IMPACT OF FLOW OF FLUIDS IN FOOD PROCESSING AND PRESERVATION

Lecture49

LECTURE 49 : VARIABLE FLOW

Good afternoon, my dear students, boys and girls, and friends. So, we have come to the sonic velocity, and we defined a non-dimensional parameter called Mach number (M), and we have shown that when the Mach number is 1, then the velocity of sound—or rather, the velocity through the medium—is the velocity of sound, right? This we have done. Now, we will do certain—yeah.





Prob. Air flows through a nozzle of diameter 1.3 mm having a discharge coefficient of 0.95, from a pressure of 5 atm to a pressure of 1 atm at 25 °C. What is the maximum velocity and mass flow rate? Ans. We know that, $\rho = \frac{pM}{RT} = \frac{5x101325x29}{8314x298} = 5.93 kg m^{-3}$ Now, upstream pressure is 5 times greater than that of down stream pressure. Hence pressure ratio is at critical condition

$$v_{0} = \sqrt{\frac{2\gamma p}{(\gamma - 1)\rho}} \left[1 - \left(\frac{p_{0}}{p}\right)_{c}^{\frac{\gamma - 1}{\gamma}} \right]$$
$$= \sqrt{\frac{2x1.4x5x101325}{(1.4 - 1)5.93}} \left[1 - (0.528)^{\frac{1.4 - 1}{1.4}} \right]$$
$$= 315.83 \, ms^{-1}$$

Now, we will do certain things, as we have said that unless you do some problem-solving, you are not actually doing the subject right; you are not grasping the subject. So, here we have shown the Mach number is 1. When the Mach number is 1, the velocity through the medium is the velocity of sound, and that is only possible when the critical pressure ratio is 0.528—that means the discharge is also then maximum, right? So, with this, let us do a problem—or, OK, before going to that, let us do another thing.



This is called variable flow, right? This is called variable flow. Now, variable flow means—I hope during your childhood, during your childhood, you have done—you have done certain things like, during your childhood, yeah, in the birthday party and many others, you have blown balloons, right? I do not know whether you have experienced it or not.

- Variable Flow
- Variable flow is a flow which occurs when upstream pressure varies. The important thing to remember is that p₀/p will always be at critical level until p_a/p increases. This will happen only when p has dropped to a value of p = p_a/0.528 = 1.894 atm. So above 1.894 atm p₀/p will be maintained at critical value. =

• Now,

$$W = C_D A_0 \sqrt{\frac{2\gamma p\rho}{(\gamma - 1)}} \left[\left(\frac{p_0}{P}\right)_{cr}^2 - \left(\frac{p_0}{p}\right)_{cr}^{\frac{\gamma + 1}{\gamma}} \right]$$

When you are blowing a balloon, when it became bigger, suddenly you missed and the balloon went like somewhere and then dropped there. I hope this you have experienced right. So, if you have not experienced do it today that you take a balloon, blow it and suddenly lose it or allow it to go anywhere. You will see it will move like anything and then it will drop. This is called variable flow.

Our container is that balloon. It had certain volume and pressure because you have pumped through your mouth, you have pumped some air inside. So pressure is up. right and the balloon mouth is very very small right. So, when it is coming out from there you will hear some sound also some sound like that and it will drop somewhere.

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• Now,

$$W = C_D A_0 \sqrt{\frac{2\gamma p\rho}{(\gamma - 1)}} \left[\left(\frac{p_0}{P}\right)_{cr}^{\frac{2}{\gamma}} - \left(\frac{p_0}{p}\right)_{cr}^{\frac{\gamma + 1}{\gamma}} \right]$$

So, this comes under this category of variable flow. So, variable flow is a flow which occurs when upstream pressure varies because you have you have released. So, the pressure is gradually falling right. So, upstream pressure is varying. So, the important thing to remember is that the P_0 by P will always be a critical level

 P_A over P increases, P_A means surrounding atmosphere, over P increases. This will happen only when P has dropped that is the pressure has dropped to a value of P is equal to P_A over 0.528 that is 1.894 atmosphere right, P_A over 0.528 is 1.894. So, above 1.894 atmosphere P_0 by P will always be maintained at critical value right. we can say that the discharge, the rate of discharge, W is $C_D A_0$ to gamma P rho by gamma minus A_0 , gamma minus 1 into P_0 by P to the power 2 by

gamma under critical condition, critical pressure ratio condition, minus Po by P to the power gamma plus 1 by gamma under critical pressure ratio condition that is the discharge right. Now, since P is dropping, right we had told already that you are taking a balloon having high pressure volume you released it. So, pressure is gradually falling right. Since p is dropping we can write w is equals to k 1, k 1 is a constant under root p rho.

