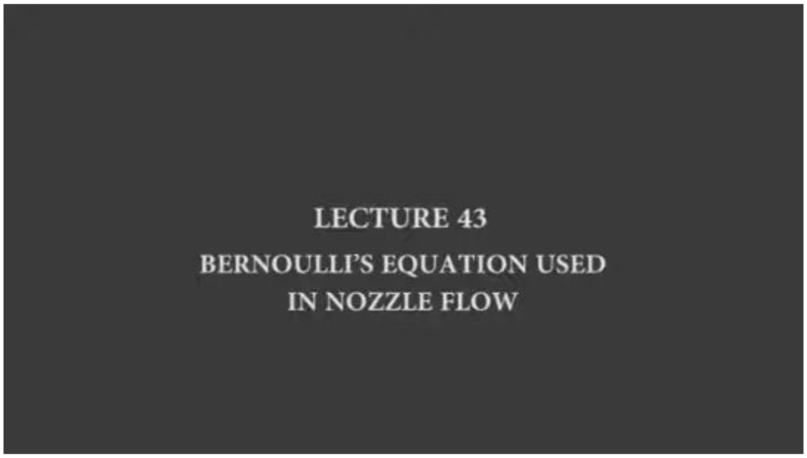
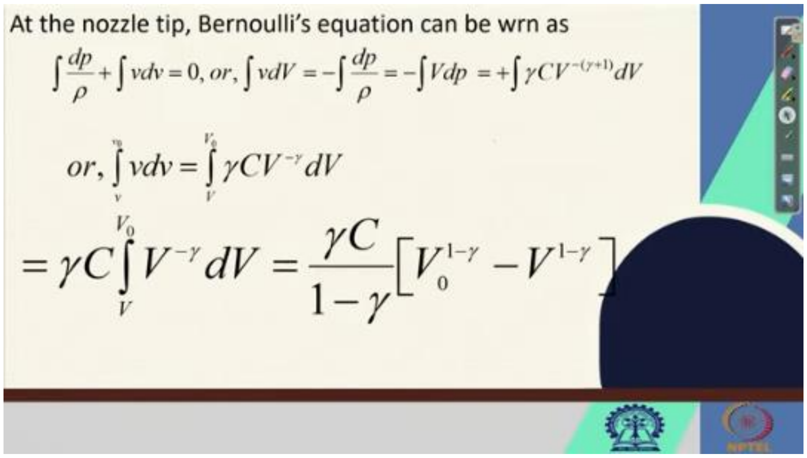
IMPACT OF FLOW OF FLUIDS IN FOOD PROCESSING AND PRESERVATION

Lecture43

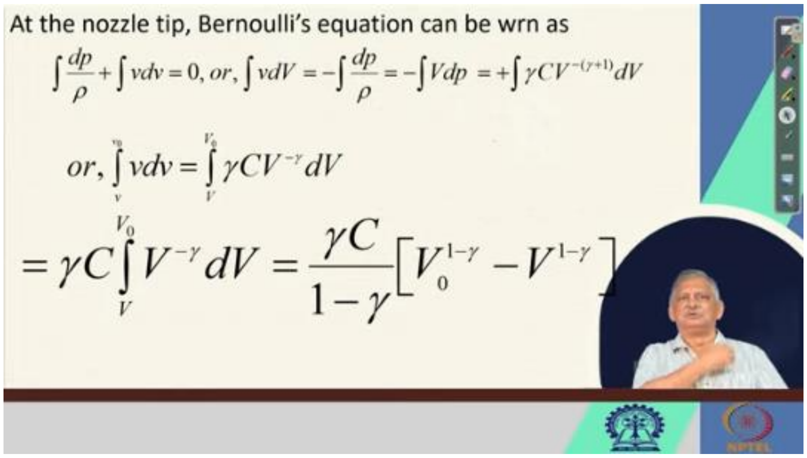
LECTURE 43 : BERNOULLI'S EQUATION USED IN NOZZLE FLOW

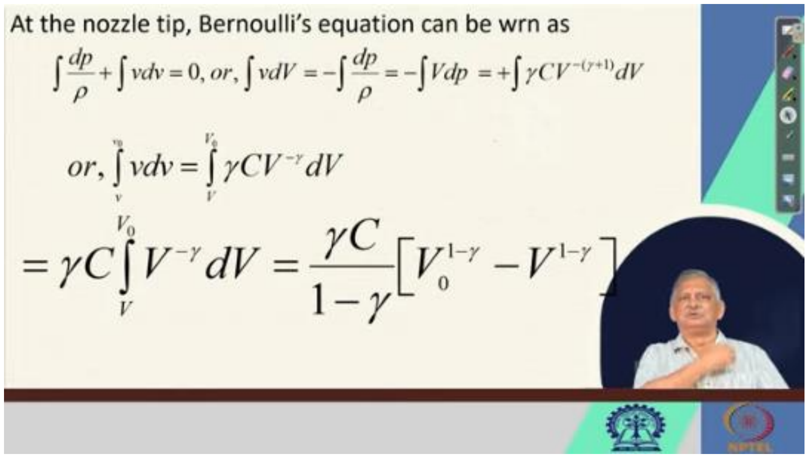
Good morning, my dear boys and girls, students, and friends. We are in the flow of fluid through nozzles, right? And we said it to be adiabatic because from a high pressure to a low pressure, when it it makes some change in temperature, right? So, up to this point, we have come to v dv between

