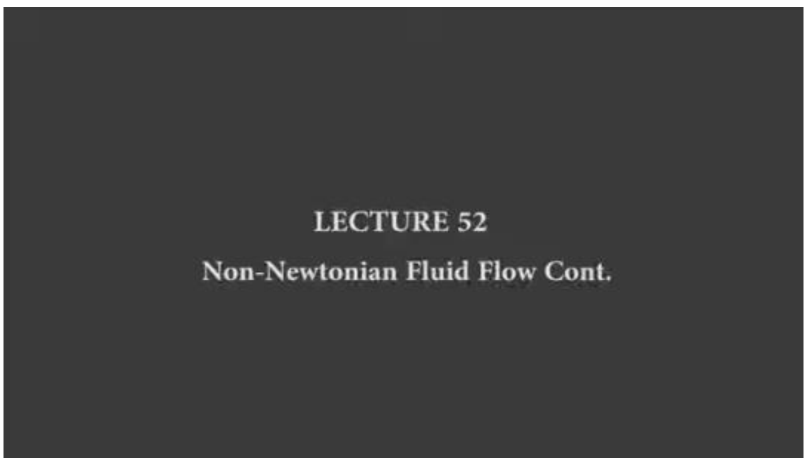
# IMPACT OF FLOW OF FLUIDS IN FOOD PROCESSING AND PRESERVATION

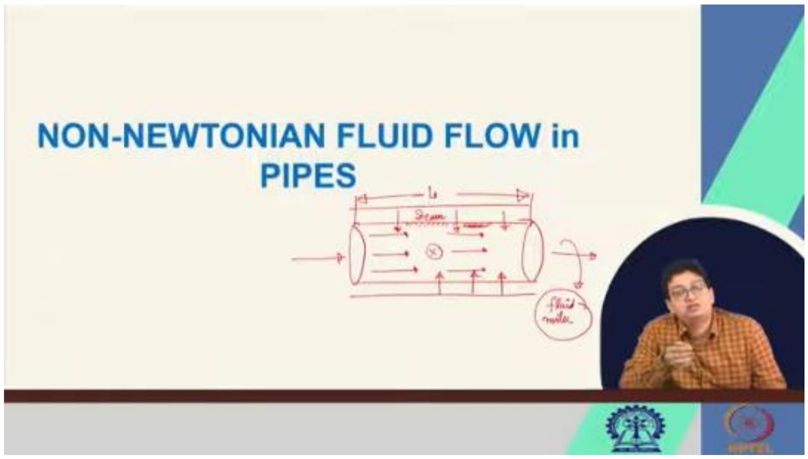
## Lecture52

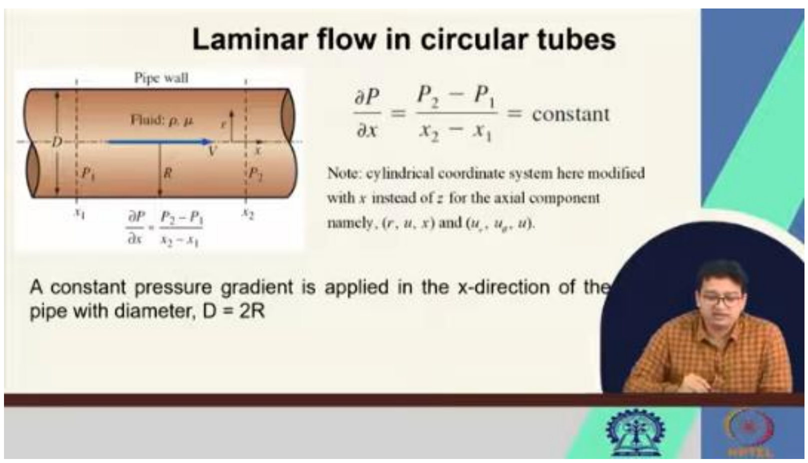
## LECTURE 52 : Non-Newtonian Fluid Flow Cont.

Friends, welcome back. So, in the last class, we looked at different kinds of non-Newtonian fluids and how they behave under shear stress and shear rate conditions. Now, we will be looking again at the flow of non-Newtonian fluids through the pipe, okay? So, here So, non-Newtonian fluid flow in pipes. So, this is very important because many food industries are mostly involved with the flow of fluids in pipes.



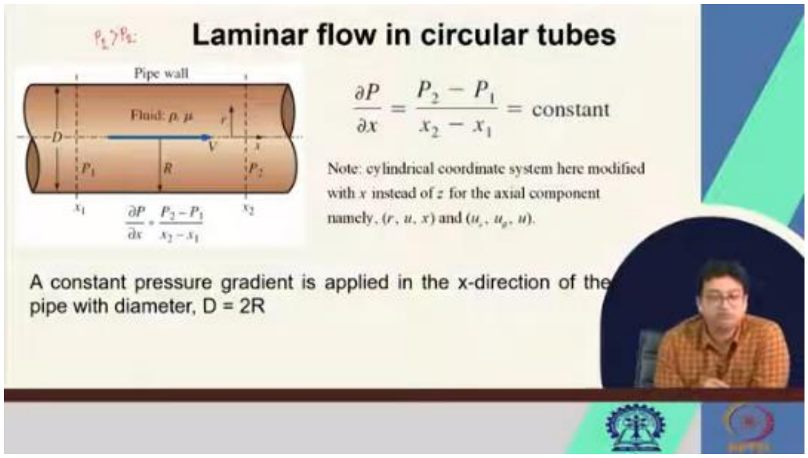

So, now we will focus on how to derive the velocity profile and the pressure drop equation because, ultimately, the pressure drop equation gives us an idea of how much pumping power is required. And the velocity profile gives us, you know, finally, if you look at what could be the temperature distribution within the pipe when the fluid is flowing in the processing industry, okay? Let us first consider laminar flow in a circular tube. So, here we have a pipe valve. We have a pipe wall over here, this one.



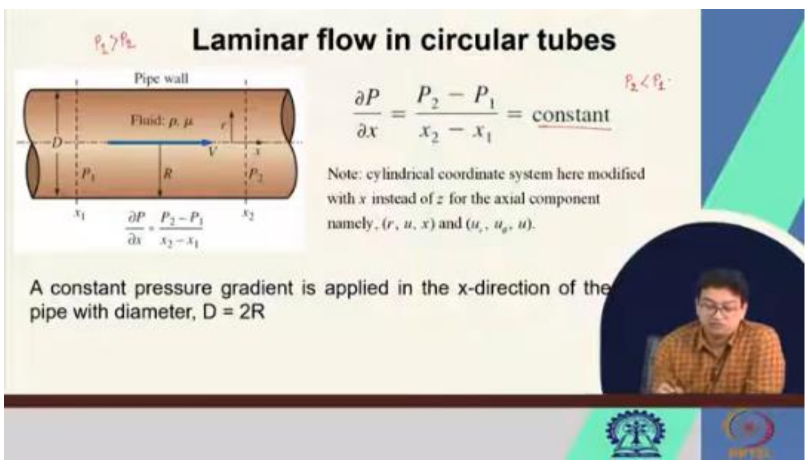




So, this is actually a pipe, and the fluid is flowing through this pipe, okay? It is a circular cross-section. Now, you have to remember this is a pressure-driven flow. So, over here at this upstream end, we have a pressure of $P_1$, and here we have $P_2$ towards the downstream. Now, $P_1$ is

