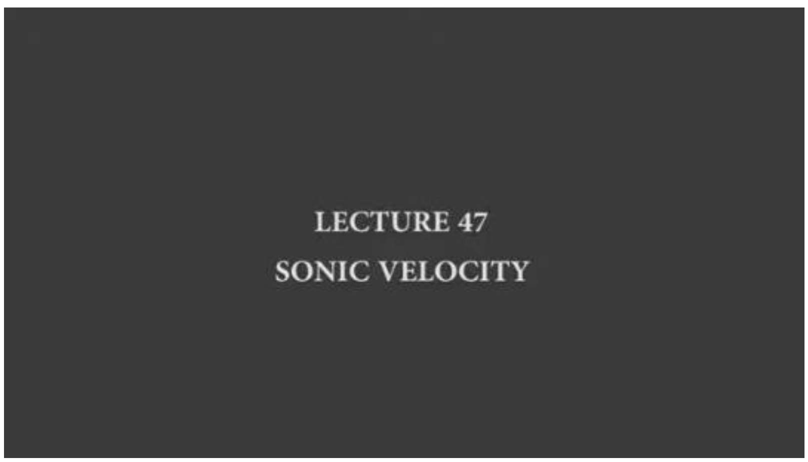
IMPACT OF FLOW OF FLUIDS IN FOOD PROCESSING AND PRESERVATION

Lecture47

LECTURE 47 : SONIC VELOCITY

Good morning, my dear students, boys and girls, and friends. We are continuing with the problems of nozzle flow, right? We have done one problem, and we said that the problem solution is very, very important for finding out the solution, okay. Now, we will go to another problem.



Now, at critical pressure ratio, i.e., $p_0/p = 0.528$, the velocity and discharge will be maximum

$$v_0 = \sqrt{\frac{2\gamma p}{(\gamma-1)\rho}\left[1-\left(\frac{p_0}{p}\right)_{cr}^{\frac{\gamma-1}{\gamma}}\right]} = \sqrt{\frac{2X1.4X3X101325}{(1.4-1)X3.44}\left[1-(0.528)^{\frac{1.4-1}{1.4}}\right]}$$

$$= 321.2\ m\,s^{-1}$$

$$and, W = C_D A_0 \sqrt{\frac{2\gamma p\rho}{(\gamma-1)}\left[\left(\frac{p_0}{P}\right)_{cr}^{\frac{2}{\gamma}} - \left(\frac{p_0}{p}\right)_{cr}^{\frac{\gamma+1}{\gamma}}\right]}$$

$$= 0.91X1.1309X10^{-6}\sqrt{\frac{2X1.4X3X101325X3.44}{(1.4-1)}\left[(0.528)^{\frac{2}{1.4}} - (0.528)^{\frac{1.4+1}{1.4}}\right]}$$

$$= 7.2058X10^{-4}\ kg\,s^{-1} = 2.594\ kg\,h^{-1}$$

I think in this problem, we have v_0—yeah, we have found out the discharge also: 0.528, yeah. So, this is another problem: air flows through a nozzle of diameter 1.3 millimeters, having a discharge coefficient of 0.95. You see, earlier the discharge coefficient was 0.91.

Example: A nozzle of 1.2 mm dia with a coefficient of discharge of 0.91 is to deliver air from 3 atm pressure to 2 atm pressure at 35 ^{0}C. Calculate the velocity, mass flow rate and the maximum velocity and mass flow rate corresponding to the downstream pressure at a critical value. Assume molecular weight of air to be 28.97.

Solution: $We\ know\ that,\ v_0 = \sqrt{\frac{2\gamma p}{(\gamma-1)\rho}\left[1-\left(\frac{p_0}{p}\right)^{\frac{\gamma-1}{\gamma}}\right]}$

