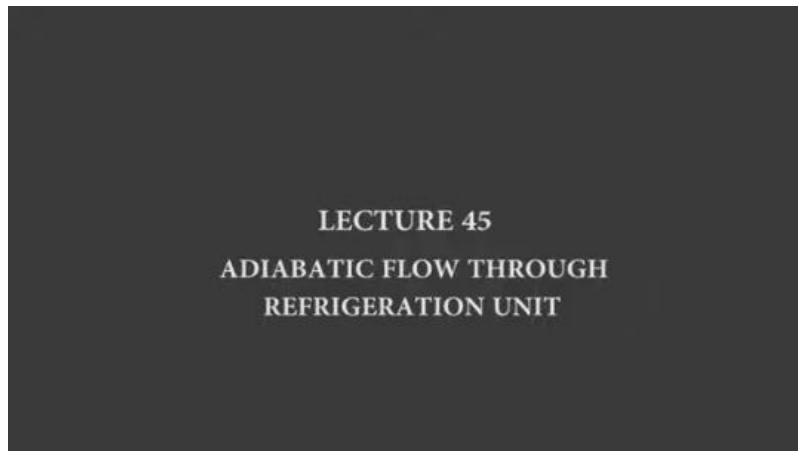


IMPACT OF FLOW OF FLUIDS IN FOOD PROCESSING AND PRESERVATION

Lecture45

LECTURE 45 : ADIABATIC FLOW THROUGH REFRIGERATION UNIT

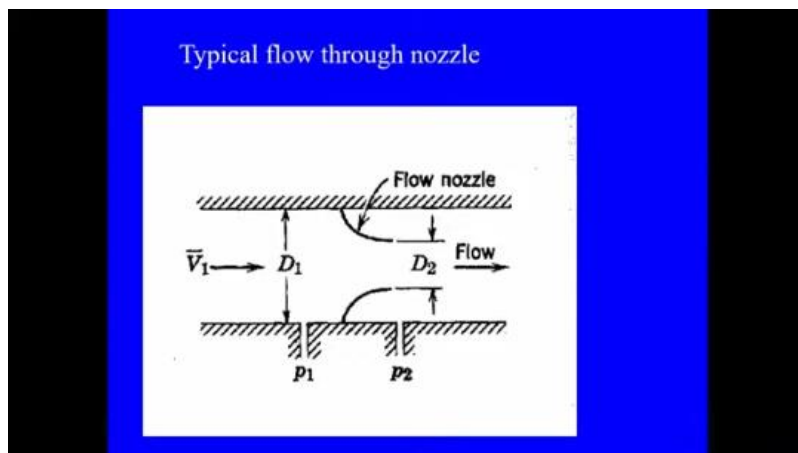
Good morning, my dear students, boys and girls, and friends. We are on the topic of nozzle flow, right? We have done a little, but I also said earlier that I would like to show you what the nozzles look like. Right? This is that nozzle, a typical nozzle.



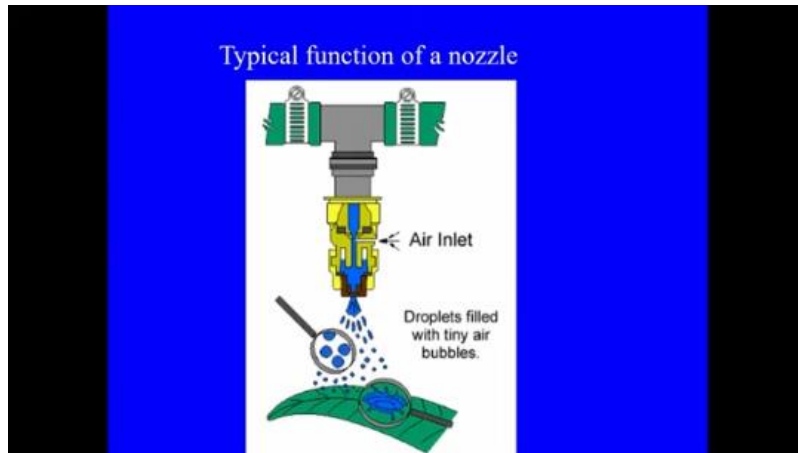


You see, from high pressure, it is coming down to low pressure. Right? This is one kind of manufacture. It is not that all nozzles are exactly like this. There are different types of nozzles.