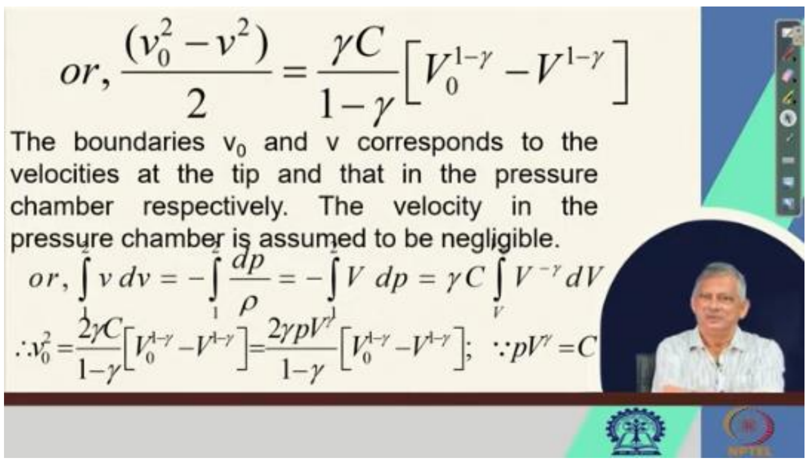




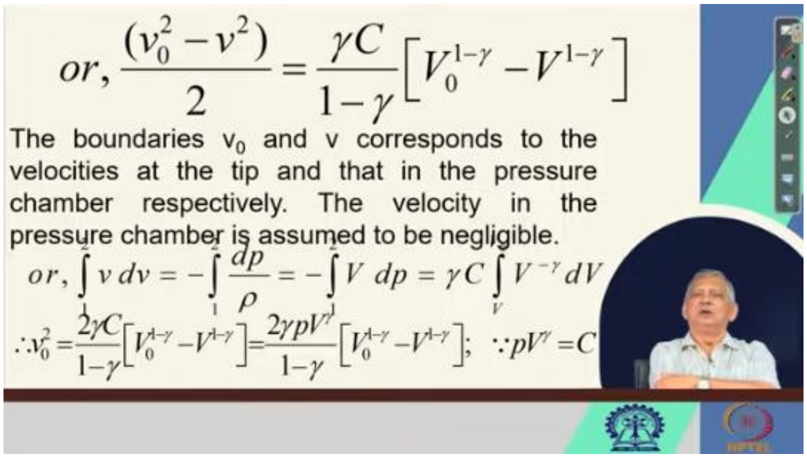
v and v_o or v outlet or v_o, whatever we call it, or v at the tip, that is equal to capital V to V_o gamma C V minus gamma d V. So, by substituting that value of dV from the previous dp, we have shown it is gamma C by 1 minus gamma into Vo to the power 1 minus gamma by 1 minus gamma V to the power 1 minus gamma, ok? Then we can write that the integration of v to v_o dv is, v from v to v_o, that is nothing but v square by 2 and the two boundaries, where v and vo. So, from there, if we take common, then it becomes vo square minus v square by 2 is equal to





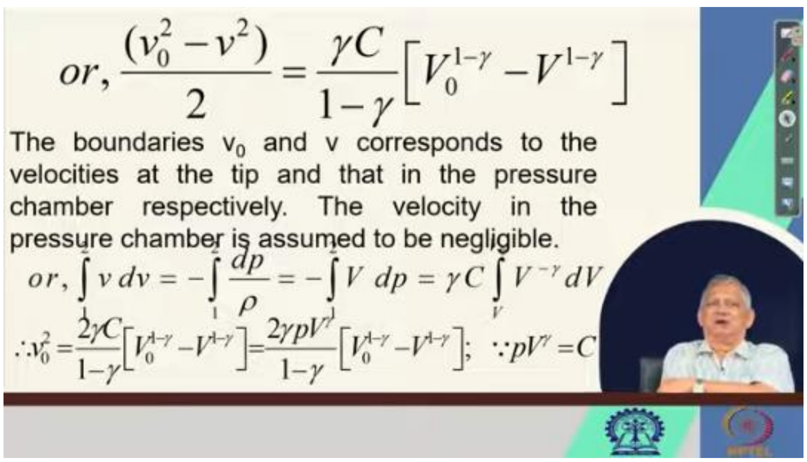


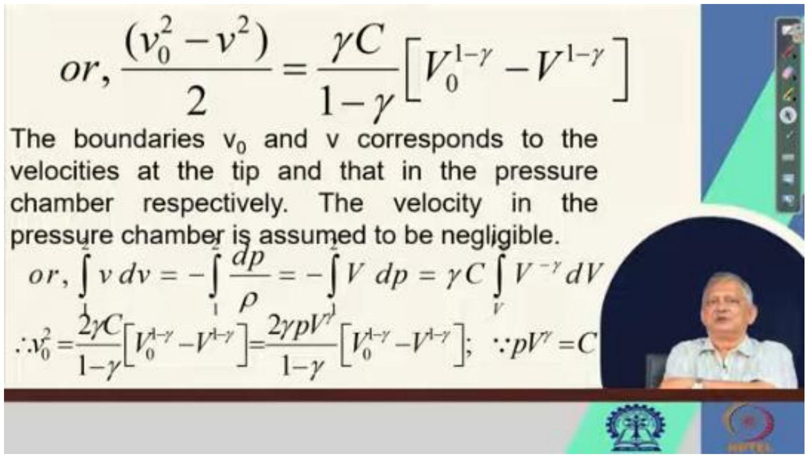
gamma C by 1 minus gamma V_o to the power 1 minus gamma minus V to the power 1 minus gamma, right? So, the boundaries vo and v correspond to the velocities at the tip, that is vo and that in the pressure chamber, that is the reservoir, v, respectively, right? The velocity in the pressure chamber is assumed to be obviously negligible, why? As I said, when you are making your mouth like this, the inside air velocity is very low because the area is very high, right. When you are making it very small like this, then you are blowing. The velocity at your tip or at your mouth end is very high compared to that inside your mouth, right. The same is true in the nozzle, from the reservoir, whatever is coming, then what we are saying. The velocity inside the reservoir is very low, right, because the area is much larger.