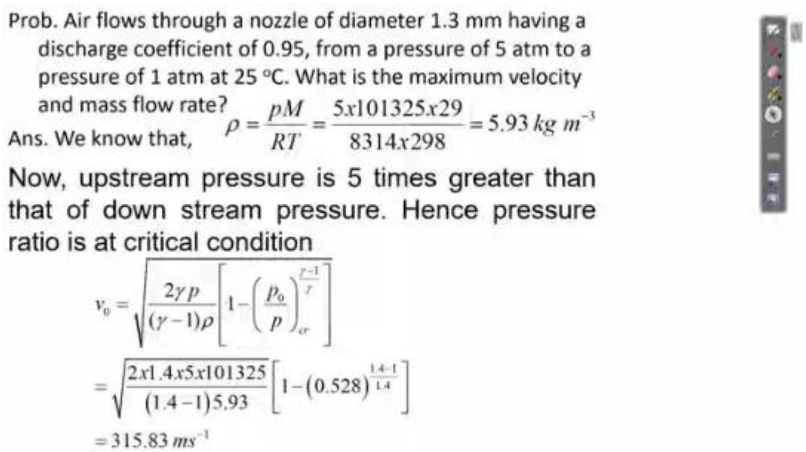
$$Now,\ \rho = \frac{pM}{RT} = \frac{3X101325X28.97}{8314X308} = 3.44\ kg\,m^{-3}$$

$$and,\ A_0 = \frac{\pi D^2}{4} = \frac{\pi X(0.0012)^2}{4} = 1.1309X10^{-6}\ m^2$$

So, this time the discharge coefficient is 0.95, which means, depending on—I have shown you different types of right, that you remember. So, different types of nozzles I had shown you. So, here it is coming to 0.95. So, what is happening?

It is flowing through a nozzle from a pressure of 5 atmospheres to a pressure of 1 atmosphere. At 25 degrees centigrade. What is the maximum velocity and maximum flow rate? Now, to know the maximum flow rate and maximum velocity, we have to first prove what? That the pressure ratio is critical, right?

We have been given one pressure at 5 atmospheres and another pressure at 1 atmosphere, right? So, with respect to 5 atmospheres, let us find out what is the value of rho. So, the value of rho is P M by RT, that is 5 into 101325 into 29 by 8314 into 298. That is 5.93 kg per meter cube, right? Now, the upstream pressure is 5 atmospheres or 5 times greater than the downstream pressure.



Therefore, the pressure ratio is at a critical condition. Why? Why? Because we said P_o by P, if it is 0.528, then it is under critical condition. Also, we said if P by P_o is equal to 1.98 something.

So, then also it is under critical pressure ratio, right. So, in this case, this P is 5, P_o is 1, so that means it is 5, which is under critical pressure ratio condition, right. So, that means we can apply the condition of critical pressure ratio, and that will give us the velocity or discharge, whatever we determine under critical pressure ratio or maximum velocity or maximum discharge, right.

Therefore, vo we can write equal to under root 2 gamma P by gamma minus 1 into rho into 1 minus P_o by P to the power gamma minus 1 by gamma under critical pressure ratio. In this respect, one more thing I would like to highlight is that for simplicity, we have done the density with respect to pressure P, which is the upstream velocity upstream pressure rather. Instead of doing upstream pressure, we could have done P_{in} plus P_{out} divided by 2, which is $P_{average}$ we could have done. So, that would have been 5 plus 1 divided by 2, which means 3 atmosphere it could have been right, and 3 atmosphere upstream to downstream pressure ratio is also under critical pressure, then this rho could have been much closer to the real right, but for simplicity, because this is nothing but a mathematical relation.

I did not change, but you can try as I have shown you that taking the $P_{average}$, right. Therefore, as we said, the velocity v or vo that is under root 2 gamma P by gamma minus 1 into rho into 1 minus P_o by P to the power gamma minus 1 over gamma under critical pressure ratio condition. So, this we substitute with the values 2 gamma is 1.4, though it is not given, but generally, as we said, for diatomic gases, gamma is 1.4 right, and air is a diatomic gas.

So, it is 1.4. So, 2 into 1.4 into 5 into 101325 divided by 1.4 minus 1 into the density, which we have found out, 5.93 into 1 minus pressure ratio 0.528 into to the power 1.4 minus 1 by 1.4, right? That is equal to 315.83 meters per second. Shall we try once again whether it is coming true or not? We can try with our calculator once more.