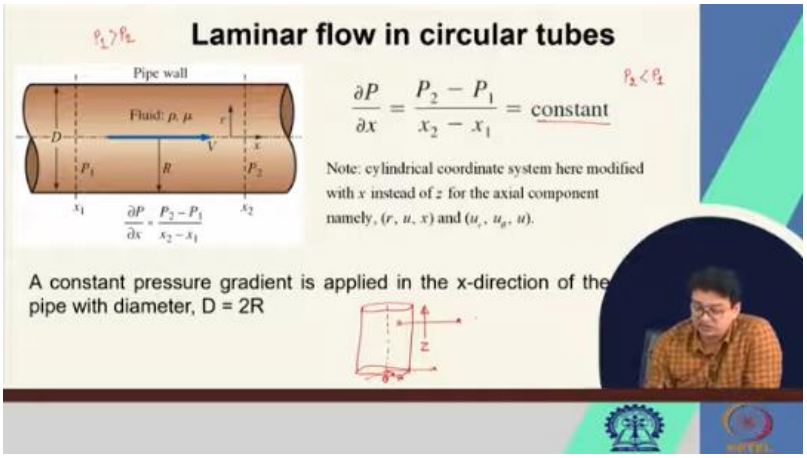
$P_1$ is greater than $P_2$, ok. So, we have a pressure over here, and when the fluid flows through the pipe, there will be frictional losses. Now, we want to calculate how much pumping power is required to overcome that. Now, the pressure gradient over the length of the pipe in the differential form can be written as del p / del x, and if we look at the final expression, it is $P_2$ minus $P_1$ divided by $x_2$ minus $x_1$, which is actually constant. Now, you have to see $P_2$ is again, you know, less than $P_1$, ok.



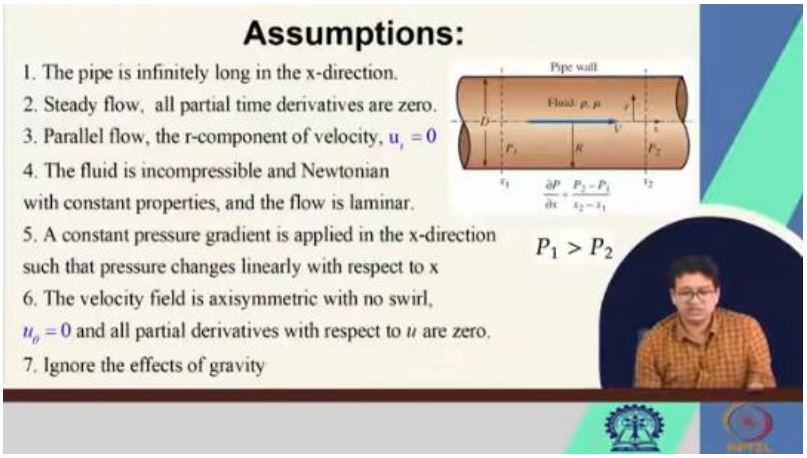

So, we will have a negative pressure gradient over here in order to get $P_1$ minus $P_2$, ok. Now, these circular tubes, if you imagine this one in a cylindrical coordinate system, ok, the circular tube. So, we have a circular tube. That means it is a cylindrical coordinate. So, that means we have in the radial direction one, axial direction—let us say in the z-direction—and another one is the azimuthal angle, which is theta.




So, when we have the fluid flow through the pipe, we have to consider the flow in all three directions. So, that is actually r, theta, z. Let us say we want to get the locations over here. Here, let us say this is point P; it can be written as r, theta, z. Ok. The cylindrical coordinate system, ok, it is a model, it is modified as r, theta, z system, ok.




Now, for our analysis, so instead of z, so this is the axial coordinate ok, we will be using x ok from now on all right. So, constant pressure gradient is applied in the x direction ok and this tube has the diameter of d equals to 2 r ok. So, that is our prior assumption Now, what would be our assumptions when the fluid is flowing through the pipe? So, perhaps you have learnt, but I want to make sure that ah when I will be ah slowly moving to the the non-Newtonian ah flow type.



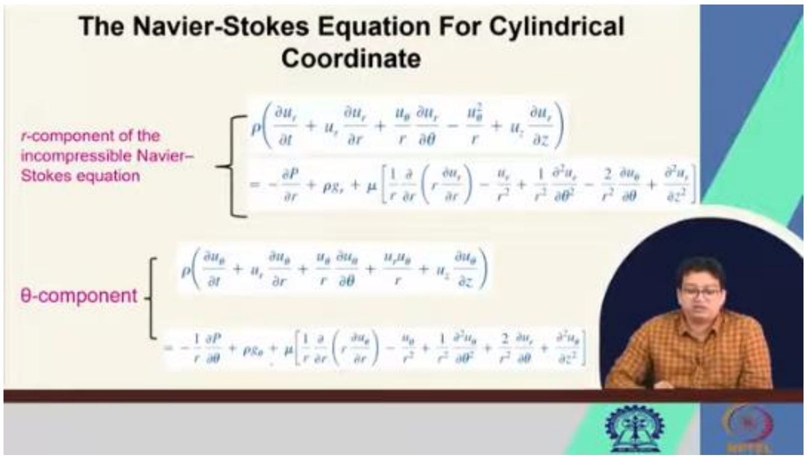

So, this kind of ah background is little bit required. So, that is why I prefer to have this one ah to discuss over here. The pipe is infinitely long in the x direction. So, in the x direction the pipe it infinitely long ok. so that we can ignore the you know effect of ah entrance effect over ok.

The steady flow all the partial time derivatives are 0. So, steady flow means any parameter or the things are changing with respect to time ok it is becoming 0. Now, the third assumption is the parallel flow. So, parallel flow means the you know there is no flow in the radial direction the parallel flow is like that ok. So, r component of the velocity equals to 0.

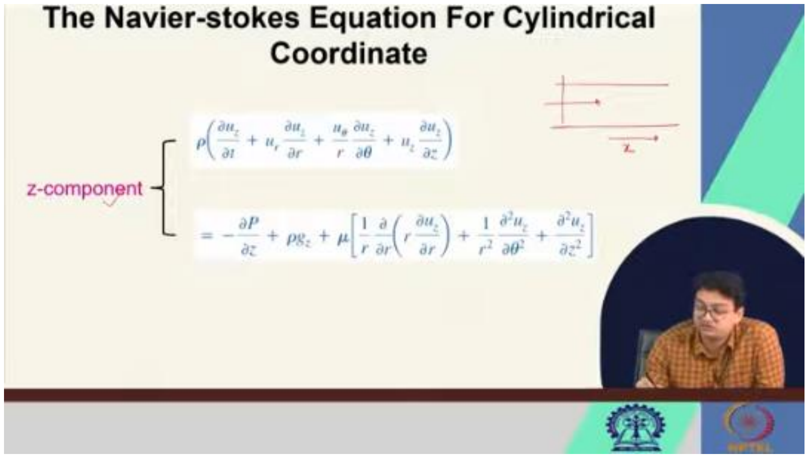
So, u equals 0, the r-component of the velocity equals 0. The fluid is incompressible and Newtonian with constant properties. So, we will first start with the non-Newtonian nature, and from there, we will go further with the non-Newtonian, okay? So, if I do not explain this, how does this come with the Newtonian part, okay? In this aspect, you will have a little problem understanding the non-Newtonian. So, of course, you have learned, but it will be good to recall what you have learned in the past, okay?