So, there the velocity is very low. When the area becomes narrow or very small, then the velocity becomes high, right. That is why it is also happening here, as I told you about your mouth, when you are whistling, it is also happening like that, etc. Many similar situations may arise. Now, the velocity in the pressure chamber is assumed to be negligible, because, as I said, it is very low.

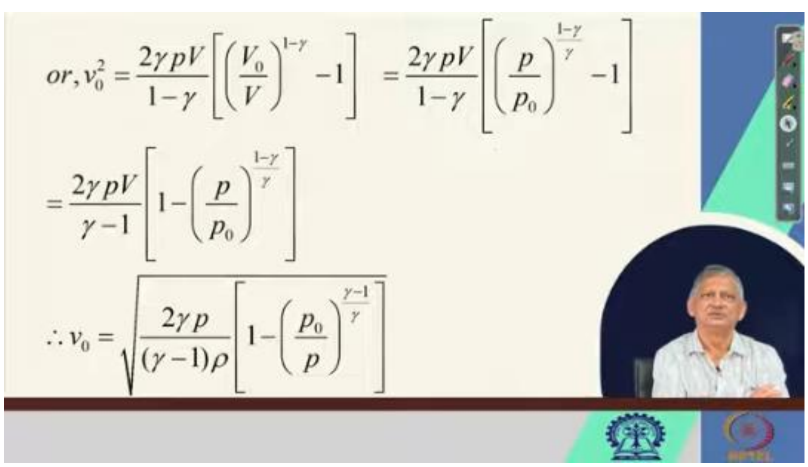
So, it can be neglected, that is, v squared by 2 can be taken as 0, then we get vo squared by 2 only, right. Then, rewriting it. That v dv, the integration of v dv, is equal to minus, or between point 1 to 2, okay, dp over rho, that is true, minus. So, that is again between point 1 to 2, the integration of minus capital V dp. Yeah, that is equal to gamma C between V to V_o, V to the power minus gamma dV, which we have already done.



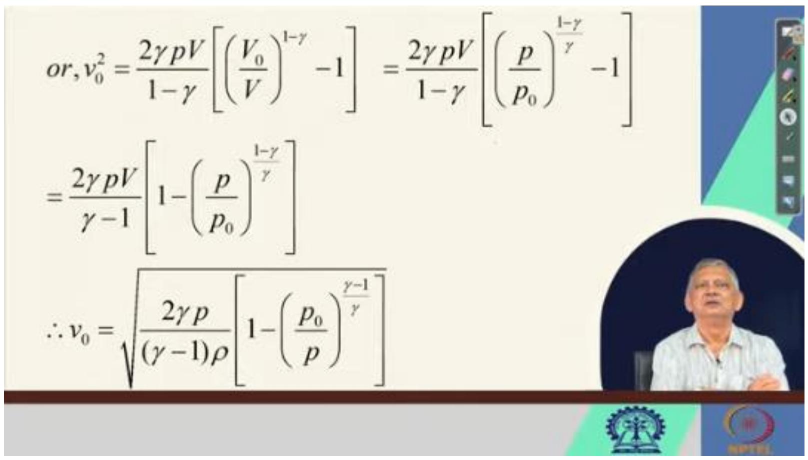


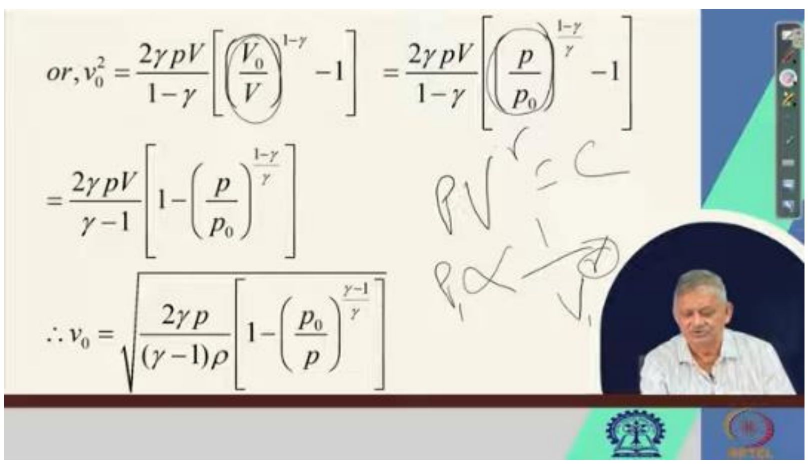
So, we can write that vo square from the top equation only it could have been that vo square is equal to 2 gamma C over 1 minus gamma. Into V_o to the power 1 minus gamma minus V to the power 1 minus gamma, right. So, this we can rewrite as that C, we can replace with P V right because C is P V gamma. So, we can replace with that as 2 gamma P V gamma by 1 minus gamma Vo to the power 1 minus gamma minus 1, V to the power 1 minus gamma, yeah. This is because we know P V gamma is constant, C. So, we have just replaced C with P V gamma, right.