• Since p is dropping

$$W = K_1 \sqrt{\rho p}$$
• Where,

$$K_1 = C_D A_0 \sqrt{\frac{2\gamma}{(\gamma - 1)}} \left[\left(\frac{p_0}{P}\right)_{cr}^{\frac{2}{\gamma}} - \left(\frac{p_0}{p}\right)_{cr}^{\frac{\gamma + 1}{\gamma}} \right]$$

Now, Mass of Gas in a container can be expressed as,
 m = V_c ρ where V_c is the volume of the container.
 Now,

$$pV = \frac{RT}{M}; \quad or, \frac{p}{\rho} = \frac{RT}{M}$$

or, $\rho = \frac{pM}{RT} = K_2 p$ (since, M,R,T are constants)

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• Now,

$$W = C_D A_0 \sqrt{\frac{2\gamma p\rho}{(\gamma - 1)}} \left[\left(\frac{p_0}{P}\right)_{cr}^{\frac{2}{\gamma}} - \left(\frac{p_0}{p}\right)_{cr}^{\frac{\gamma + 1}{\gamma}} \right]$$



right. Obviously, that constant k1 is nothing, but equal to $C_D A_o$, rest of the part under root 2 gamma by gamma minus 1, because P we have taken out, rho you have taken out, right 2 gamma by gamma minus 1. P_o by P to the power 2 by gamma under critical condition, minus P_o by P to the power gamma plus 1 by gamma by critical, under critical condition, right. Now, mass of gas at a container can be expressed as m, that is equal to V_C into rho, where V_C , it is not that specific volume, V is the volume. V_C into rho where V_C is the volume of the container, right that is why it is denoted as V_C , volume of the container.

This is capital V; do not mix it up with the earlier capital V, which was specific volume, but here it is the volume. Real volume, that is why it is denoted as V_C, right. So, the mass of that is expressed as V_C into rho, and therefore, we can write from PV is equal to RT by M, or P by rho is equal to RT by M, or rho is equal to PM by RT. Therefore, we can say that rho is equal to k_2P , right, because M, R, T are constants. So, we can write it to be k_2P , where k_2 is also a constant, and we can define it as k_2 is equal to.



Your m by k_2 is equal to M by RT, right. Therefore, we can write m is equal to V_C into k_2 into P, right, and dm, change of mass, dm, small change of mass. That we can write is nothing but V_C k_2 dP. V_C is the container volume, k_2 is the constant, dm & dP, any change in mass, small change in mass, also corresponds to a small change in pressure. Right, that may be increasing or that may be decreasing. If it is increased as it is coming, it is directly proportional; as the mass is increasing, dP is also increasing, right, as the mass is decreasing.

$$\therefore m = V_c K_2 p; \quad and, dm = V_c K_2 dp$$

$$Now, W = K_1 \overline{)pp} = K_1 \sqrt{K_2 p^2} = K_1 \sqrt{K_2} p$$

$$\therefore dW = K_1 \sqrt{K_2} p dt$$

$$Now, dW = -dm$$

$$or, K_1 \sqrt{K_2} p dt = -V_c K_2 dp$$

$$or, \int_1^2 dt = -\frac{V_c \sqrt{K_2}}{K_1} \int_1^2 \frac{dp}{p} = -\frac{V_c \sqrt{K_2}}{K_1} \ln \frac{p_2}{p_1} = \frac{V_c \sqrt{K_2}}{K_1} \ln \frac{p_1}{p_2}$$

$$or, t = \frac{V_c \sqrt{K_2}}{K_1} \ln \frac{p_1}{p_2}$$

Therefore, we can write that the discharge rate, W, is equal to k_1 . Under root p rho. So, that is equal to k_1 under root k_2 P square, right. That means, k_1 under root k_2 , but P is outside the root, right. So, a small differential like dw.

that should be equals to, that is dW is the rate, rate of flow, that is per unit time, right. So, much either volume or kg per unit time right. So, dW is equal to k_1 under root dt, dt is the time through which the rate has been changed right. Therefore, we can write easily dW

must be equals to minus dm. So, the rate, as the m is decreasing rate is also decreasing that is why the negative sign.

So, we can write k_1 under root k_2 P dt is nothing, but equals to minus V_C k_2 dP, right, minus V_C k_2 dP, or we can write integration of dt between point 1 and 2 is equal to V_C into under root k_2 by k_1 right into integral of dP over P, between point 1 to 2. So, that is equal to minus V_C into under root k_2 by k_1 this dP by P is ln P₂ by P₁ with the limit 1 to 2. So, we can rewrite that negative sign we can remove by changing P₂ by P₁ to P₁ by P₂. So, that becomes V_C under root k_2 over k_1 ln of P₁ by P₂.



