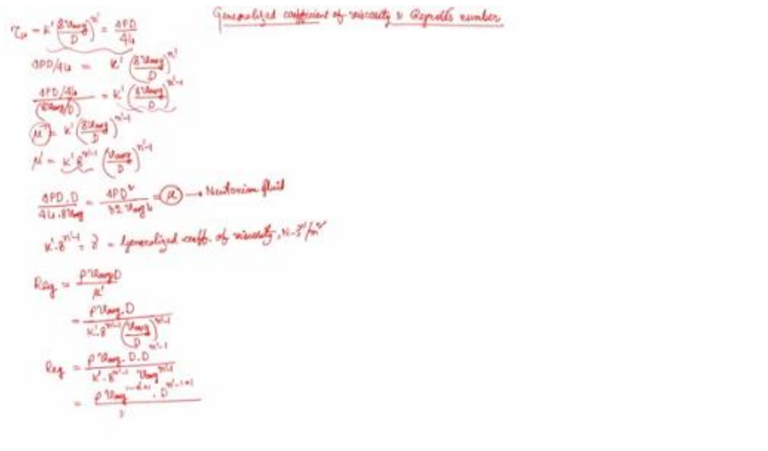




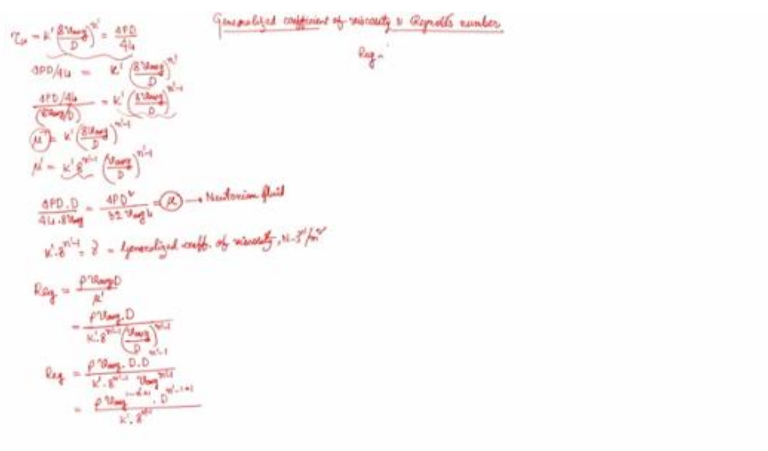
So, this D will go on top k prime in prime minus 1. We will adjust the average also, ok. So, generalized nearest number. So, what will we do? So, $rho_{average}$

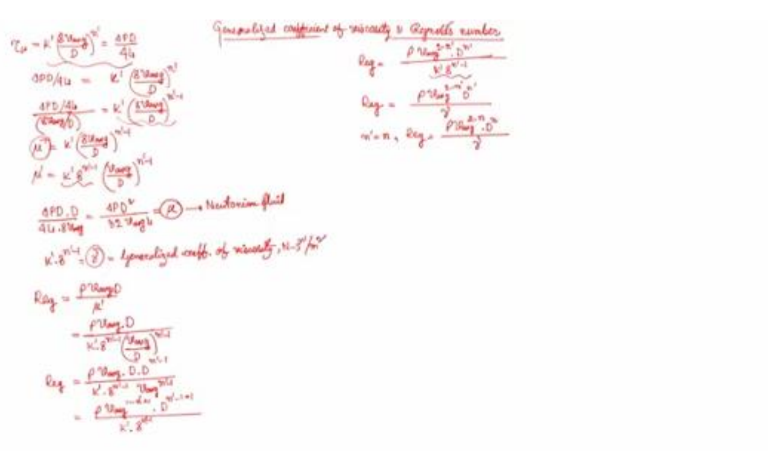


So, what will we be having? So, 1 minus n plus 1. So, here we will be having n prime. So, here we will be having minus 1 plus 1. So, k prime minus 1.