It is very interesting and not boring at all. Therefore, vo square is 2 gamma P V, only you remember small v is the velocity and capital V is the specific volume, right. So, vo square we can write to be equal to 2 gamma P V by 1 minus gamma into V_o by V to the power 1 minus gamma by minus 1, right. So, this is equal to 2 gamma P capital V by 1 minus gamma



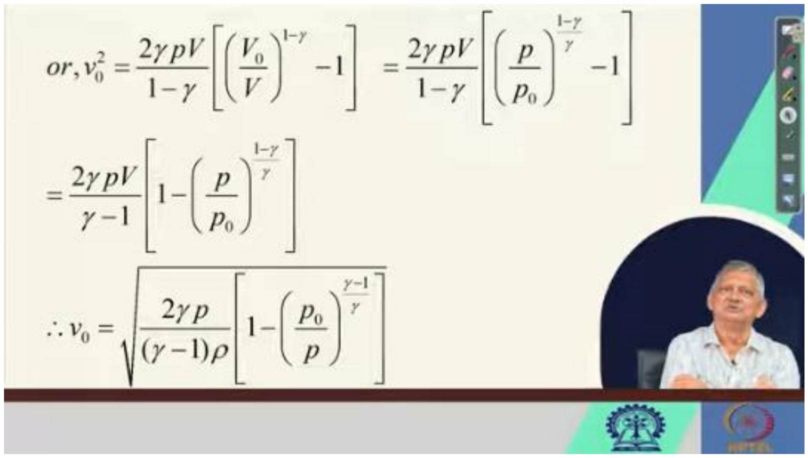
into P by P_o because V_o by V is nothing, but P by Po, because P is inversely proportional to V, that we have shown many times, right. So, we can write, ok, for those who do not come to the class regularly, for them, yeah. P V gamma is equal to constant, right, that means P is proportional to 1 by V, right, or V to the power gamma, whatever we write, that means if we write it P_1, then it is P_1 that is in inverse right, that is what explicitly here we have done V_o by V to the power 1 minus gamma as P by P_o to the power 1 minus gamma by gamma because this gamma is in the denominator, right. So, this we have written and we can say that

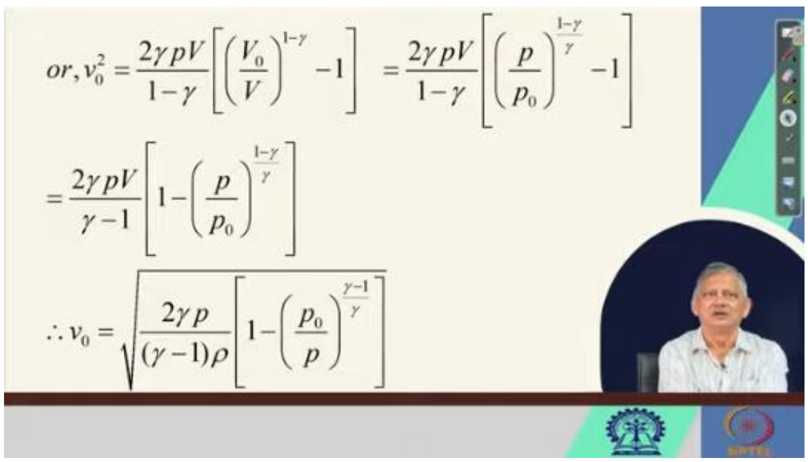




PV^{γ} is equal to a constant, implying that we wrote $2\gamma PV / (1 - \gamma)$. I say that, okay, it is capital V_2, comma, P capital V / (1 - γ), because our left side is small v, right? So, small v_o squared. is 2γPV, P capital V / (1 - γ) * (capital V_o / capital V)^(1 - γ) - 1 = 2γP capital V / (1 - γ) * (P / P_o)^((1 - γ)/γ) - 1. So, this is equal to 2γPV / (γ - 1) * (1 - (P / P_o)^((1 - γ)/γ)).

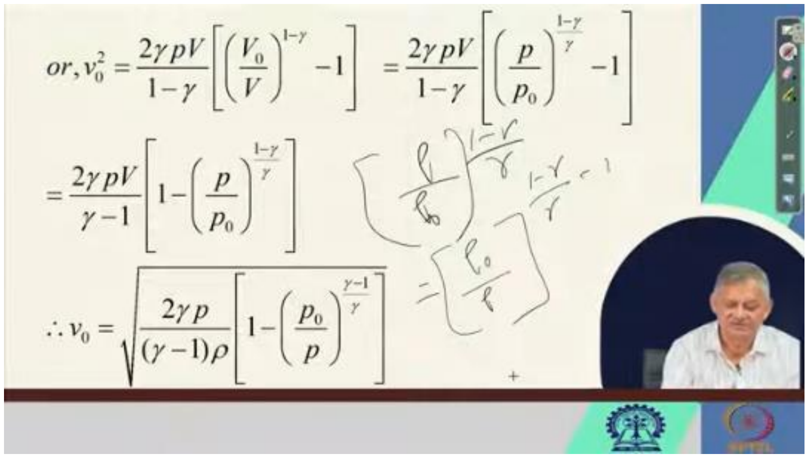
This is simple, just a rearrangement, nothing more, you see, because 2γPV was there. Now, 1 - γ has been changed to γ - 1. So, one negative is there. So, that negative is taken inside as 1 - (P / P_o)^((1 - γ)/γ), right? So, we can write v_0 as nothing but √(2γp / (γ - 1))



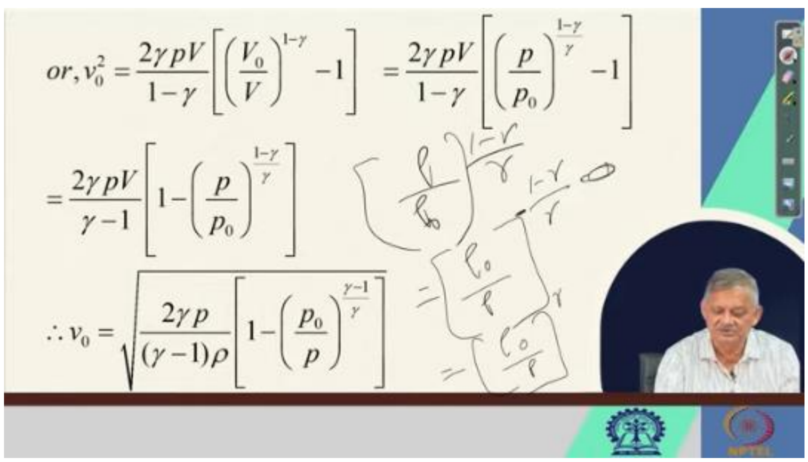


into ρ * (1 - (P_o / P)^((γ - 1)/γ)). So, earlier it was, as you see, earlier it was (P / P_o)^((1 - γ)/γ), right? Now we have inverted it. So, this is equal to P_o / P, we have 1 /, we have made it. So, that becomes 1 - 1 / (1 - γ) / (γ - 1), right.