So, it has come. Let us try 1 minus 0.528 is So, x to the power y, so to the power 1.4 minus 1, that is 0.4, divided by 1.4, divided So, this goes out. So, equal to this much, okay. This into 2 into 1.4 into 1 0 1 3 2 5

is equal to this much divided by 1.4 minus 1, that is 0.4, into 5.93. Bracket close. So, it is coming 96, etcetera. So, the square root of this is 310.67, and we are getting 315.83. Maybe somewhere we have done something wrong, or maybe not. I am not saying that all the time, okay.

Then we go for the other one, which is for the maximum velocity, right. Now, dW/dP_o is zero, right. Therefore, A_o is $\pi/4 \times 0.0013$ whole squared, which equals 1.327×10^{-6} meter squared. Therefore, W is equal to $C_D \times A_o \times \sqrt{}$

$2\gamma P\rho/(\gamma - 1) \times (P_o/P)^{(2/\gamma)} - (P_o/P)^{((\gamma + 1)/\gamma)}$, which equals $0.95 \times 1.327 \times 10^{-6} \times \sqrt{(2 \times}$ 1.4, which is γ, $\times$ 101325 $\times$ 5.93). I think one P value is missing, which is 5 divided by 1.4 minus 1, into 0.528 to the power 2/1.4 minus 0.528 to the power (1.4 + 1)/1.4, right. This becomes equal to 6.69×10^{-4} kg per second, which, when converted to kg per hour,

is 2.409 kg per hour, right. So, this way you can find out. Now, another very important topic is called sonic velocity. I gave you the example that sonic velocity is the velocity of sound in a medium.

Sonic Velocity

$$v_s = \sqrt{\frac{K}{\rho}}$$ K=bulk modulus of air

Velocity of sound in air

$$K = \frac{\Delta p}{\Delta V / V} = -\frac{dp}{dV / V} = -\frac{dp}{\rho dV}$$

$$Now, \because V = \frac{1}{\rho}; \quad or, \frac{dV}{d\rho} = -\frac{1}{\rho^2}; \quad or, dV = -\frac{d\rho}{\rho^2}$$

$$\therefore K = -\frac{dp}{\rho\left(-\frac{d\rho}{\rho^2}\right)} = \frac{\rho dp}{d\rho}; \quad or, \frac{K}{\rho} = \frac{dp}{d\rho}$$

$$\therefore v_s = \sqrt{\frac{dp}{d\rho}}$$

$$\frac{dW}{dp_0} = 0$$

$$Now, A_0 = \frac{\pi}{4}(0.0013)^2 = 1.327x10^{-6}\ m^2$$

$$\therefore W = C_D A_0 \sqrt{\frac{2\gamma p\rho}{(\gamma-1)}\left[\left(\frac{p_0}{P}\right)_{cr}^{\frac{2}{\gamma}} - \left(\frac{p_0}{p}\right)_{cr}^{\frac{\gamma+1}{\gamma}}\right]}$$

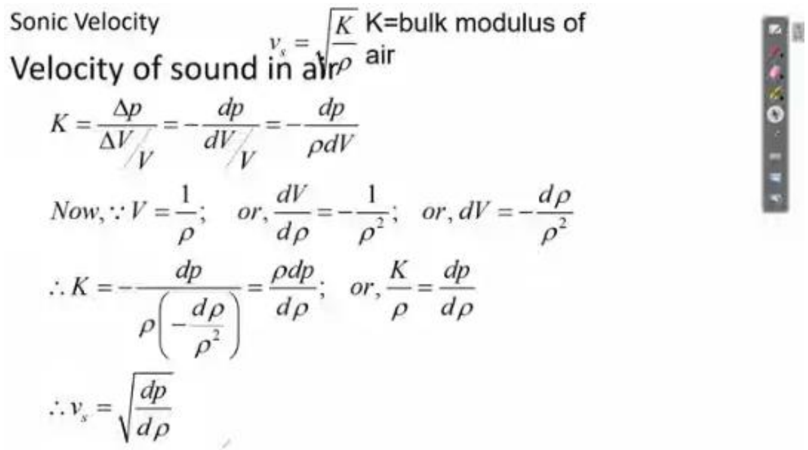
$$= 0.95x1.327x10^{-6}x\sqrt{\frac{2x1.4x101325x5.93}{(1.4-1)}\left[(0.528)^{\frac{2}{1.4}} - (0.528)^{\frac{1.4+1}{1.4}}\right]}$$