The flow is also laminar. So, of course, you already know this term: a constant pressure gradient is applied in the x-direction. So, that means there is a $P_1$ minus $P_2$ divided by across this, you know, length that is divided by L, okay? L is the length of the pipe, let us say, that means $x_2$ minus $x_1$, okay? The velocity field is axisymmetric with no swirl, that means there is no swirl like this, okay?

So, the theta component of the velocity is 0, and the partial derivative with respect to u is also 0. Since this is a pressure-driven flow, we will ignore the effect of gravity. So, with this assumption, now we will look at how to derive the pressure-drop relationship, okay? Now, here, the very basics are the Navier-Stokes equation for the cylindrical coordinate because we will derive it from here, okay? You can, of course, go through the past slides; you can look at it.

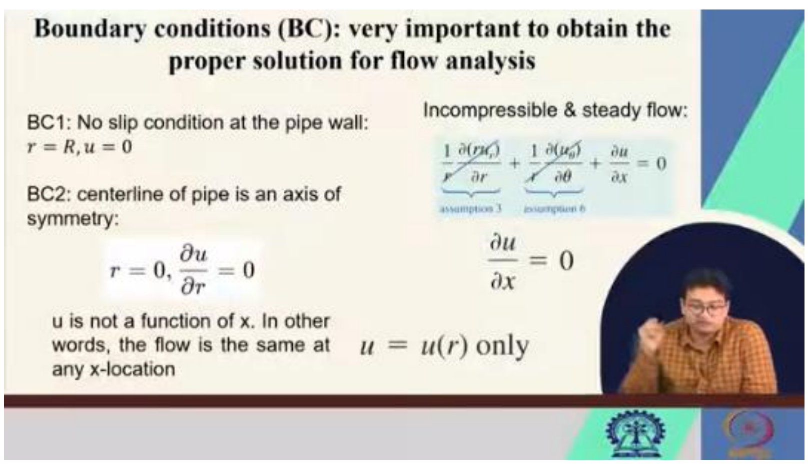



So, this one the first one is the r component of the Navier-Stokes equation, this one is the theta component. and finally, we will have the z component. So, for our case since as I said the fluid is flowing in the we are considering in the x direction. So, here we have $P_1$ and we have $P_2$ ok. So, instead of z we will be using x ok in the x direction ok.

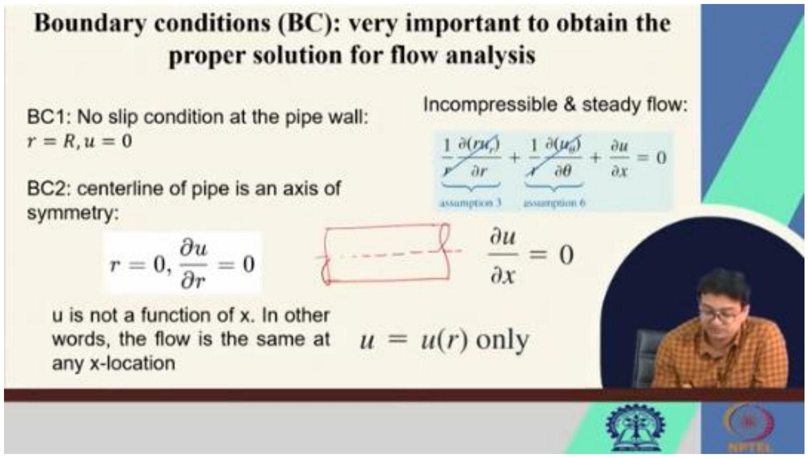



Now, also we need help of the continuity equation in the cylindrical coordinates. So, here the cylindrical coordinate continuity equation is del rho del t plus 1 by r del del r of r rho $v_r$ plus 1 by r del del theta of rho $v_{theta}$ del del z of rho $v_z$. So, you can understand over here is the z component or you can say over x component ok. It will be the $x_1$ here this is in the theta and this is the in the radial direction ok. So, this is equals to 0.

boundary conditions. Now, boundary condition is very much important in order to get the solution ok. So, what are that boundary conditions? See the boundary conditions comes from the physical interpretation of the what you are actually observing. So, fluid is flowing.

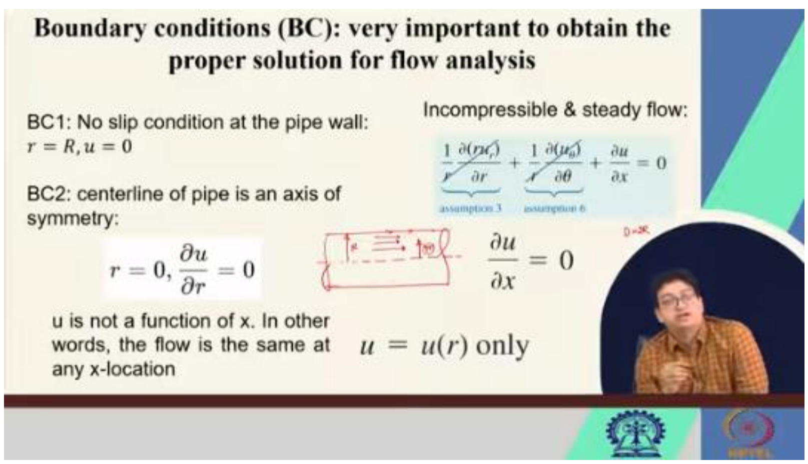



So, let us draw it over here, ok. So, the fluid is flowing, ok. Now, this one is R, capital R. The variable one is the smaller. That means, as you progress or as you move from the center line of the boundary towards the wall of the tube, ok, the variable is the smaller, ok.

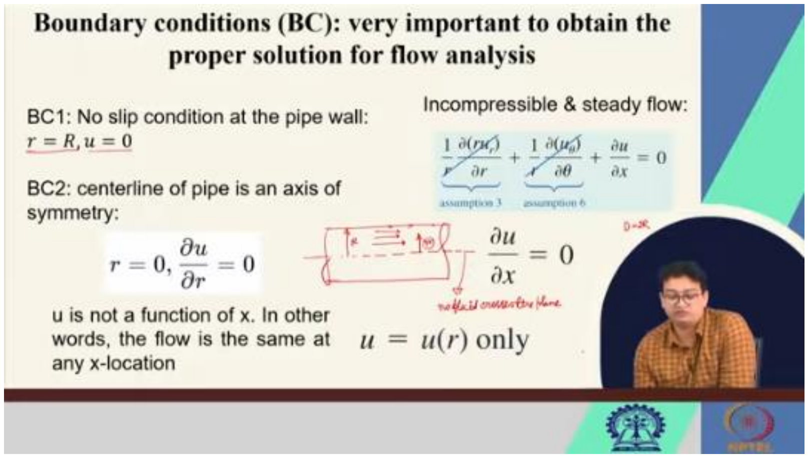



D equals to 2R. Now, what is boundary condition 1? Here, it is said that there is a no-slip condition at the pipe wall. That means, the fluid is moving like this. At the pipe wall, the fluid comes to a complete stop.

That means, there is no velocity of the fluid. The layer that is touching the surface of the internal wall of the So, that means, when r equals to small r, u equals to 0, it is called the no-slip condition. Now, another one we will make at the center line, ok. So, this is actually a symmetric structure, ok.