So, that means this is gamma, right? P_o by P s gamma minus 1 by gamma, that is with a minus. Gamma 1 minus gamma by gamma 1 negative, that is gamma minus 1 here. So, not this minus 1. So, it becomes this as minus, okay.

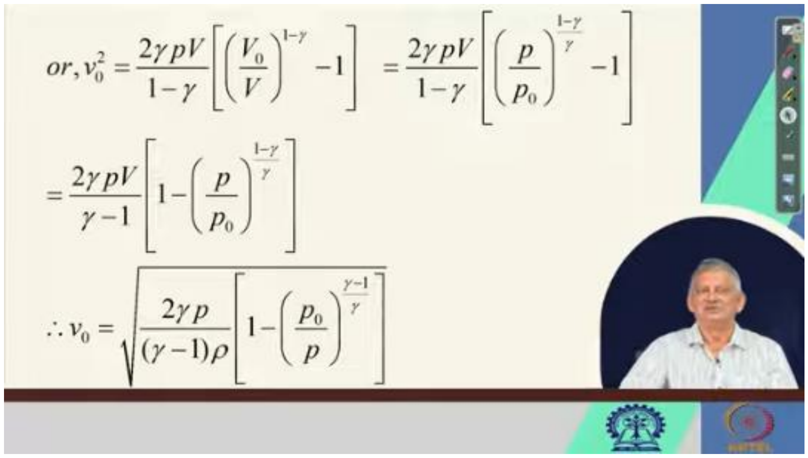


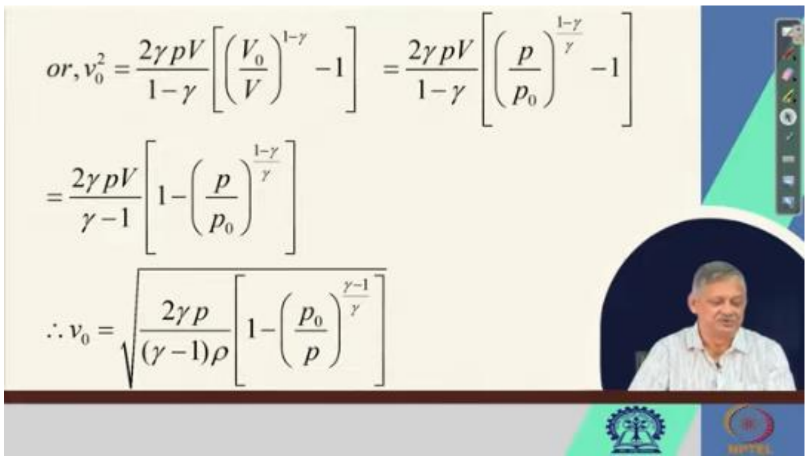
And this is P_o by P to the power, since this minus is taken. So, this becomes minus 1, this becomes plus gamma. So, gamma minus 1 by gamma, right? That is what we have done here. So, vo, that is the tip velocity, we have found out to be

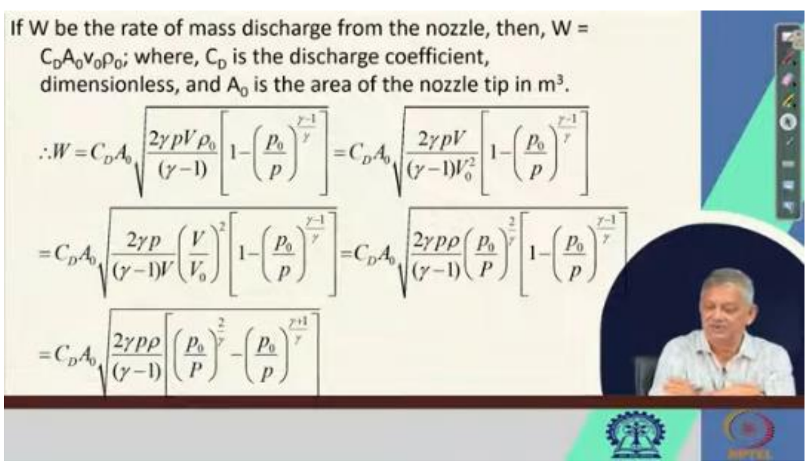


2 gamma P over gamma minus 1 into that capital V, we have replaced it with rho, because capital V is nothing but 1 by rho, right? So, instead of capital V, we have written here 2 gamma by gamma minus 1 into rho, right? Into obviously, whatever was there 1 minus P_o by P to the power, since we have inverted that P to P_o. So, one negative who is there on the top in the power. So, that power makes it gamma minus 1 by gamma, right.

So, this is the velocity of fluid in the tip, where gamma is the ratio of CP / CV, right? P is the source or reservoir pressure, and rho is the density of the fluid. And Po is the pressure at the tip. So, if vo is the velocity at the tip, then P_o is the pressure at the tip, and P is the pressure at the inlet or the reservoir, right? So, then we can say that if W is the rate of mass discharge from the nozzle. W is the mass discharge from the nozzle, then W equals C_D Ao vo rhoo, right?