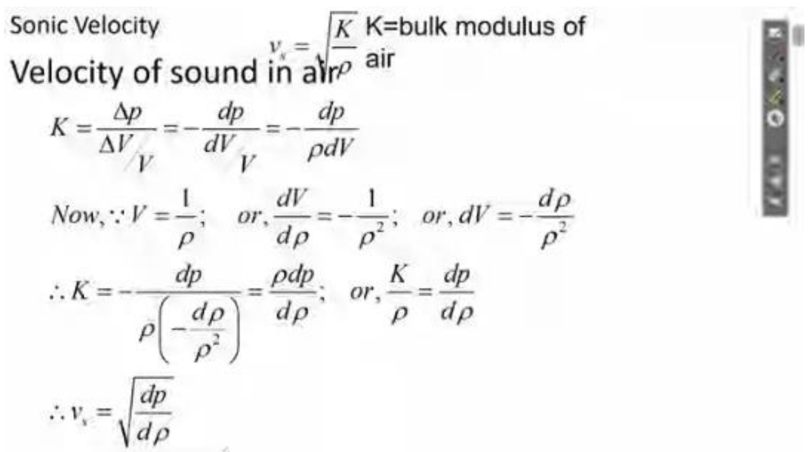
$$= 6.69x10^{-4}\ kg\ s^{-1}, \quad or,\ 2.409\ kg\ h^{-1}$$

Obviously, velocity of sound is a function of medium. If it is air it will have one, if it is water it will have I hope during swimming you might have said or you might have heard something from outside. Definitely that is different from what you have heard or you hear when your head is above the water. So, this is practical you know that you have observed.

So, velocity of sound is a function of the media through which the sound is propagating right. So, it is defined as velocity of sound as K by rho. where K is the bulk modulus of air and rho is obviously the density of air. So, vs that is velocity of sound we defined as under root K by rho, K is the bulk modulus of air and rho is the density. Now from the definition of bulk modulus of air we can write that K is equal to delta P over delta V by capital V right.

So, this is by definition, delta P over delta V per unit volume. right, that is change in pressure with the change in volume per unit volume change in not volume change in specific volume per unit specific volume right. So, this can be equated as minus dP over dV by V means as V is increasing that d V is decreasing that is why it is a negative that is equal to minus dP over now this V, which we have 1 by V in the denominator is replaced with rho. So, we write minus rho d P over rho d V.