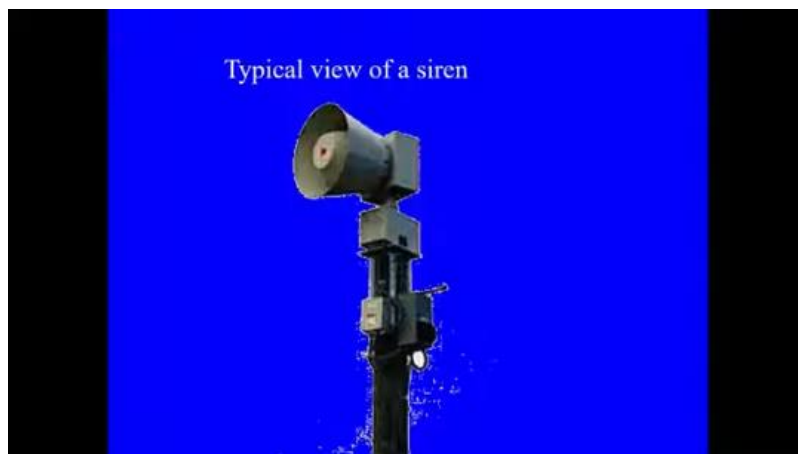
Like this, right. You see that there is a drastic difference in diameter D_1 and D_2 , right, and pressures are measured by means of a measuring device. And those pressures, which are measured by different methods, as you know, by a manometer, also pressure can be obtained, right. So, this is the flow nozzle. Where this is experimental, of course, where diameters you can measure easily, right, and velocities, this is the inside velocity, which is much less.



Inside pressure P_1 is high, and outside pressure P_2 depends on the value of D_2 , right. So, this is how it can be measured experimentally. So, this is another typical function of a nozzle. So, there are many nozzles that are blown with air, right, or it could be some fluid. So, droplets filled with tiny air bubbles are blown.



They are put or forced to flow from one end to the other end, right. So, this is another type nozzle. The third nozzle, which I mentioned, is used during war or maybe in factories where they maintain the time in, time out, etcetera.



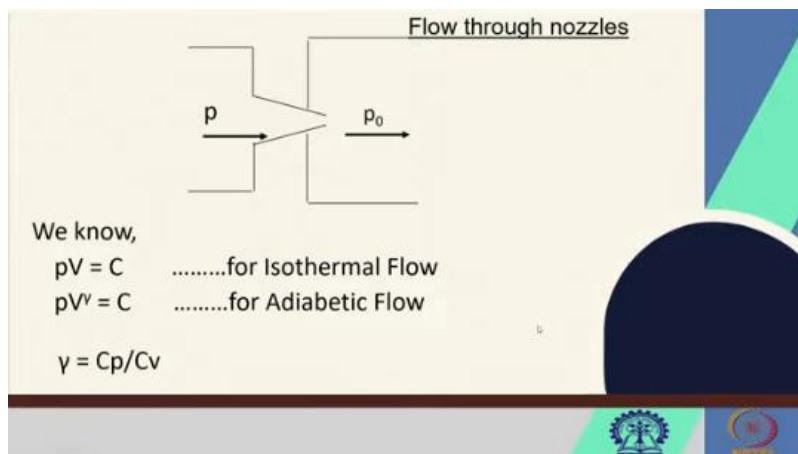
So, this is for warning the workers to get in on time, and it is also going up. So, for that, they raise the siren, and this is typically called a siren. Right, the same sound that you get from different, what should I say, different, yeah, factories, and where obviously the population of workers is much higher. So, it is only possible because people from all parts are coming. So, to let them know that the entry time is.

Coming and also before exiting, they also give a siren so that workers become alert to complete their job and get ready to exit. Right? And the other one I mentioned now, in every railway station, every school during war, this was there. So, had there been any possibility of any attack, either by air or otherwise, this would alarm people, for alarming people, this device used to be used, right.