Therefore, we have integrated dt, from point 1 to 2, that is t is equal to V_C under root k_2 by k_1 into ln of P_1 by P_2 . This tells that how much time it will take from a pressures P_1 to become pressure P_2 right given the values of constant V_C , k_2 and k_1 , which you can easily find out because you have already defined k_2 in terms of MRT and K_1 a bigger one. right, but the values are known. So, you can find out what is the time required for bringing down the pressure from given pressure P_1 to known pressure P_2 , right.





So, again, we undergo through a problem where a nozzle of 20 millimeter diameter releases the air from an air reservoir to the atmosphere. The initial pressure and temperature of the air are 6 atmospheres and 20 degrees centigrade, respectively. The volume of the reservoir or the volume of the receiver is 20 cubic centimeters.

A nozzle of 20 mm diameter releases air from an air reservoir to the atmosphere. Initial pressure and temperature of air are 6 atm and 20 °C respectively. Volume of the receiver is 20 cubic cm. If C_D is 0.98, m is 28.97, y is 1.3, how long it will take to lowen the pressure to 3 atm? Solution: $K_{2} = \frac{M}{RT} = \frac{28.97}{8314.34X293} = 1.18519X10^{-5}$ and, $t = \frac{V_c \sqrt{K_2}}{K_1} \ln \frac{p_1}{p_2} = \frac{15X\sqrt{1.18519X10^{-3}}}{2.0528X10^{-4}} \ln \frac{6}{3}$ = 174.3 sec

If C_D is 0.98, or rather, if C_D is 0.98, M is 28—it should be capital M—M is 28.97, gamma is 1.3, how long will it take to lower the pressure to 3 atmospheres? So, we are given a nozzle of 20 millimeter diameter that releases air. from an air reservoir to the atmosphere. The initial pressure and temperature of the air are 6 atmospheres and 20 degrees centigrade, respectively. The receiver is 20 cubic centimeters.

If C_D is 0.98 and capital M is 28.97—that is the molecular weight—gamma is 1.3, how long will it take to lower the pressure to 3 atmospheres, right? From 6 atmospheres to 3 atmospheres, right. Now, let us first find out the constants. The first constant, k₁, was C_D A_0 under root 2 gamma by gamma minus 1 into P₀ by P₂ under critical pressure ratio condition to the power 2 by gamma. Minus P_0 by P under critical pressure ratio condition to the power gamma minus 1, gamma plus 1 by gamma, right. So, here all the values are known. C_D has been given as 0.98. A_0 we have to find out, pi by 4 into we are said that 20. We have said 20 cubic centimeters, OK, 20 degree centigrade and nozzle of 20 millimeter diameter.



So, that means, that is also said 20. So, 20 into 10 to the power minus 3 whole square is the area. Into 2 into gamma is 1.3, fine. Therefore, divided by 1.3 minus 1, fine, P_0 by P under critical condition is 0.528 to the power 2 by 1.3, right, minus 0.528 to the power 1.3 plus 1 by 1.3, and this on simplification. Or calculation becomes equals to 2.0528 into 10 to the power minus 4.

Obviously, a question should have come to your mind that we have taken P_0 by P under critical condition to the power 2 by gamma. Is it our power? We just can take anything as we like, perhaps in science you cannot. You have to always justify. So, for this justification is what?

Justification is the pressure ratio. Your inlet pressure is reservoir at 6 atmospheres and your outlet pressure is 3 atmospheres. right. That means, P $_{tip}$ is 3 atmosphere and P $_{inside}$ is 6 atmospheres. What is the ratio?

If we make P_0 by P, then it is 3 by 6, that means 0.5. right. We earlier also said that if P_0 by P is 0.528, 28 was on the second decimal, but if it is 0.5 then it is under critical condition or the reverse if we make or the reverse if we make 6 by 3 that is P by P_0 , 6 by 3 is 2 which is more than 1.89 that value right. So, that means, it is under critical pressure ratio condition though we have not said it separately here, but it is.

that you have to also show. It cannot be that you will, you will just take it, and take it for granted. You have to establish that you are taking it because it is under critical pressure

ratio condition. So, you have found out first constant k_1 which is 2.0528 into 10 to the power minus 4.