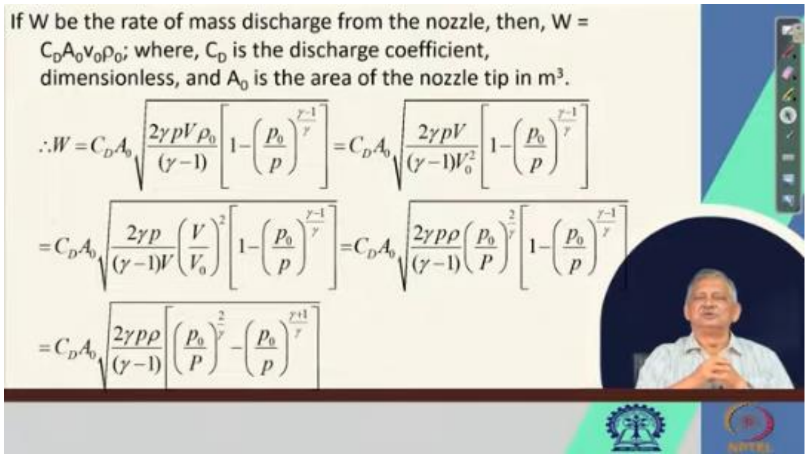




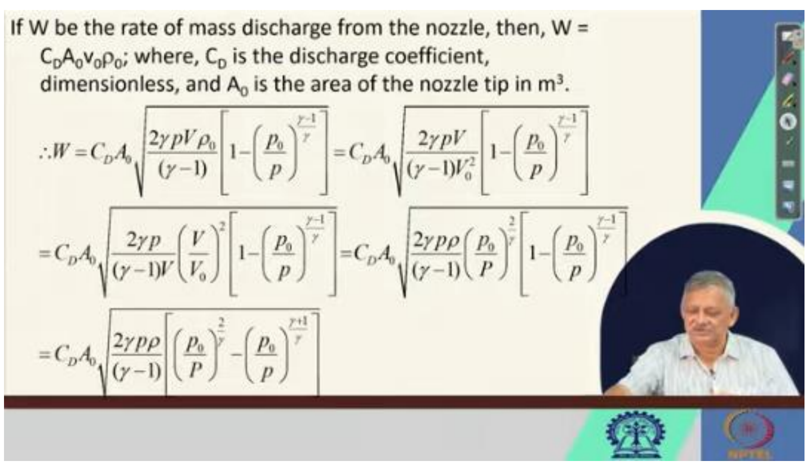


C_D is the discharge coefficient, the discharge coefficient, as, say, I am a nozzle manufacturer. I know how efficient the nozzle is. There will be some error because of the manufacturing, some of this and that. So, I will tell for my nozzle what the value of gamma is. That is, that is given, sorry, what the value of C_D is, the coefficient of discharge, how much it can discharge.

Similarly, another company that is manufacturing that. So, they will also declare that. The coefficient of discharge is this much; somebody says 0.9, somebody says 0.8, somebody says in between 0.85 or 0.89, whatever different values. So, that is given C_D. Obviously, Ao is the area of the tip, and v_o is the velocity at the tip. Rhoo is the density at the tip, right?



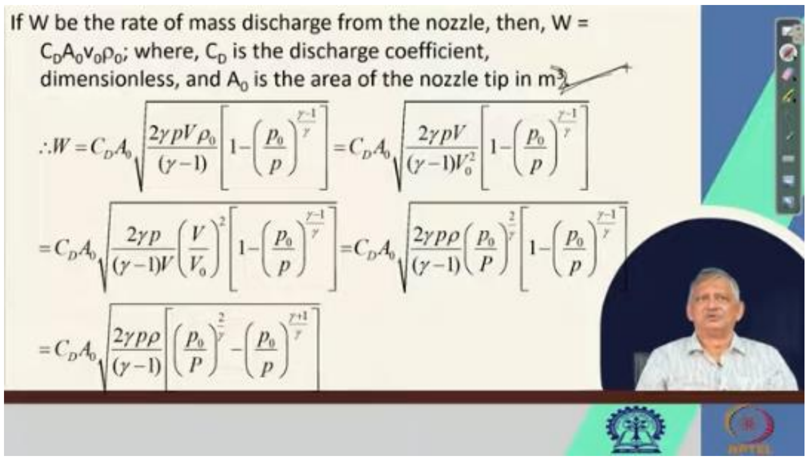
That is how much discharge or mass flow rate is happening, W, right. C_D is dimensionless, A_o is in meter square, v_o is meter per second, and rho_o is kg per meter cube. So, let us look into that: C_D is dimensionless, A_o is in meter square, v_o is in meter per second, and rhoo is in kg per meter cube. So, that means, this meter cube, this meter cube, goes out, that becomes mass flow rate, kg per second, that is, the discharge rate, right: C_D A_o v_o rho_o.



Now, if that be true, then we can say that We can say that C_D is the discharge coefficient, as I just discussed, right? Because different manufacturers will manufacture it in different ways, and their discharge coefficient is different. That is why, in the nozzle, that C_D, discharge coefficient, is written and declared by the manufacturer. Right? Yeah, you can also determine. It is not that difficult.

You can also determine. We will see in the future what we can do. So, if C_D is the discharge coefficient, which is, of course, dimensionless, A_o is the area of the nozzle tip in meter square, sorry, in meter square. A_o is the area of the nozzle, obviously, this is in meter square, meter square, mind it, right. So, this is not cube; that was a cut-and-paste mistake.