So, the basic principle is the same: from a higher diameter, high pressure, so the reservoir source where the velocity is low is going through a very narrow opening, and that is causing the development of sound, right. We will show afterwards how the velocity of sound is getting affected. I do not know how many of you are near to the air base, at least a military air base.

If you see that air base, then you will There were cases when you could hear a booming sound. It is so heavy that even the glass windows can get broken because it is not only sound but also a jerk. So that happens. So, those who are in the military area, military base, they experience it.

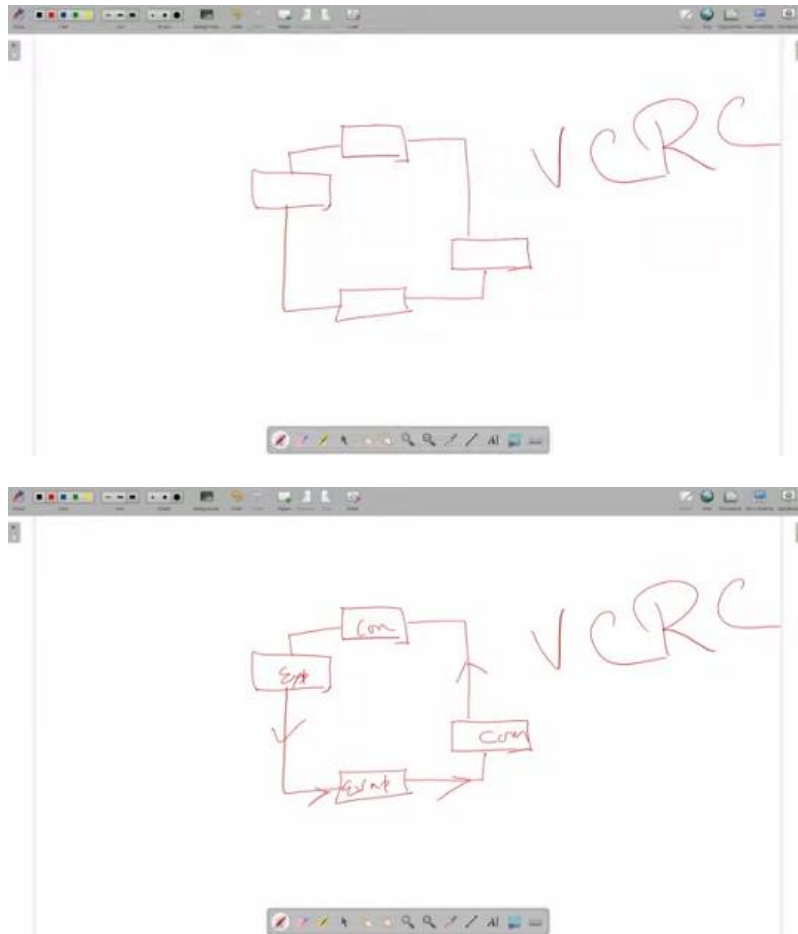
Yeah, we bring IIT Kharagpur near Salwa where there is a base, right? So, these are the sirens, okay? So, we said This is the mechanism by which the flow through nozzles occurs, right? Inside is high pressure, outside is low pressure.



And we also said that the flow is more adiabatic than isothermal. Because temperature may change. I had given an example also that in the refrigeration cycle, in the vapor compression refrigeration cycle, the fluid or liquid refrigerant flows through a valve that is called an expansion valve. When it goes through that from a very high pressure to a very low pressure, there is a change in the fluid, and this change is called

a change in temperature or throttling. The change in temperature of the fluid occurs because that fluid was at a higher temperature and higher pressure from the condenser. I told you last time, okay, if you are forgetting or if you have such a thing, then we can also show that this is the condenser this is the compressor, this is the expansion device, and I'm sorry, that is the evaporator, and this is the expansion device.