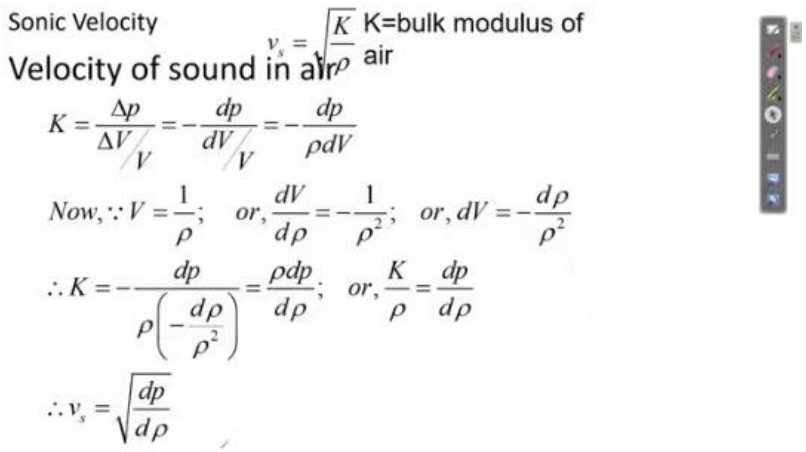




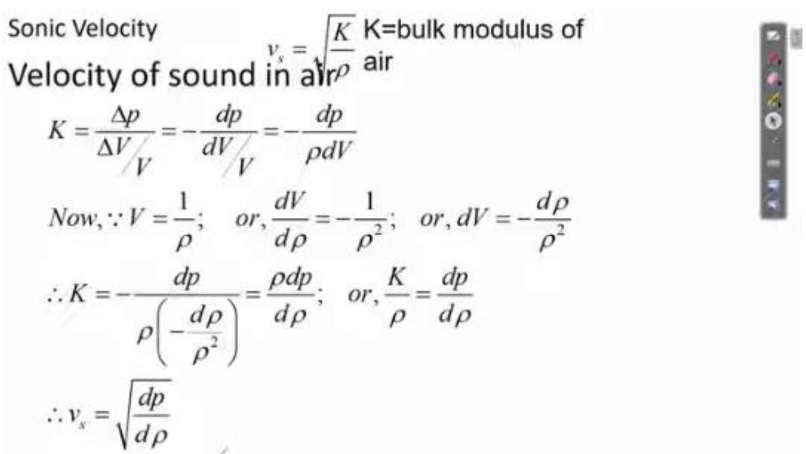
Right, since V capital is 1 by rho, that is known. Therefore, we can write dV over d rho, right? dV over d rho means we are differentiating specific volume with respect to density. Right. If you differentiate, if you want to see what is the change of specific volume with respect to density, then you have to differentiate V with respect to rho, or rather capital V with respect to rho, then that becomes dV over d rho. That is equal to minus 1 by rho square because it is rho to the power minus 1.

So, on differentiation, that minus comes outside, and it is minus 1 minus 1, which means minus 2. So, that becomes 1 by rho square. Or, dV can be said to be equal to minus d rho by rho square, right. So, if we substitute in terms of K, then K is equal to minus dP over rho into minus rho by rho square, right. So, K becomes equal to minus dP over rho times minus d rho over rho square, right.

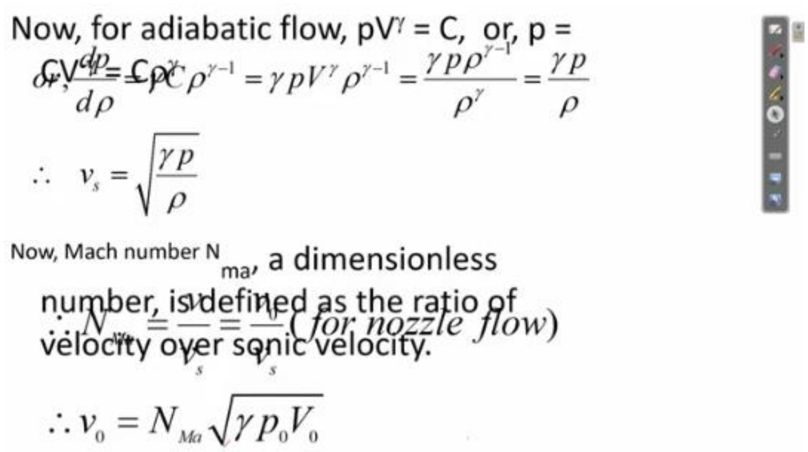
This, on simplification, we can write to be equal to rho into dP. Over d rho because these two rhos in the denominator, one is canceling, and the other goes to the top, and one negative from here and one outside negative, they are also canceling out. So, we get rho dP over d rho. Right, or this can be said to be K by rho is equal to dP over d rho, right. So, we write that the velocity of sound is nothing but under root dP over d rho.