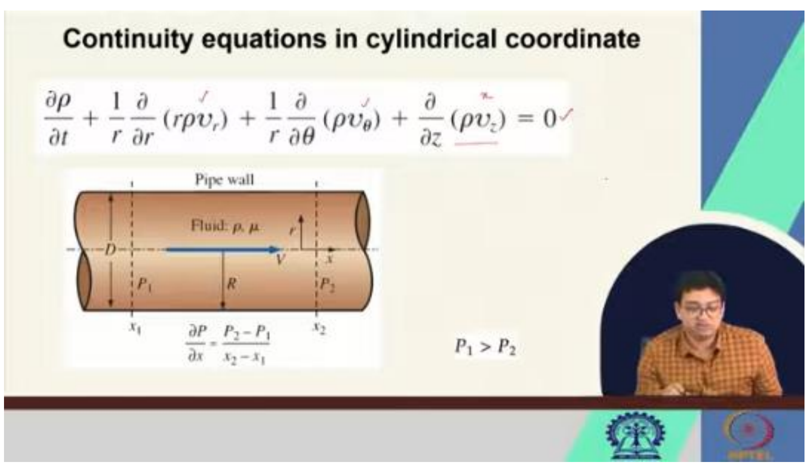



Now, at the center line, what we are saying is the axis of symmetry over here. So, no fluid crosses the plane. Sometimes, it is also known as the no-flux boundary condition. No-flux boundary condition means here at r equals to 0. So, r equals to 0, del u / del r equals to 0.

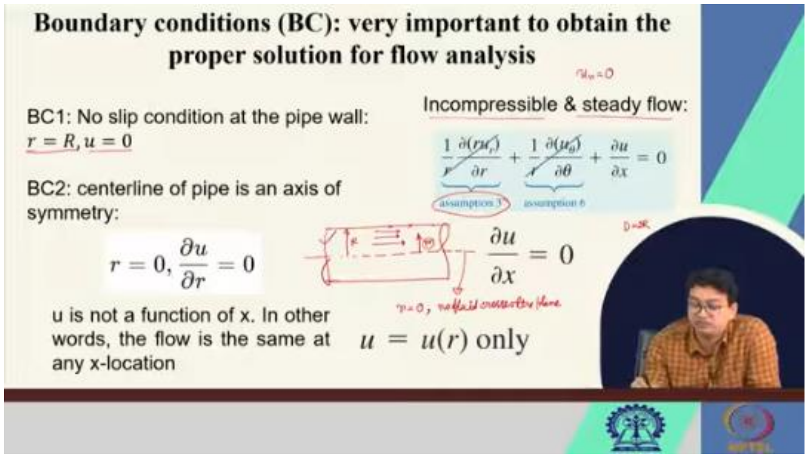



So, no-flux boundary condition means this is actually acting as an impermeable surface, and this part is basically a mirror of this one. So, at r equals to 0, del u / del r equals to 0. So, this boundary condition will be handy for deriving the equations. Now, if you look at the continuity equation, okay. So, if you have a look at it.

So, this is 1 by r del del r of r $u_r$ plus 1 by r del del theta of utheta plus del u / del x. So, over here, okay. So, of course, it is a steady state, meaning this one will be 0, steady state okay, that is what we are considering. And 1 by r del del r of r rho $v_r$. So, when we are saying incompressible and steady flow, we can take the rho out, okay, we can take the rho out. Now, what was assumption 3? It is a parallel flow, you remember.

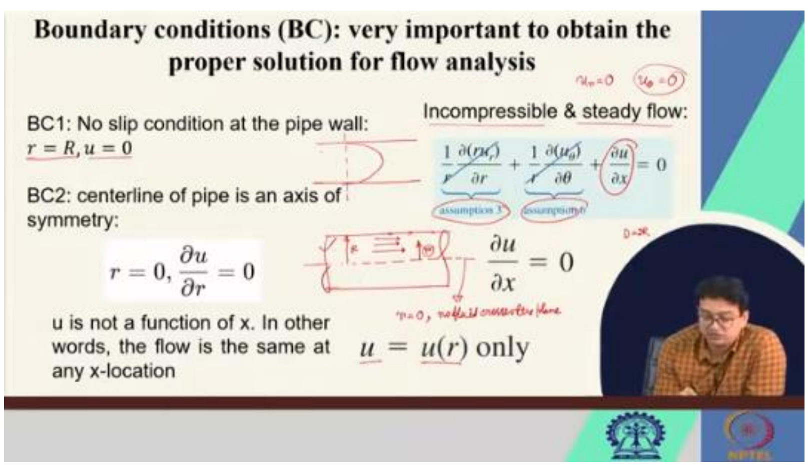



So, the radial component will be 0 ok, the parallel, it is a parallel flow, that means, ur equals to 0. ok. And also the utheta wall also we have considered 0. So, in those cases, so if you apply this, this is for the assumption 6, if you apply them then we will be left with only this part del u / del x equals to 0 ok. Now, del u / del x equals to 0 means

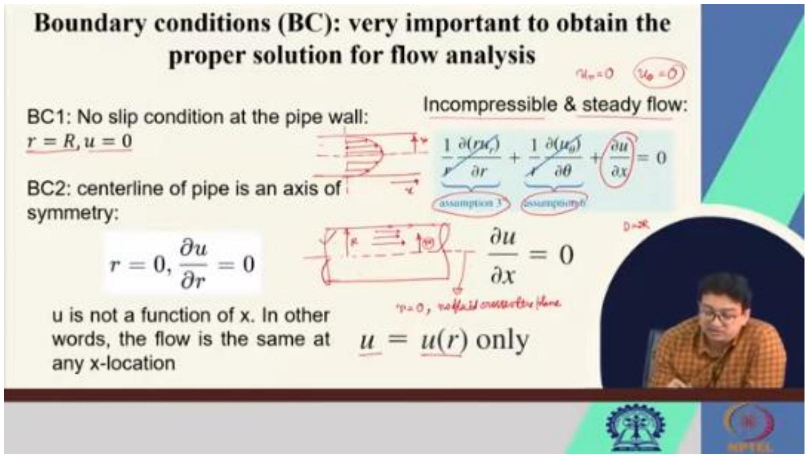


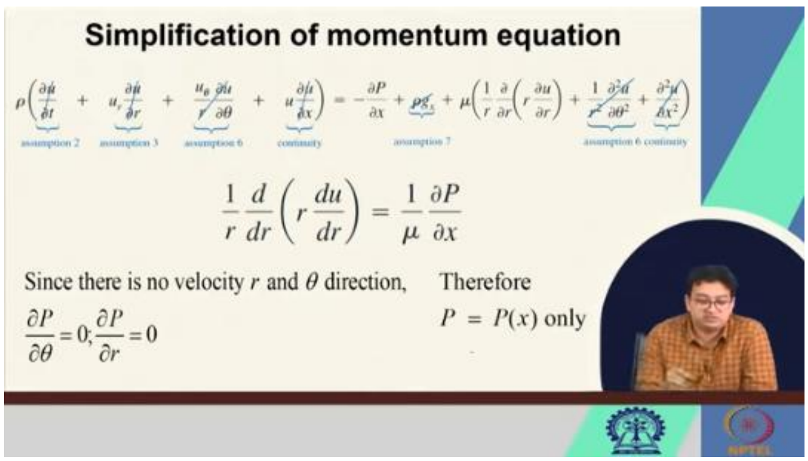

So, if you want to remove this partial differential part that means, u has to be something that is u equals to u is a function of r only. That means, you cannot see if del u / del x equals to 0 that means, it is constant with the respect of the x. Now, if you visualize so, we are talking about the fully developed flow ok. Now, if you have the fully developed flow I will draw it over here let me see some spaces So, we will also see. So, we will have the fully developed for laminar profile looks like this ok.