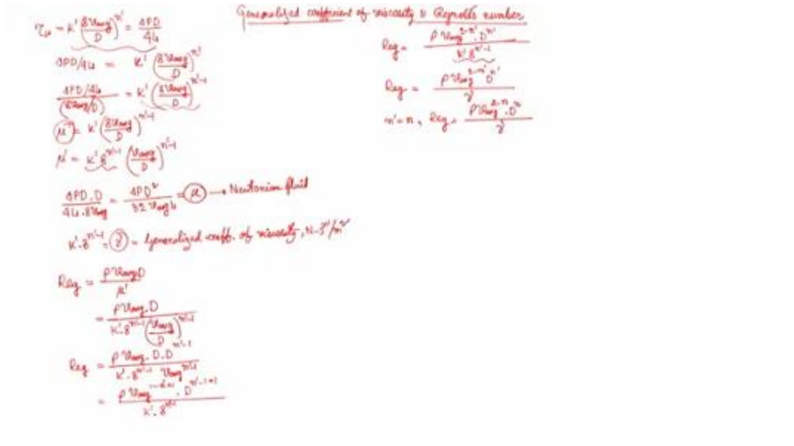


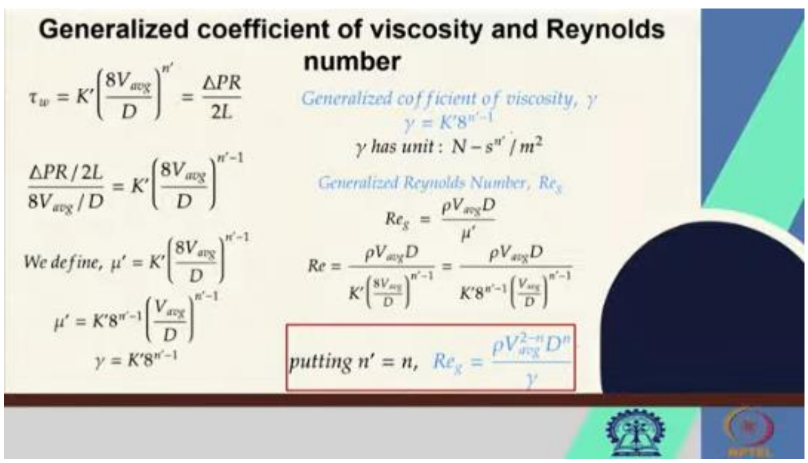
So, finally, Re_g equals to $rho_{average}$ $rho_{average}$ 2 minus n prime D to the power n prime k prime a to the power n prime minus 1, right? Okay, so what is this? The denominator is what we are having, gamma. So Re_g to $rho_{average}$ 2 minus n prime d n prime gamma we have. Many times n prime is also written as n equals to n Re_g, therefore, will be $rho_{average}$ 2 minus n D to the power n Ok.

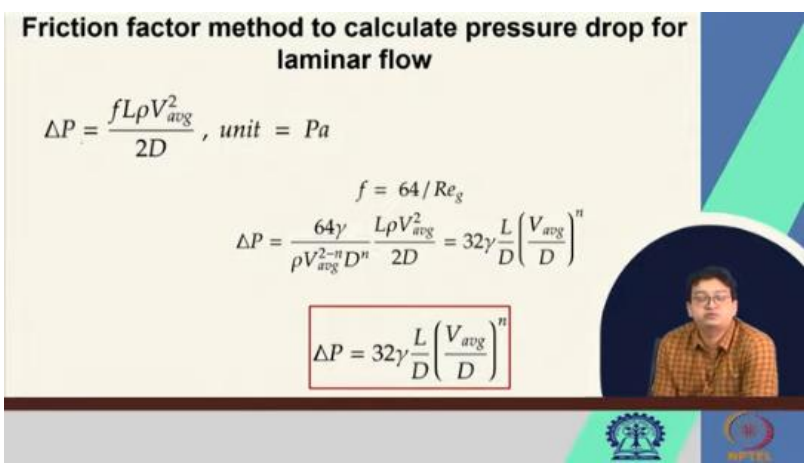




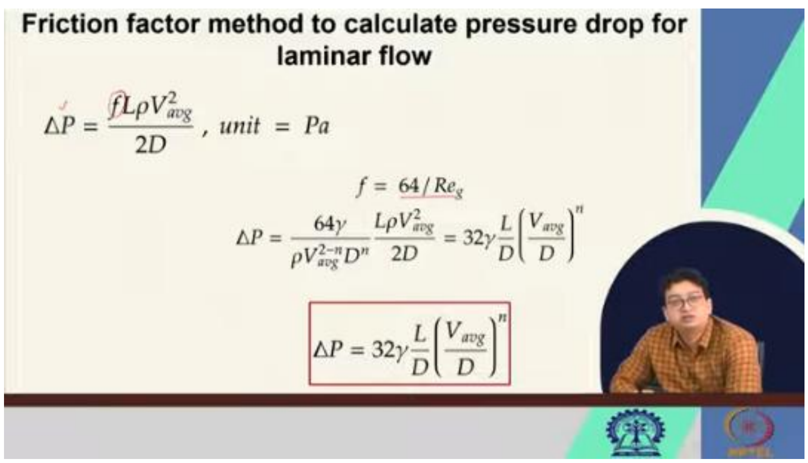
So, we have this form: generalized Reynolds number equals to rho $v_{average}$ to the power of 2 minus n, D to the power of n, divided by gamma, ok. So, here it is. So, now, we will calculate the delta P using the friction factor method. So, you remember, that the friction factor method is, you know, the main method of calculating pressure drop for laminar flow.