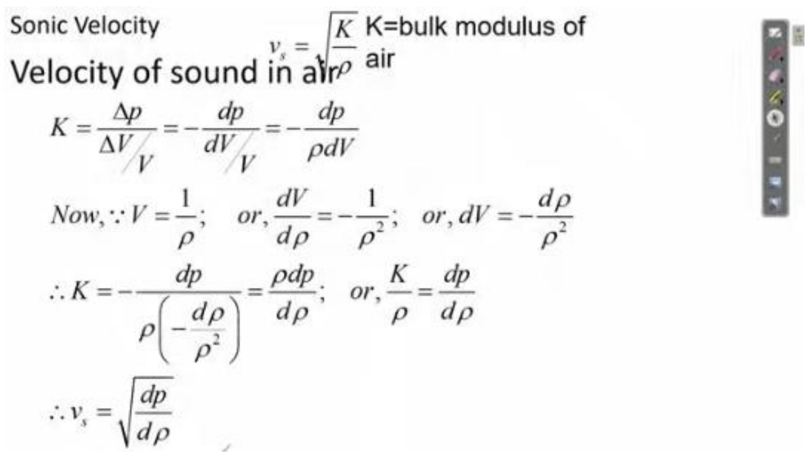


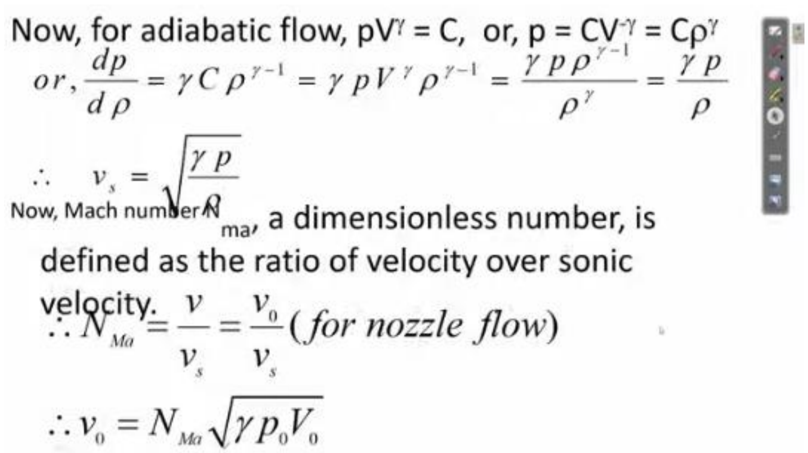
we wrote in the beginning at velocity of sound is K by rho, K being the bulk modulus of air if air is the medium. Then we arrived at the relation of velocity of sound with pressure and density under root dP over d rho right. vs is under root dP over d rho that is velocity of sound in medium air. vs is under root dP over d rho that means, under root change in density or change in pressure with respect to change in density right.



Now, we say that the flow is adiabatic that right from the beginning we are saying ok, right from the beginning we are saying that the flow is adiabatic. So, for adiabatic flow P V gamma is equal to constant right. Now, say P V gamma is constant if it is the adiabatic flow and P V gamma being constant we also can write P is equal to C V capital V to the power minus gamma that means, C into rho to the power gamma because V capital V to the power minus gamma means rho to the power gamma.

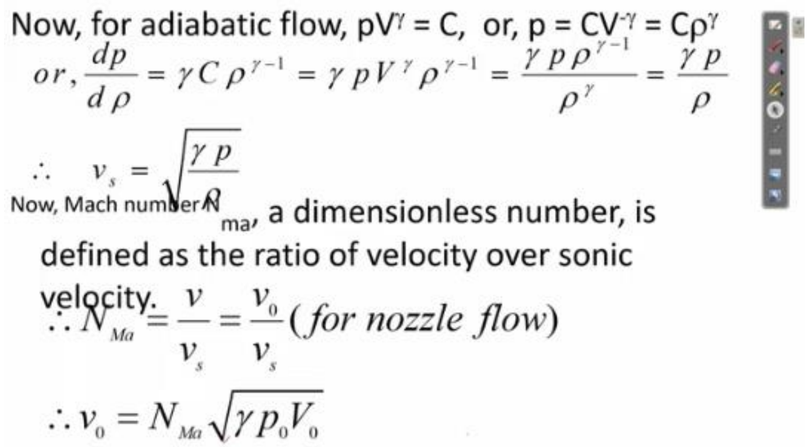






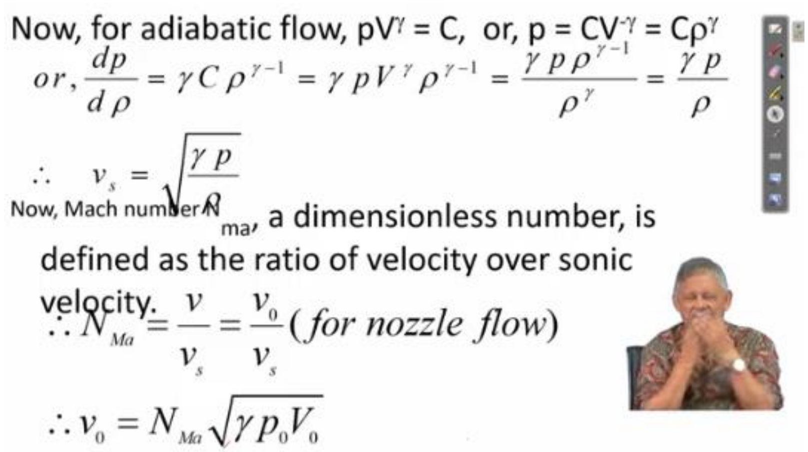
So, we write P is C into rho to the power gamma or dP d rho that is equal to gamma C rho to the power gamma minus 1 that is equals to gamma P capital V to the power into rho to the power gamma minus 1. This is equal to gamma P again rho to the power gamma minus