So, laminar profile looks like this. Now, the velocity profile develops over the radial direction. So, let us take this is in the x direction and this is r direction. So, here let us say let us take this point.



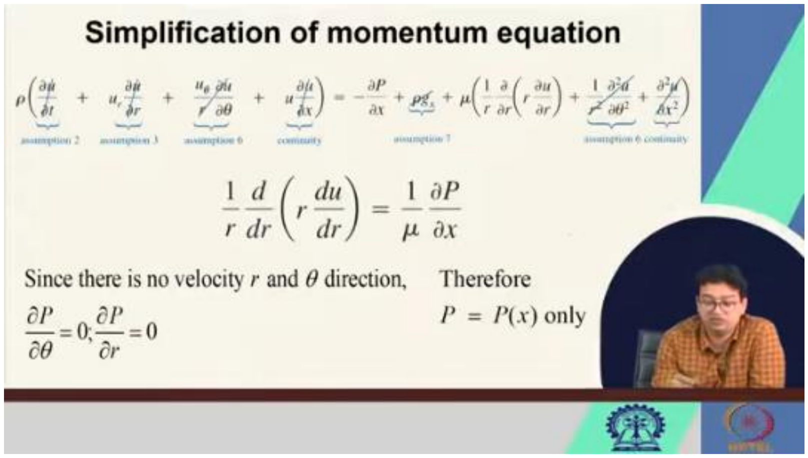

Now, at this point, if you move along the x, so let us say small r, if you move along the x, the velocity is not changing; it remains constant. But if you move along the r, you take a fixed x; if you move along the r, the velocity is changing because there is a velocity gradient due to the viscosity effect. So, that is why u is a function of r only. Now, coming to the next part, we will go through the simplification of the momentum equation, ok. So, this is the momentum equation.



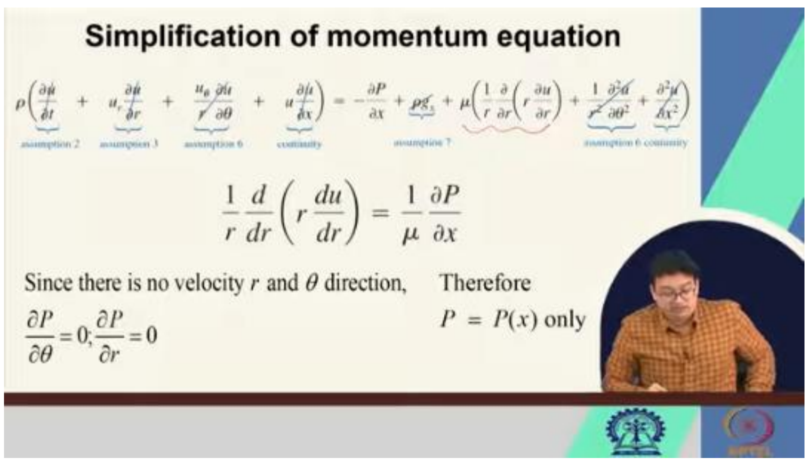




So, rho multiplied by del u / del t plus $u_r$ del u / del r plus u theta by r del u / del theta plus u del u / del x equals minus del p / del x, this is the pressure gradient. rho $g_x$ we will ignore the gravity, plus this is the viscous term, ok. If you apply all the assumptions, del u / del t, that means steady, we will make it 0. Assumption 3, no radial part, ok, the $u_r$ equals 0. Assumption 6, u theta equals 0; here it is coming from the, if you go back, it is coming from the continuity equation, ok.

del u / del x equals 0. Now, we will be left with minus del p / del x. We will ignore the effect of gravity. Same thing over here. Now, this part will be over here. Now, what we will be having?

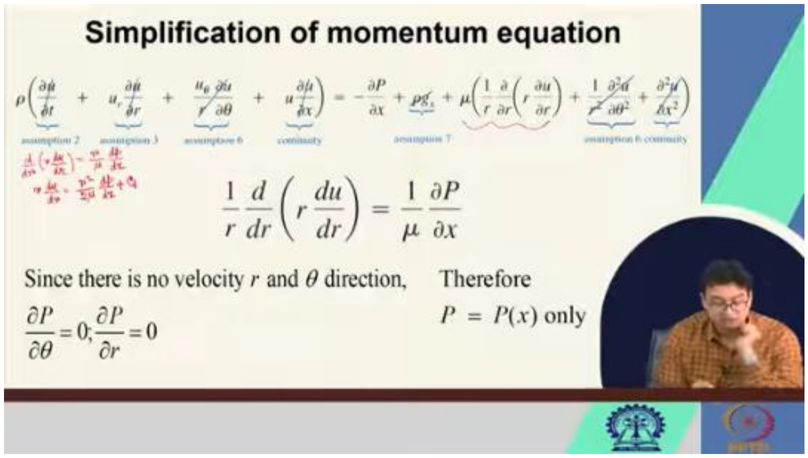


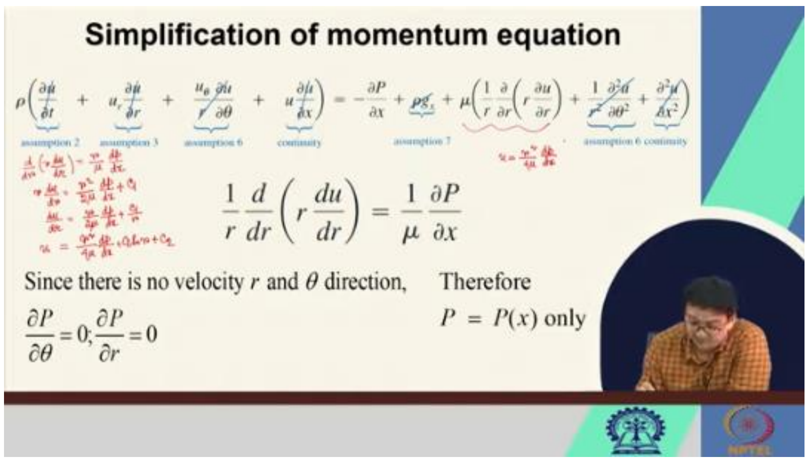

1 by r d dr of r multiplied by du / dr equals to 1 by mu del p / del x. Now, what we have to do? We have to integrate it. So, what we will do? Let us see we will do it over here.



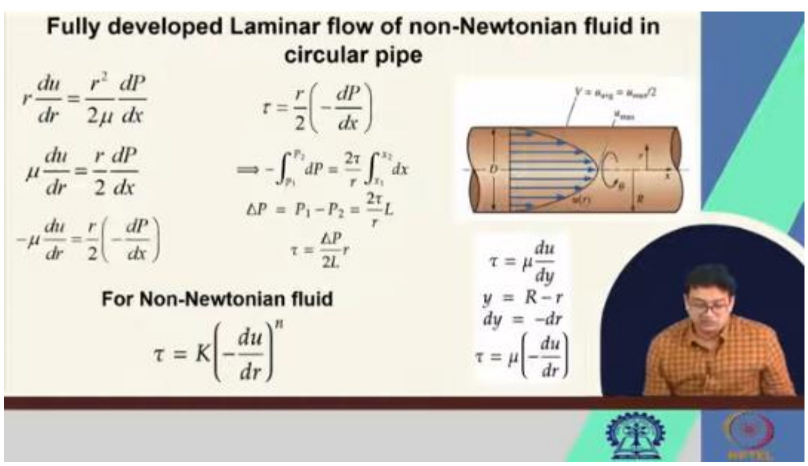

So, what we can do that we can write this one as a d dr of r du / dr equals to r by mu r by mu dp / dx. Now, r du / dr equals to r square by, so we will have to integrate 2 mu dp / dx plus $C_1$, it is a constant. It is a constant. So, r du / dr equals to r goes over the r square by 2 mu dp / dx plus $C_1$.