Second constant k_2 , that is M by RT, M is given 28.97 temperature given 20 degree centigrade that means, 273 plus 20, is 293 right. So, it is 1.1859 into 10 to the power minus 5. Therefore, the time required, which we have been asked from 6 to 3, time required is equal V_C under root k_2 by k_1 into ln of P₁ by P₂.

So, if you substitute the values, then it becomes V_C was given 15 volume of the container no not 15, 20. I think here we have done that mistake not mistake or whatever. So if it is here 15 then you change the volume of the receiver from 20 to 15 in the problem. Because solution is based on 15. So, the problem also has to be made on the basis of 15.

So, change that 20 cubic centimeter to 15 cubic centimeters So, time required is 15 into under root 1.8519 which you have found out. into 10 to the power minus 5 over 2.0528 into 10 to the power minus 4 ln of 6 by 3 right. If you do the solution of it through calculator. You will see that you are getting 174.3 seconds, right?



reservoir to the atmosphere. Initial pressure and temperature of air are 6 atm and 20 °C respectively. Volume of the receiver is 20 cubic cm. If C_p is 0.98, m is 28.97, γ is 1.3, how long it will take to lower the pressure to 3 atm?



=174.3 sec

You are getting 174.3 seconds. Now, does not it come to your mind that whether it will be second or hour or minute? why second, why not hour, why not minute? Because that you can find out from the previous one right. Here you see $\ln P_1$ by P_2 .

So, pressure ratio has no dimension right. This is the volume of the container that is it has no, it is, rather it has, it has in meter cube or centimeter cube or whatever it is. Then k_1 , k_1 is not here k_1 we have found out there ok. k_1 is here.



So, that is $C_D A_o$, A_o is meter square, gamma dimensionless. So, 2 gamma by gamma minus 1 dimensionless, Po by P dimensionless because they are cancelling out, then may next P_o by P is cancelling out. That means, k_1 has unit that is meter square right. Now, k_2 , k_2 is what?

 k_2 is here. It is shown that k_2 is M by RT, if you remember right. So, M by RT is k_2 from there. M is kg mole per kg, T is degree centigrade, and another R. So, that also you take in terms of mole, or here you have taken kilojoules per kg in that unit, right? So, if you look at all these, you will see that time t is becoming only in seconds.

Since our time is also going out, we have only a few minutes to solve another problem: that is, a reservoir of oxygen is maintained at 900 millimeters of mercury pressure and 25 degree centigrade temperature. A 10-millimeter nozzle fitted to this reservoir releases oxygen to a pressure of 650 millimeters of mercury. If the molecular weight of oxygen is 32, what is the rate of release of oxygen? If the downstream pressure falls to 200 millimeters of mercury, or what is the 200, 200 millimeters of mercury, then what is the percentage increase in the rate of supply of oxygen, assuming C_D is 0.98?

Molecular weight is 32, gamma 1.4. So, quickly repeat: a reservoir of oxygen is maintained at 900 millimeters of mercury pressure and 25 degree centigrade temperature. A 10-

millimeter nozzle fitted to the reservoir releases oxygen to a pressure of 650 millimeters of mercury. If the molecular weight of oxygen is 32, what is the rate of release of oxygen? If the downstream pressure falls to 200 millimeters of mercury, what is the percentage increase in the rate of supply of oxygen?



Molecular weight 32, gamma 1.4, right. It is perhaps from the previous, not from variable; it is normal, right. W 650, we can find out the distance z directly as $C_D A_0$ to gamma P rho by gamma minus 1 into P_0 by P to the power 2 by gamma minus P_0 by P to the power 1 gamma plus 1 by gamma. So, by substituting the values, we get that it becomes 0.0186594.

Similarly, W 21 red also, here also, you see we have written the previous. The discharge pressure and inlet pressure 650, 900, we have taken in, right. And W 200 is 0.98 into 5 pi by 4 into 10 into 10 to the minus 10 to the minus 3 whole square into 2 into 1.4 into 900 into 101.325 1.4 minus 1 into 760 into 200 by 900 to the power 2 by 1.4 minus 200 by 900 to the power

This comes to 0.0192386, and the percent increase is 3.01, OK. Our time is up. So, I thank you all for listening to the class. Thank you.

Prob.:-A reservoir of oxygen is maintained at 900 mm of Hg pressure and 25 °C temperature. A 10 mm nozzle, fitted to this reservoir releases oxygen to a pressure of 650 mm of Hg. If molecular weight of oxygen is 32, what is the rate of release of oxygen? If the down stream pressure falls to 200 Hg of, what is the percentage increase in rate of supply of oxygen? Assume C_p=0.98, m=32, γ =1.4.