1 divided by rho to the power gamma because, that V to the power gamma is now taken as rho to the power gamma right. Therefore, this is nothing, but gamma P by rho because rho to the power gamma minus 1 minus gamma. So, gamma gamma goes out.



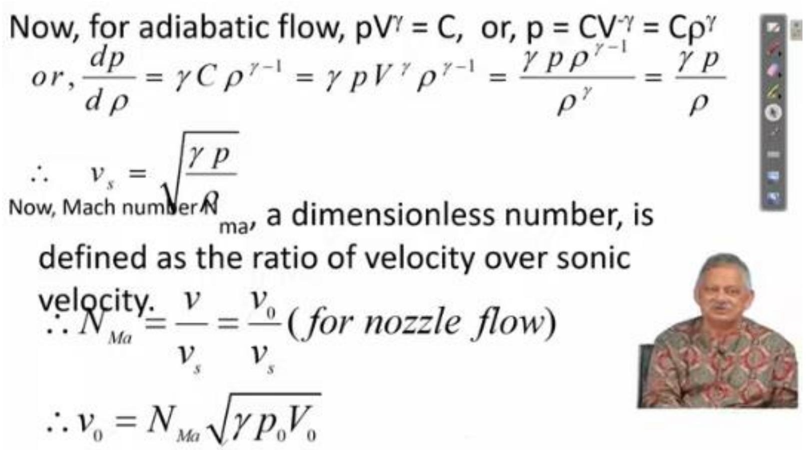
So, it becomes rho inverse. So, rho inverse means rho is in the denominator. So, gamma P by rho is the dP over d rho. Hence, the velocity of sound, we defined as, if you remember, gamma P by rho, right, we defined it as dP over d rho as the velocity of sound, and now

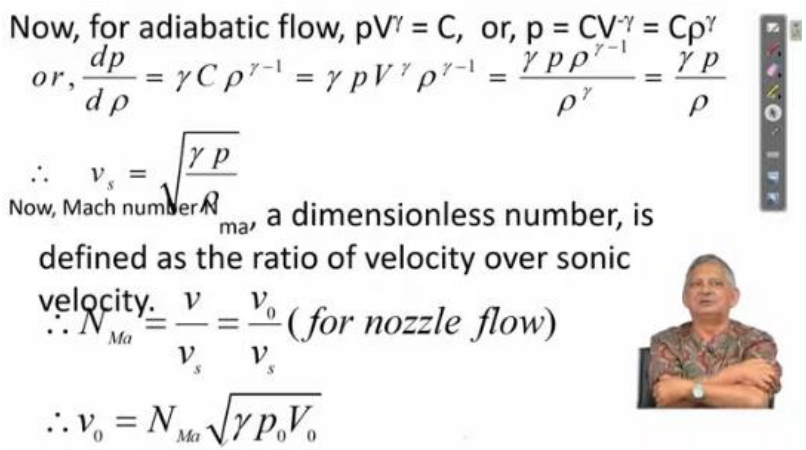
we have found out that dP over d rho for air medium is gamma P by rho. That means, the velocity of sound is expressed in terms of gamma P by rho, right. We now define a new number called the Mach number. That is N_{Ma}, Mach number, which is only used for sound. The Mach number is defined as a dimensionless number and can be defined as the ratio of velocity over sonic velocity.