So, if you further do it du / dr equals to r by 2 mu dp / dx plus $C_1$ by r. Now, what would be the u? So, u will be again r square by small r square by 4 mu r square by 2 dp / dx plus $C_1$ ln r plus $C_2$. So, this is this one we have over here u equals to r square by 4 mu dp / dx plus $C_1$ ln r plus $C_2$. Now, how to get this $C_1$ and $C_2$ value?



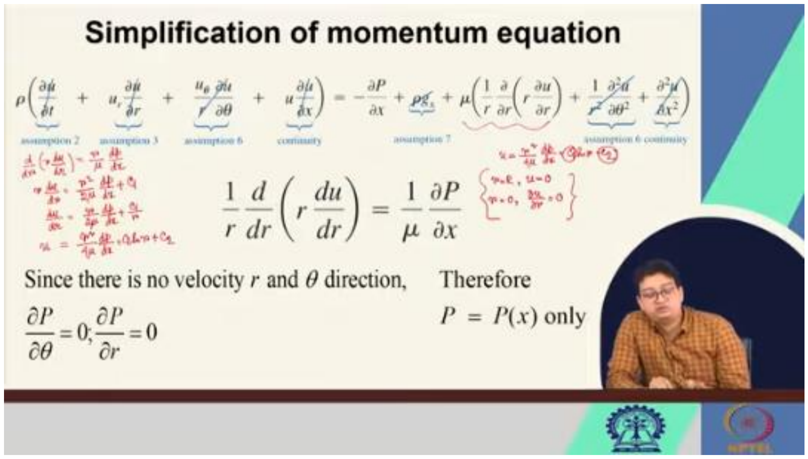



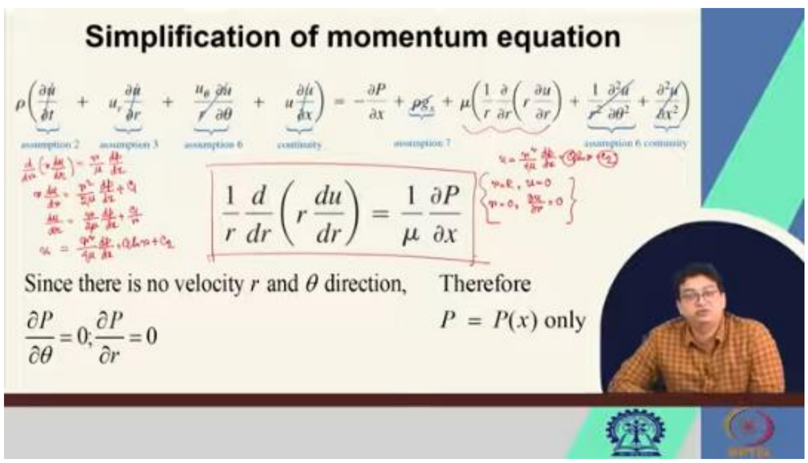

We have to apply the boundary conditions, ok. If you remember, we have discussed it, ok. So, if we apply those, ok. So, we will be able to get the values, ok. So, if you look at it, u equals to.



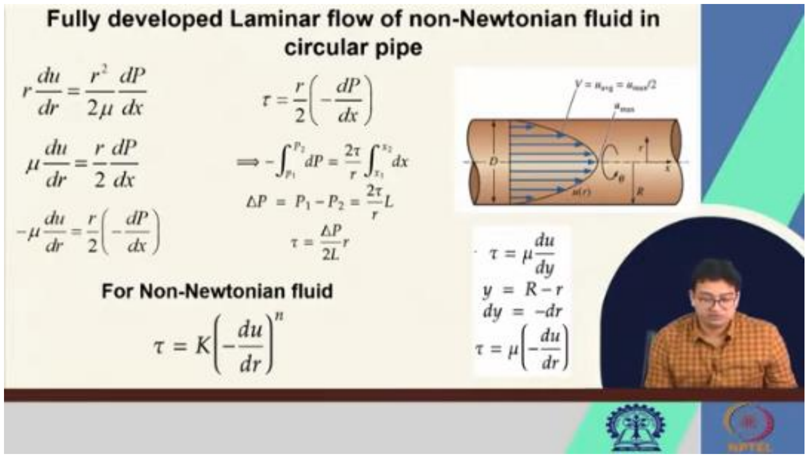

So, if you have the boundary condition, ok. So, I will just So, at r equals to capital R, you will have u equals to 0, and at r equals to 0, you will have del u / del r equals to 0, ok. So, if you do it, if you apply the boundary condition, you will be able to get the velocity profile for the Newtonian fluid in the laminar flow region. This part you have looked at.

Now, we will move to the part that we will not cover here. So, since you have already done it, now we will move to how it looks like for the non-Newtonian one, ok. Now, what is the purpose? We want to use this for the non-Newtonian one. We want to use this one, ok.



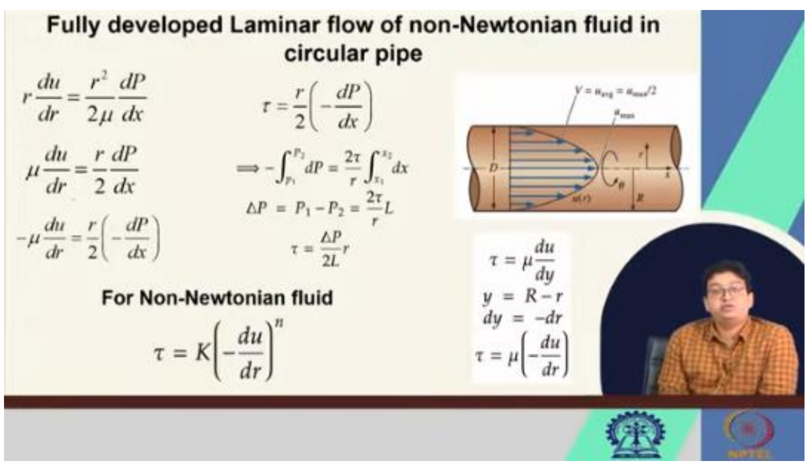




Now, how to do it? So, now for the non-Newtonian one, So, for the fully developed laminar flow, the non-Newtonian one. So, if you look back from here. So, r du / dr equals to r square by 2 mu dp/dx plus $C_1$.

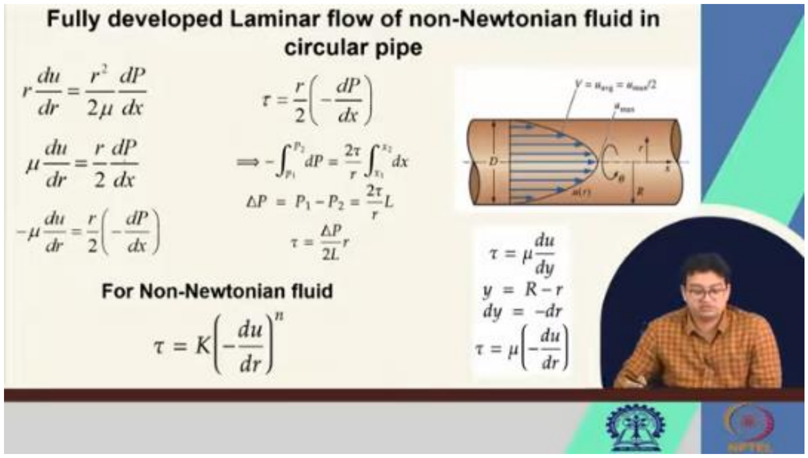



So, if you apply this boundary condition, eventually you will get $C_1$ equals to 0, ok. Now finally, this one will yield to this equation: r du / dr equals to r square by 2 mu multiplied by dp / dx. So, here this one So, r du/dr equals to r square by 2 mu dp/dx plus $C_1$ equals to 0, ok. Now, what we will do?