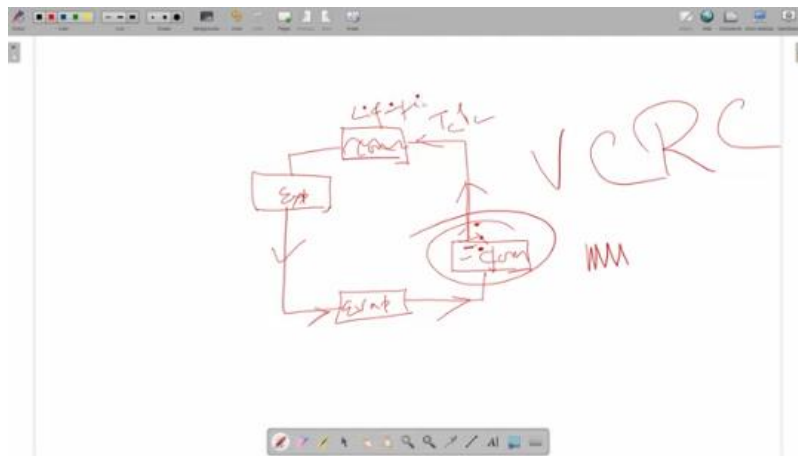
This is the complete cycle called the vapor compression refrigeration cycle. So, the compressor, this is the condenser, this is the expansion device, and this is the evaporator, right? So, the flow of the fluid is like this. From this, the compressor is making high pressure, right? And at high pressure, because it is under a normally reciprocating compressor, normally by and large, right? So, when it is



compressing, at that time, a valve here opens, and it goes to the line of the condenser. There is a non-return valve here which, when this pressure is less than this pressure, it gets closed, and again, when this pressure is more than this pressure, then it opens, right? So, that happens; this closing happens when it is in the return cycle, and the opening happens in the compression cycle. So, high pressure, high temperature, $T_{\text{condenser}}$, or $T_{\text{compressor}}$, and $P_{\text{compressor}}$, or $P_{\text{condenser}}$, they go through the condenser, and this vapor gets liquefied, right?

And not only that, from there, this liquid goes into the expansion valve. Generally, it is very, very narrow; we call it a capillary tube. Right, we call it a capillary tube; narrow

capillary tubes are used, and many other devices are there. So, from this high pressure to the low pressure, P evaporator, T evaporator, when it gets converted, then there is a



Change in temperature, a drop from T_c to T_e , happens, and it is definitely a drastic drop—a lot of temperature drop depending on what your evaporator temperature requirement is, right. Then we come to this point, okay. We have shown you that in the refrigeration cycle, from high pressure to low pressure, fluid is moving, right. And there, we have just shown that the temperature is being dropped drastically.

Flow through nozzles

We know,
 $pV = C$ for Isothermal Flow
 $pV^\gamma = C$ for Adiabatic Flow
 $\gamma = C_p/C_v$

There is no heat flow, of course, but because of throttling—that high pressure to low pressure through a narrow opening—this phenomenon is called throttling. I repeat, right. This phenomenon is called throttling, t h r o t l i n g, throttling, right, sorry. So, when throttling is happening, that time your temperature is reducing, and you are able to get low temperature. I gave this example also, if you remember that you make your mouth orifice very small and blow some air—I am not saying for sound whistle—blow some air, you will see that

the outlet of the air which you are blowing from your mouth is getting a little colder than what is in your mouth. It may not be substantial because your pressure drop is not substantial. That is why in this flow through the nozzle, the flow is adiabatic, right. And that is why we are justifying that why PV^γ is constant is used, not the PV is equal to constant, that is isothermal, because the process does not remain isothermal; it becomes adiabatic.

There is no heat flow, right. So, if we remember that, then we have also shown that with the PV^γ , we have derived it. I am not going into detail of it, right. We have derived it like this, where using Bernoulli's equation, we have integrated the velocity from point 1 to 2, v , say that is inlet velocity to the tip velocity.

In this case, the flow is adiabatic, since, the gas after being released from the pressure side is likely to undergo a temperature change.

∴ $pV^\gamma = C$ (constant), where, γ is the ratio of heat capacities at constant pressure and at constant volume respectively.