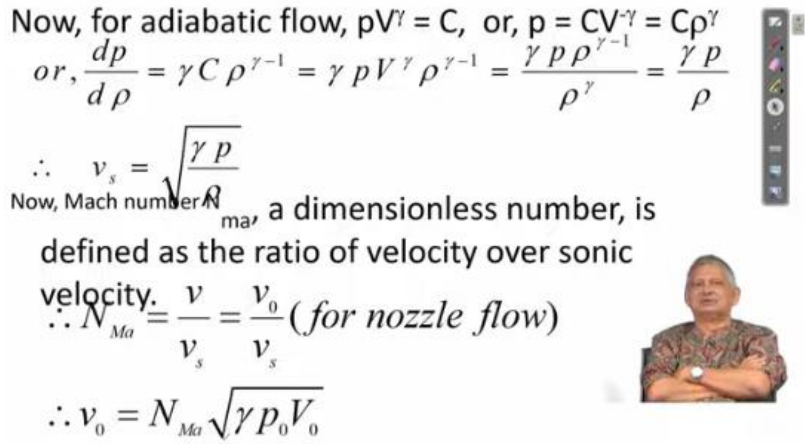
That ratio of the velocity of sound in the medium over sonic velocity, that is, the velocity of sound or velocity of the medium, rather, or maybe the velocity in the medium, velocity ratio over the velocity of sound. Right, or sonic velocity. So, N_{Ma}, we write as v by v_s, right, where v is the velocity of the medium, right. Then we write N_{Ma} equals v by v_s, right. Therefore, we can write this as equal to v_o by v_s, right, where v is nothing but the tip velocity or velocity of the medium at the tip, and V_s is the velocity of sound.

So, this is true for nozzle flow because we are taking the velocity at the tip, right. Therefore, we can simply write the velocity at the tip, v_0, is equal to N_{Ma}. Now, we have already shown earlier, what is the value of v_s. So, we found out v_s to be $\sqrt{(dP/d\rho)}$, which we have also shown that $\sqrt{(dP/d\rho)}$ is nothing but $\sqrt{(\gamma P/\rho)}$. Right.



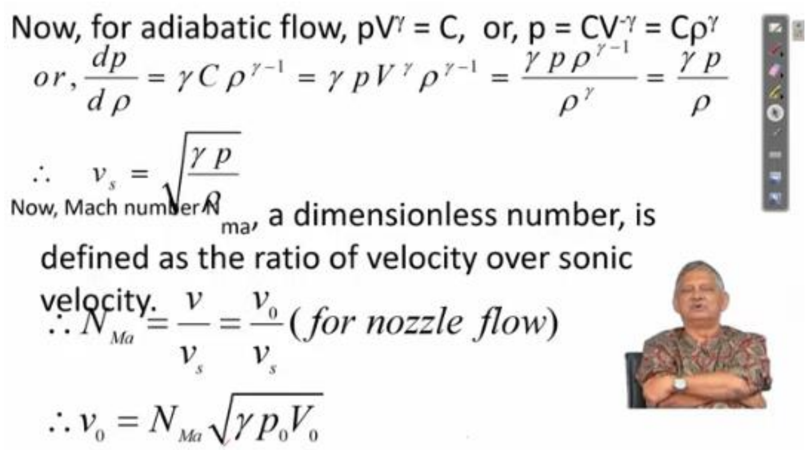


So, that means v_0 is Mach number N_{Ma} into $\sqrt{(\gamma P/\rho)}$, right? So, that means your ρ we have taken to the numerator, γ, that P is P_0 and ρ, we have taken to the numerator as capital V_0, all considering to be at the tip. This is the fundamental. You again see that we have already defined v_s as $dP/d\rho$ in the medium, right.



We also have said or found out that the velocity of sound can be related as $\sqrt{\gamma P/\rho}$, right. Now, we have defined a new number called Mach number N_{Ma}. defined as the ratio of the velocity of the medium to that of the sound, equal to v_0 or v_{naoht} to v_s, where v_0 is the velocity of the medium through the nozzle. We have now come to the relation that the velocity at the tip is nothing but the Mach number, that is, Mach number N_{Ma} into $\sqrt{\gamma P}$. Now, since it is v_0, so P also has become P_0, and that ρ we have taken as V, capital V. So, capital V_0 to the power rather V_0.

Vnaught is N_{Ma} under root gamma Pnaught Vnaught, right. So, with this, we have come to the end of this class. So, I thank you all for listening, and in the next class, we will perhaps complete this, or maybe we would like to complete it in the next class.



Thank you. Thank you for listening.