We want to get some relationship with the tau. Tau, what is tau? Tau is the shear stress acting on the wall, ok. That is, we denote it as the tauw in generic term; tau is the shear stress, ok. So, what we have to do?



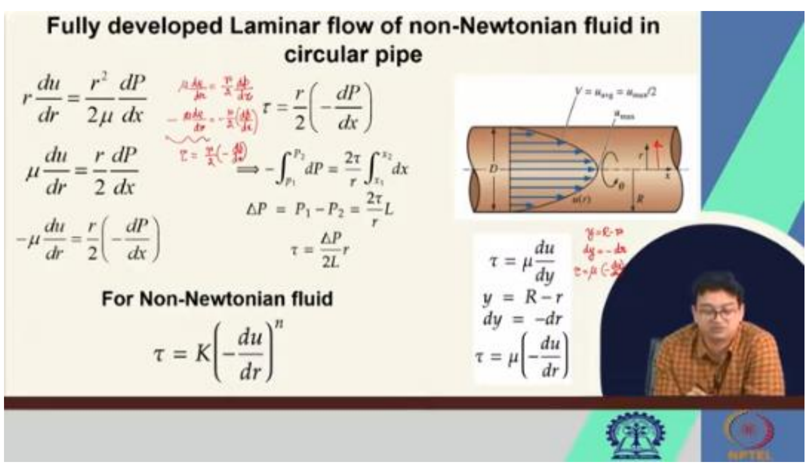

So, what will we do? So, you have du/dr equals to r squared by 2 mu dp / dx. du/dr will be small r by 2 multiplied by dp / dx. This is very clear, ok. So, now, we will multiply both sides. So, once you have both sides by mu, mu du/dr equals to, so that means, your mu will be omitted over here.



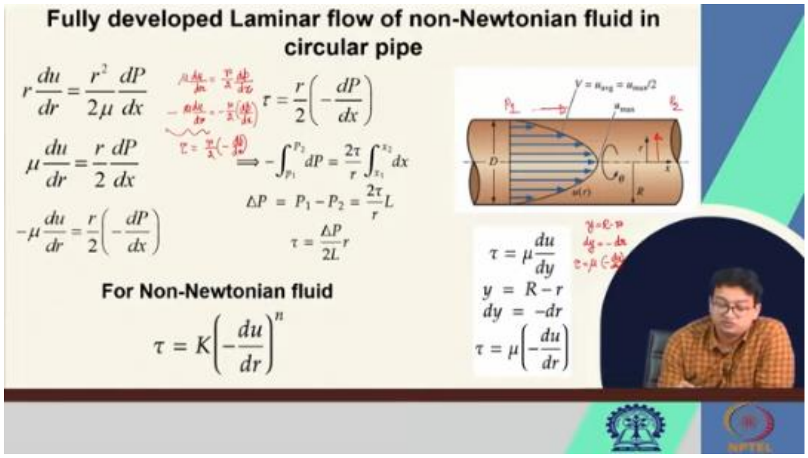

So, r by 2 dp / dx, ok. Now, if you look at it, remember what our relationship was: tau equals to mu du / dy. Now, what will we do? Put y equals to R minus small r. So, dy equals to minus dr. That means, when you have the relationship tau equals to mu minus du/dr. So, we are moving in this direction.

Ok. We are moving in this direction; you see the velocity is going down, ok. So, we will finally have this negative velocity gradient, ok. Now, once we have mu du/dr, we will put a negative sign equals to minus r by 2 dp / dx, ok. So, this one will give you tau.

Here we will have r by 2 minus dp / dx. See, the r cannot be negative. So, this is the negative. So, dp / dx is negative over here to make it the positive value when the overall value comes. It is actually $P_2$ minus $P_1$ because, see, from upstream here it is $P_1$, here it is $P_2$, ok. So, when the flow is occurring in this direction, ok, the pressure is decreasing.

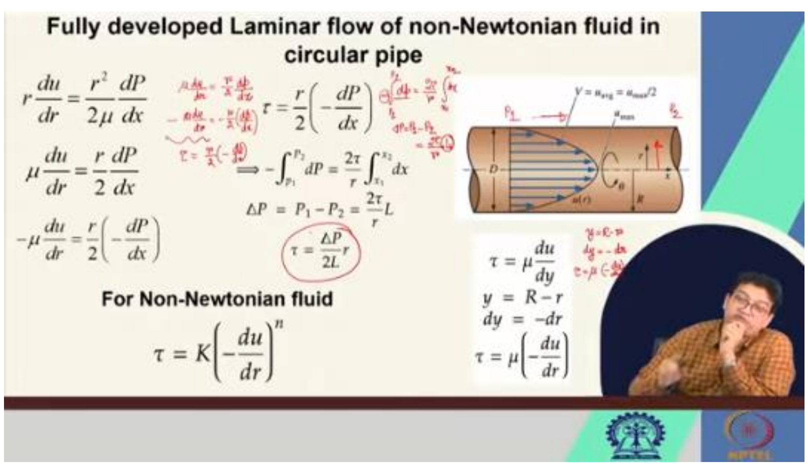



So, that is the significance over here. So, minus dp / dx is given, ok. So that means, tau equals to r by 2 minus dp / dx. So, if we integrate this one, integrating dp equals to 2 tau by r dx from $x_1$ to $x_2$, here we have $P_1$ to $P_2$. So, if you integrate it,

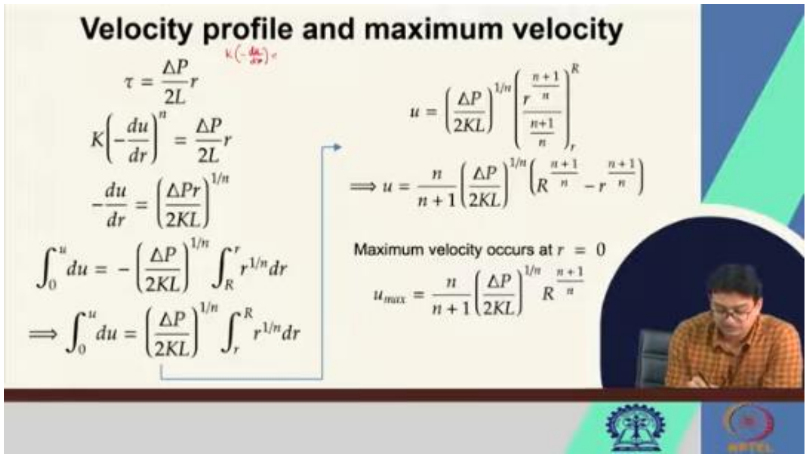



So, what we will be having is dP means, so, upper limit minus lower limit $P_2$ minus $P_1$. So, the negative will be adjusted. So, you will have $P_1$ minus $P_2$, that is denoted as delta P. $P_1$ minus $P_2$ equals to 2 tau by small r multiplied by capital L. So, let us say $x_2$ minus $x_1$ equals to capital L, that is the length of the tube. Length of the pipe. So, we now have this relationship: tau equals to delta P r by 2 L. Now, we will exploit this one.