We can write, $p = CV^{-\gamma}$

or, $\frac{dp}{dV} = -\gamma CV^{-(\gamma+1)}$; or, $dp = -\gamma CV^{-(\gamma+1)}dV$

The tip velocity we have said it to be, say, v_0 or v_0 , whatever we call v_{tip} . So, we have shown it to be v square is equal to $2\gamma C$ by $1 - \gamma$ into v_0 , v_0 , rather here also small v_0 square is equal to $2\gamma C$ by $1 - \gamma$ into capital V_0 to the power $1 - \gamma$ minus small v , rather, capital V to the power $1 - \gamma$, and this on simplification we have said. to be $\gamma P V$ divided by $1 - \gamma$, rather $\gamma p v$ gamma divided by $1 - \gamma$ into V_0 to the power or V_0 to the power $1 - \gamma$ minus v , capital V to the power $1 - \gamma$, right. And then we found out that the velocity at the tip is nothing but equals to under root $2\gamma P$ divided by γ minus 1 into ρ into $1 - P_0$ by p to the power γ minus 1 by γ , right? This we have shown.

At the nozzle tip, Bernoulli's equation can be wrn as

$$\int \frac{dp}{\rho} + \int v dv = 0, \text{ or, } \int v dV = - \int \frac{dp}{\rho} = - \int V dp = + \int \gamma C V^{-(\gamma+1)} dV$$

$$\text{or, } \int_v^{v_0} v dv = \int_V^{V_0} \gamma C V^{-\gamma} dV$$

$$= \gamma C \int_V^{V_0} V^{-\gamma} dV = \frac{\gamma C}{1-\gamma} [V_0^{1-\gamma} - V^{1-\gamma}]$$

$$\text{or, } \frac{(v_0^2 - v^2)}{2} = \frac{\gamma C}{1-\gamma} [V_0^{1-\gamma} - V^{1-\gamma}]$$

The boundaries v_0 and v corresponds to the velocities at the tip and that in the pressure chamber respectively. The velocity in the pressure chamber is assumed to be negligible.

$$\text{or, } \int v dv = - \int \frac{dp}{\rho} = - \int V dp = \gamma C \int V^{-\gamma} dV$$

$$\therefore v_0^2 = \frac{2\gamma C}{1-\gamma} [V_0^{1-\gamma} - V^{1-\gamma}] = \frac{2\gamma p V_0^\gamma}{1-\gamma} [V_0^{1-\gamma} - V^{1-\gamma}]; \therefore p V^\gamma = C$$

And also we have shown the discharge rate of discharge, W , right. From the relation, rate of discharge, W is equal to $C_D A_0 v_0$, that is the area at the tip $C_D A_0$, v_0 , velocity at the tip, and ρ_0 , that is velocity & density at the tip. So, from this relation, we found out that the discharge rate W is definitely discharge rate will be kg per second in terms of mass velocity.

If W be the rate of mass discharge from the nozzle, then, $W =$

$C_D A_0 v_0 \rho_0$; where, C_D is the discharge coefficient, dimensionless, and A_0 is the area of the nozzle tip in m^2 .

$$\therefore W = C_D A_0 \sqrt{\frac{2\gamma p V \rho_0}{(\gamma-1)}} \left[1 - \left(\frac{p_0}{p} \right)^{\frac{\gamma-1}{\gamma}} \right] = C_D A_0 \sqrt{\frac{2\gamma p V}{(\gamma-1) V_0^2}} \left[1 - \left(\frac{p_0}{p} \right)^{\frac{\gamma-1}{\gamma}} \right]$$

$$= C_D A_0 \sqrt{\frac{2\gamma p}{(\gamma-1) V} \left(\frac{V}{V_0} \right)^2} \left[1 - \left(\frac{p_0}{p} \right)^{\frac{\gamma-1}{\gamma}} \right] = C_D A_0 \sqrt{\frac{2\gamma p \rho \left(\frac{p_0}{p} \right)^{\frac{2}{\gamma}}}{(\gamma-1)}} \left[1 - \left(\frac{p_0}{p} \right)^{\frac{\gamma-1}{\gamma}} \right]$$

$$= C_D A_0 \sqrt{\frac{2\gamma p \rho}{(\gamma-1)}} \left[\left(\frac{p_0}{p} \right)^{\frac{2}{\gamma}} - \left(\frac{p_0}{p} \right)^{\frac{\gamma+1}{\gamma}} \right]$$