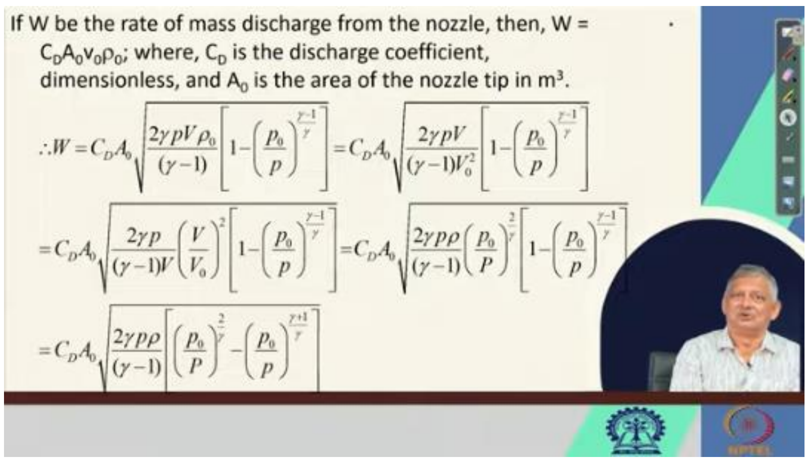
So, W is C_D A_o vo. So, vo we are replacing with under root 2 gamma P etc. Now, that we had 1 by rho. So, that rho is converted into capital V, right by gamma minus 1, right. Now, we have 1 rhoo outside.



So, if it is to be taken inside then what does it do? If it is to be taken inside, then it is, it should be rhoo square, right? It should be rho$_o$ square that one square is missing because this rho when it is going within the, within the square root, right, then it should be rho$_o$ square. I told you in the beginning that in many, many areas, many, many places there could be some or other mistake which I have not corrected, knowingly, that if you can identify, that will be better. Otherwise, obviously, I am identifying as here we are seeing that it is rho$_o$ square, right. Because again, why not to see again because, here you see v_o is 2 gamma P by gamma minus 1 into rho. So, this rho when it goes to the top, so it becomes capital V into 1 minus P_o by P to the power gamma minus 1 by gamma, that is what here you have taken, 1 minus P_o by P to the power gamma minus 1 by gamma, is there and is also V is that rho is taken into V, capital V on the top, and we have introduced the outside rho$_0$ inside that is it is supposed to be rho$_0$ square, right. Therefore, we can write that this is equal to C_D A_o. Under root 2 gamma P V by gamma minus 1 into since it is it was rhoo square. So, now, it has become V_o square. I told you also earlier that there that mistake could be there, but subsequently, that mistake I have taken care of.

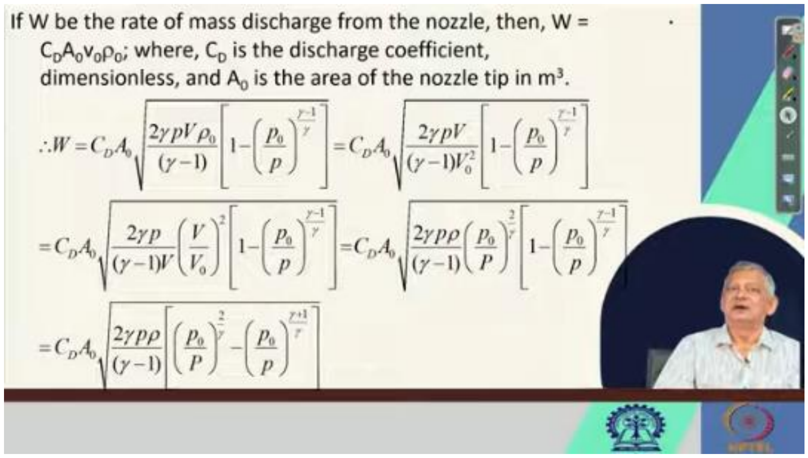
There is no mistake remaining, right. So, I hope that you could have found out that rho $_0$ when it went inside there should have been 1 square. Now, where did that square go? So, I said that this was a mistake and that we have taken into the next step as when we changed it from rhoo to V_o square capital V_o square, ok. Then C_D Ao remains like that under root 2 gamma P capital V into.

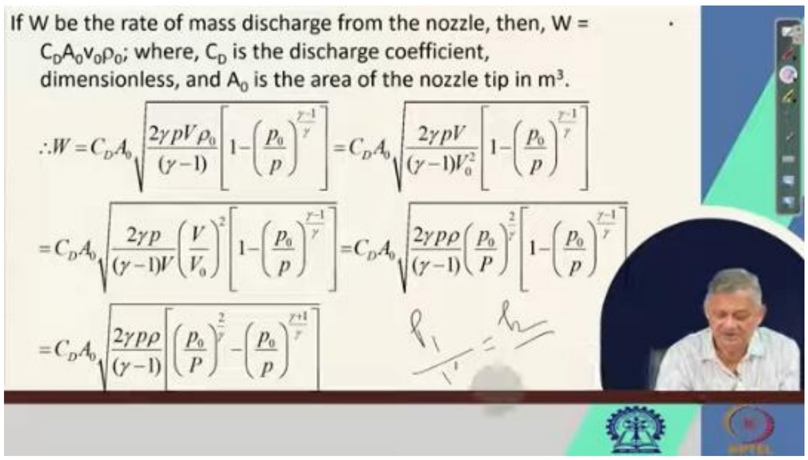
Or 2 gamma P capital V by gamma minus 1 into V_o square all into 1 minus Po by P to the power gamma minus 1 by gamma, right. Therefore, we can say that this is equal to C_D Ao under root. 2 gamma P, it was already there, by gamma minus 1 that is also there, right. Now, we introduced one V square inside. So, V by V_o square there one additional V we have used.