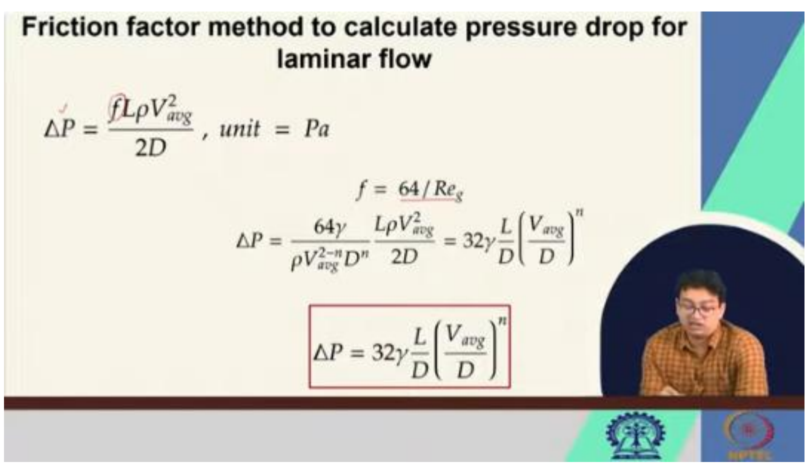




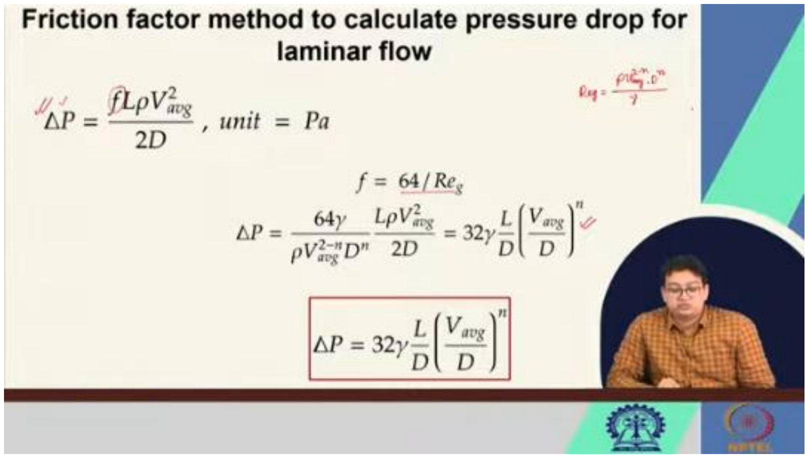


So, in that case, we know that delta P equals to f L rho $v_{average}$ squared by 2 D. So, the f value we can calculate as f equals to 64 divided by the generalized Reynolds number. This is similar to the Newtonian fluid, and then delta P, if you put all those things, 64 gamma divided by rho vaverage 2 minus n D to the power of n L rho $v_{average}$ squared by 2 D. If you rearrange this, you will finally have 32 gamma L by D multiplied by vaverage by D to the power of n. So, if we have to calculate the pressure drop for non-Newtonian fluid flow under the laminar region, we can directly use 32 gamma L by D vaverage by D to the power of n. In case we may not remember it always, I suggest you come from this one mother equation. Delta P equals to f rho L v squared by 2D, where, you know, f is 64 by Re_g, and Re_g, the Reynolds number, is nothing but rho $v_{average}$ 2 minus n to the power of n divided by gamma. Gamma is the generalized coefficient of viscosity, ok.





So, what we have seen in this class, in this topic, is that we have developed the equation using the average velocity of the non-Newtonian fluid flow through the pipe. And finally, we have seen that in the generalized Reynolds number, we are using that the gamma, one gamma, is the generalized coefficient of viscosity, that is, gamma equals to k prime a to the power of n minus 1, ok. So, that is how you can calculate.



So, this is the form that is a little bit different from the Newtonian flow, which in the Newtonian flow is rho vaverage D by mu. In this case, we are using the generalized coefficient of viscosity. So, as I said, if you want to calculate the delta P, you always start from this mother equation: f L rho v square by 2 D, where f is calculated as 64 by Reynolds number. And finally, if you put all those things, you will get 32 gamma L by D vaverage D to the power n. So, we will stop here. Thank you.