How that also is shown, but still how W is equal to $C_D A_0 v_0 \rho_0$, right? C_D is dimensionless. But A_0 is meter squared, v_0 is meter per second, and ρ_0 is kg per meter cubed, right? So, this meter squared, this meter, and this meter cubed cancel out, resulting in kg per second, which is W . Right. So, we can say that the discharge rate, right, discharge rate is W equal to $C_D A_0 \sqrt{2 \gamma P \rho_0^{\frac{\gamma}{\gamma-1}}}$.

If W be the rate of mass discharge from the nozzle, then, $W = C_D A_0 v_0 \rho_0$; where, C_D is the discharge coefficient, dimensionless, and A_0 is the area of the nozzle tip in m^2 .

$$\begin{aligned} \therefore W &= C_D A_0 \sqrt{\frac{2 \gamma p V \rho_0}{(\gamma-1)} \left[1 - \left(\frac{p_0}{p} \right)^{\frac{\gamma-1}{\gamma}} \right]} = C_D A_0 \sqrt{\frac{2 \gamma p V}{(\gamma-1) V_0^2} \left[1 - \left(\frac{p_0}{p} \right)^{\frac{\gamma-1}{\gamma}} \right]} \\ &= C_D A_0 \sqrt{\frac{2 \gamma p}{(\gamma-1) V} \left(\frac{V}{V_0} \right)^2 \left[1 - \left(\frac{p_0}{p} \right)^{\frac{\gamma-1}{\gamma}} \right]} = C_D A_0 \sqrt{\frac{2 \gamma p \rho \left(\frac{p_0}{p} \right)^{\frac{2}{\gamma}}}{(\gamma-1)} \left[1 - \left(\frac{p_0}{p} \right)^{\frac{\gamma-1}{\gamma}} \right]} \\ &= C_D A_0 \sqrt{\frac{2 \gamma p \rho}{(\gamma-1)} \left[\left(\frac{p_0}{p} \right)^{\frac{2}{\gamma}} - \left(\frac{p_0}{p} \right)^{\frac{\gamma+1}{\gamma}} \right]} \end{aligned}$$

Into P_0 by P to the power 2 by γ minus P_0 by P to the power γ plus 1 by γ , right. This we have developed, right. Then we also have developed this: if we differentiate W , that is discharge rate, with respect to P_0 , right. Then we get dW / dP_0 , and it is a complex one which has come up like $C_D A_0 \sqrt{2 \gamma P \rho_0^{\frac{\gamma}{\gamma-1}}}$ under root dP_0 under root

If W be the rate of mass discharge from the nozzle, then, $W = C_D A_0 v_0 \rho_0$; where, C_D is the discharge coefficient, dimensionless, and A_0 is the area of the nozzle tip in m^2 .

$$\begin{aligned} \therefore W &= C_D A_0 \sqrt{\frac{2 \gamma p V \rho_0}{(\gamma-1)} \left[1 - \left(\frac{p_0}{p} \right)^{\frac{\gamma-1}{\gamma}} \right]} = C_D A_0 \sqrt{\frac{2 \gamma p V}{(\gamma-1) V_0^2} \left[1 - \left(\frac{p_0}{p} \right)^{\frac{\gamma-1}{\gamma}} \right]} \\ &= C_D A_0 \sqrt{\frac{2 \gamma p}{(\gamma-1) V} \left(\frac{V}{V_0} \right)^2 \left[1 - \left(\frac{p_0}{p} \right)^{\frac{\gamma-1}{\gamma}} \right]} = C_D A_0 \sqrt{\frac{2 \gamma p \rho \left(\frac{p_0}{p} \right)^{\frac{2}{\gamma}}}{(\gamma-1)} \left[1 - \left(\frac{p_0}{p} \right)^{\frac{\gamma-1}{\gamma}} \right]} \\ &= C_D A_0 \sqrt{\frac{2 \gamma p \rho}{(\gamma-1)} \left[\left(\frac{p_0}{p} \right)^{\frac{2}{\gamma}} - \left(\frac{p_0}{p} \right)^{\frac{\gamma+1}{\gamma}} \right]} \end{aligned}$$