How? Now, for a non-Newtonian fluid, you know that is how the relationship looks like: tau equals K multiplied by minus du/dr to the power n. What will we do? See, this is also tau; this is also tau. That means this also equals delta P by 2L multiplied by r. What are we interested in? At first, we are interested in the velocity profile. From there, we will go to the pressure drop calculation, okay?

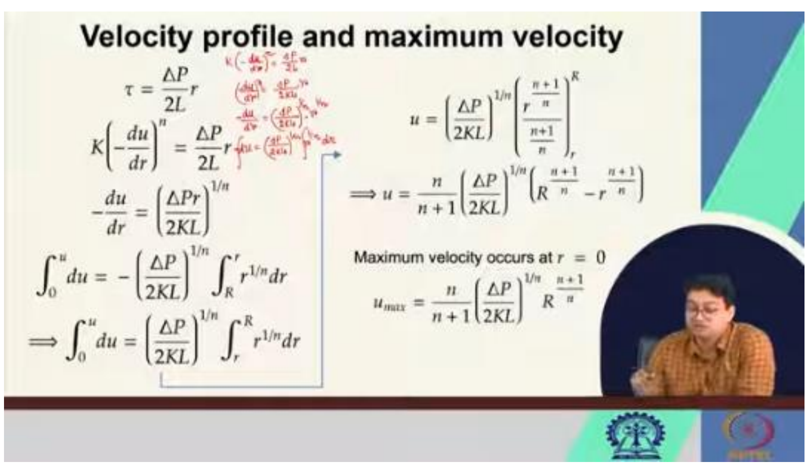



So, how do we do it? So, let us see, okay. So, K multiplied by minus du / dr equals delta P by 2L multiplied by r, okay. Now, we want to have this du out, okay, so to the power n. So, if you look at it, du / dr equals delta P by 2KL




Let us put n negative multiplied by r. Now, from here, if you want to have du / dr equals delta P by 2KL to the power 1 by n, r to the power 1 by n. So, negative sign. Now, what will we do with u? So, u means, in order to get u, we have to integrate. So, delta P by 2KL by 1 by n to the power 1 by n, r to the power 1 by n dr. So, du. So, negative. So, let us integrate.

Now, what would be the limit? See, you are having flow. Okay, so this is capital R. OK, and here we have, say, r is changing. It is a variable, OK? It is a radial direction. Fluid is flowing, flowing like that. So, at small r equals to capital R, you will have a complete stop. That means u equals to 0, and at some location r, it will have some velocity, let us say u. So, then what we will do, OK?




We will put 0. We will follow this one. So, at small r equals to capital R, u equals to 0. Here, u equals to u at r equals to small r. So, we can write this one. So, that means, so u equals to, so I will rather we will go from to this side.

So, we will go to this side. So, minus u equals to delta p by 2kL to the power 1 by n. So, if you integrate this one, you will have r to the power n plus 1 by n. So, x to the power n dx, x to the power n plus 1 by n divided by n plus 1 by n. So, if you have this from, I mean, if you have x to the power n, that means x to the power n plus 1 divided by n plus 1. So, we will have over here n plus 1 by n. We will have small r to, no, we have capital R. So, we have capital R and here it is small r.

Now, if you rearrange this one, we will get, let us say u, we will adjust a negative sign, delta p by 2kL to the power 1 by n. So, n plus 1 will go out. Then what will we have? See, it is supposed to be upper limit minus lower limit. So, when the negative sign is adjusted, we will have r to the power n plus 1 by n plus 1 by n minus r to the power n plus 1 by n, ok.

So, we can take this capital R out; we can have a different form also, ok. So, that means u is a function of r here. So, u will be delta P by 2kL to the power 1 by n multiplied by n by n plus 1, multiplied by here in the parenthesis we will have capital R to the power n plus 1 by n minus small r to the power n plus 1 by n. Now, this will give us a parabolic profile. So, in the center at r equals to 0, you will have the max, that means velocity is maximum, ok.

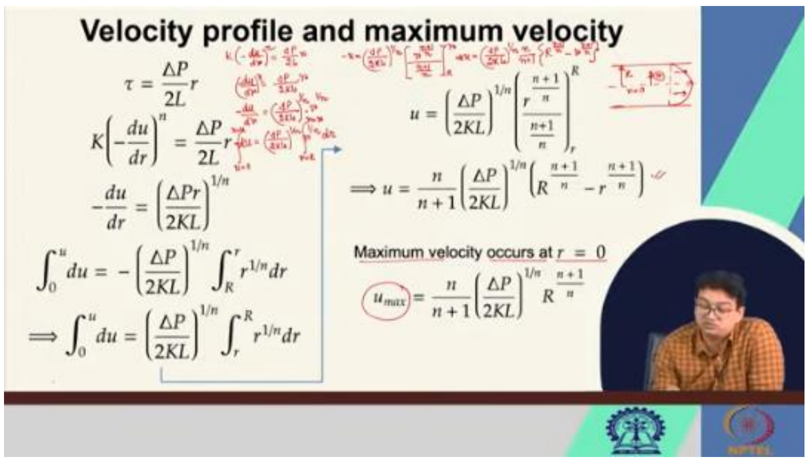
So, maximum velocity occurs at small r equals to 0. So, therefore, umax will be, so you will put small r equals to 0, that means n by n plus 1 delta P by 2kL to the power 1 by n multiplied by r to the power n plus n plus 1 by n. So, that is the u max. So, now we have the knowledge of $u_{max}$. Now, how do we calculate the average velocity?

So, average velocity we will calculate using this formula. So, $v_{average}$ equals 2 by r square integral of 0 to r, $u_r$ r dr. Now, let us have a look at what we have learned so far. What we had looked at in this particular topic of the lecture is the velocity profile of a non-Newtonian fluid, that is, a non-Newtonian fluid flowing through a pipe, ok. We have taken help from the Navier-Stokes equation and the continuity equations, and we made a few assumptions, ok. So, the assumptions are that when the fluid is flowing through the pipe,

under a constant pressure gradient. There is no net flow in the theta direction, meaning there is no swirl, and the flow is parallel, ok. The r-component of the velocity, $u_r$, equals 0, and the flow occurs only in the axial direction, ok. We have also ignored the effect of gravity, ok. With these assumptions, we derived a shorter version of the momentum conservation and continuity equations. Using these and the boundary conditions, we obtained the velocity profile.

Now, you see, we have the velocity profile here, ok. As I said, it is a power law in nature, ok. Here, n is the flow behavior index. Now, you could have, for example, Newtonian flow, ok.

For the Newtonian flow, n equals 1, which means k becomes the viscosity, ok. So, if you set n equal to 1. So, you will see what you get for the Newtonian. So, n equals 1 by 2, ok? Delta P by 2 k L multiplied by r to the power 1 plus 1, which means 2 divided by 1. So, that is capital R square.




So, that gives us the Newtonian nature. So, it is a very generic one. When you set n equal to 1, you will get the non-Newtonian, and if you set n equal to 1, you will get the velocity profile of the Newtonian fluid, ok. So, this is very basic and simple. So, we will stop here. Thank you.