Differentiating W with respect to p_0
we get,

$$\frac{dW}{dp_0} = C_d A_0 \sqrt{\frac{2\gamma p \rho}{(\gamma-1)}} \frac{d}{dp_0} \left[\left(\frac{p_0}{P} \right)^{\frac{2}{\gamma}} - \left(\frac{p_0}{P} \right)^{\frac{\gamma+1}{\gamma}} \right]$$

Now, let $x = \frac{p_0}{P}$, and $\frac{dx}{dp_0} = \frac{1}{P}$ and $\frac{d}{dp_0} = \frac{d}{dx} \frac{dx}{dp_0}$

$$\therefore \frac{dW}{dp_0} = C_d A_0 \sqrt{\frac{2\gamma p \rho}{(\gamma-1)}} \frac{d}{dx} \sqrt{x^{\frac{2}{\gamma}} - x^{\frac{\gamma+1}{\gamma}}} \frac{dx}{dp_0}$$

$$= C_d A_0 \sqrt{\frac{2\gamma p \rho}{(\gamma-1)}} \frac{1}{P} \frac{1}{2} \sqrt{x^{\frac{2}{\gamma}} - x^{\frac{\gamma+1}{\gamma}}} \left(\frac{2}{\gamma} x^{\frac{2}{\gamma}-1} - \frac{\gamma+1}{\gamma} x^{\frac{\gamma+1}{\gamma}-1} \right)$$

P_0 by P to the power 2 by γ minus P_0 by P to the power γ plus 1 by γ , right. We substituted for its simplicity to find out; we substituted x instead of P_0 by P , and by that we can write dx / dp_0 , which is nothing but 1 by P , and $d dp_0$ is equal to $d dx$ of dp_0 dx / dp_0 , right. So, from dW / dp_0 , we have found out that by substituting the values of x , then we could get this relation that P_0 by P is equal to 2 by γ plus 1 to the power γ by γ minus 1 , right, by substituting the value of x .

In terms of P_0 and P , right. Now, it is known that for diatomic gases, the value of γ is 1.4 . So, if you substitute γ as 1.4 , then we can say that P_0 by P is equal to nothing but 0.528 , which is known as the critical pressure ratio, right. The critical pressure ratio for maximum discharge because we started with

Now, for maximum discharge $\frac{dW}{dp_0} = 0$

$$\therefore \frac{2}{\gamma} x^{\frac{2}{\gamma}-1} - \frac{\gamma+1}{\gamma} x^{\frac{\gamma+1}{\gamma}-1} = 0; \text{ or, } 2x^{\frac{2}{\gamma}-1} - (\gamma+1)x^{\frac{\gamma+1}{\gamma}-1}$$

$$\text{or, } \frac{2}{\gamma+1} = \frac{x^{\frac{\gamma+1}{\gamma}-1}}{x^{\frac{2}{\gamma}-1}}; \text{ or, } \frac{2}{\gamma+1} = x^{\frac{\gamma-1}{\gamma}}; \text{ or, } x = \left(\frac{2}{\gamma+1} \right)^{\frac{\gamma}{\gamma-1}}$$

Hence, $\frac{P_0}{P} = \left(\frac{2}{\gamma+1} \right)^{\frac{\gamma}{\gamma-1}}$ when the discharge is maximum

For diatomic gases, such as air, γ is equal to be 1.4 .

$$\therefore \frac{P_0}{P} = 0.528; \text{ or, } \frac{P}{P_0} = 1.893$$

maximum discharge, when it occurs, that time $d p d w$, rather, $d W / d P_0$ is equal to 0 , right. So, that is happening when the pressure ratio is 0.528 , right, that is tip pressure to the internal pressure, tip pressure to the internal pressure is equal to 0.528 , or if it is inverse, then P by P_0 , that is interior pressure to the tip pressure, that should be equal to 1.893 . So,

this, though a recapitulation, a little, but understanding of what a siren is, how it looks like. Then why we are considering only, as of now, only the adiabatic process, etc., we have established and justified.

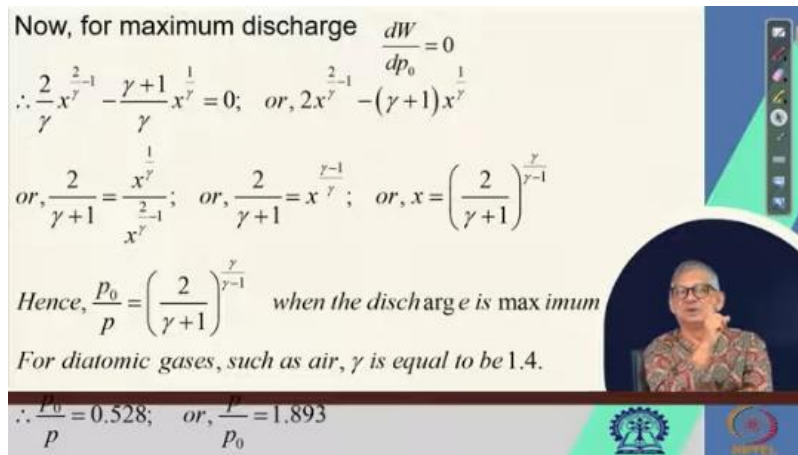
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$$\text{or, } \frac{2}{\gamma+1} = \frac{x^{\frac{1}{\gamma}}}{x^{\frac{2}{\gamma}-1}}; \text{ or, } \frac{2}{\gamma+1} = x^{\frac{\gamma-1}{\gamma}}; \text{ or, } x = \left(\frac{2}{\gamma+1} \right)^{\frac{\gamma}{\gamma-1}}$$

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For diatomic gases, such as air, γ is equal to be 1.4.

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Your today's time is over. So, we will see in the next class subsequent derivations, right. Thank you so much for listening to this topic.

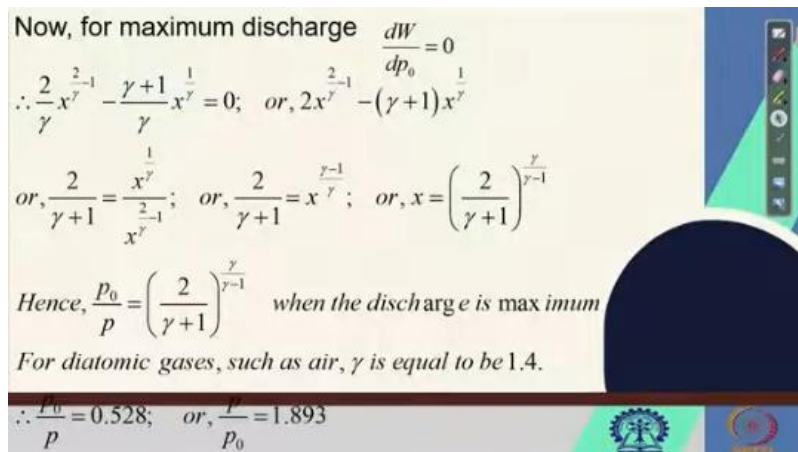
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$$\therefore \frac{2}{\gamma} x^{\frac{2}{\gamma}-1} - \frac{\gamma+1}{\gamma} x^{\frac{1}{\gamma}} = 0; \text{ or, } 2x^{\frac{2}{\gamma}-1} - (\gamma+1)x^{\frac{1}{\gamma}} = 0$$

$$\text{or, } \frac{2}{\gamma+1} = \frac{x^{\frac{1}{\gamma}}}{x^{\frac{2}{\gamma}-1}}; \text{ or, } \frac{2}{\gamma+1} = x^{\frac{\gamma-1}{\gamma}}; \text{ or, } x = \left(\frac{2}{\gamma+1} \right)^{\frac{\gamma}{\gamma-1}}$$

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$$\therefore \frac{p_0}{p} = 0.528; \text{ or, } \frac{p}{p_0} = 1.893$$


Thank you.