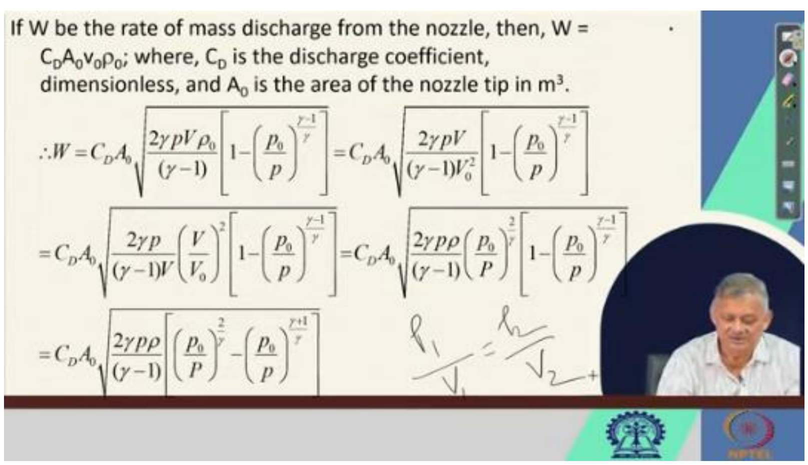


So, we get 1. 1 by V, capital V, right, and the rest remains the same as 1 minus Po by P to the power gamma minus 1 by gamma, right. Now, we are making C_D A_o remain the same under the root 2 gamma P rho. That V we have again changed it to rho 2 gamma P rho by gamma minus 1 and V by V_o to the power 2 or V by V_o square that we now convert it into P_o by P. V by V_o is proportional to P_o by P to the power 1 by gamma, right.

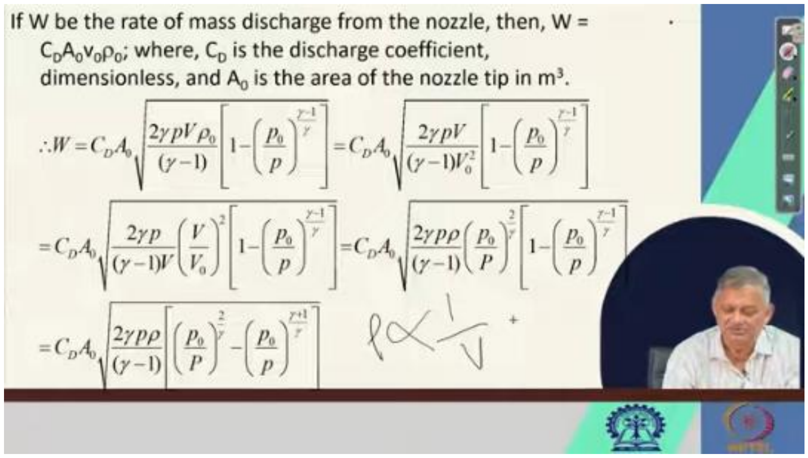
So, we write P_o by P instead of V by V_o square as P_o by P to the power 2 by gamma, right. If this is true, then Mind it, here that every time I am saying that if you take P_1 by V_1 is equal to P_2 by V_2, it is not so, right. It is not so because it is, P is inversely proportional to 1 by V, P is inversely proportional to V, right.

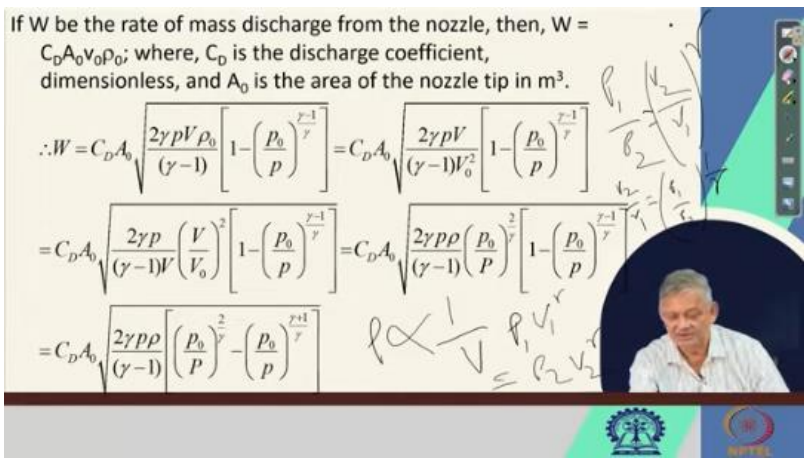




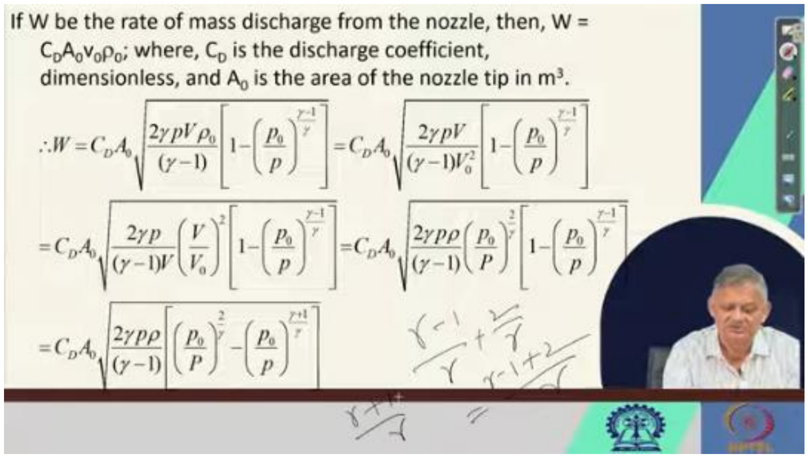


So, if we write $P_1 V_1$ is equal to $P_2 V_2$. Obviously gamma, obviously gamma. So, P_1 by P_2 is equal to V_2 by V_1 to the power gamma, right, or V_2 by V_1 is equal to P_1 by P_2 to the power 1 by gamma, right. This is exactly what we have done here. This is exactly what we have done here that we have changed that V by V_o square as P_o by P to the power 2 by gamma.

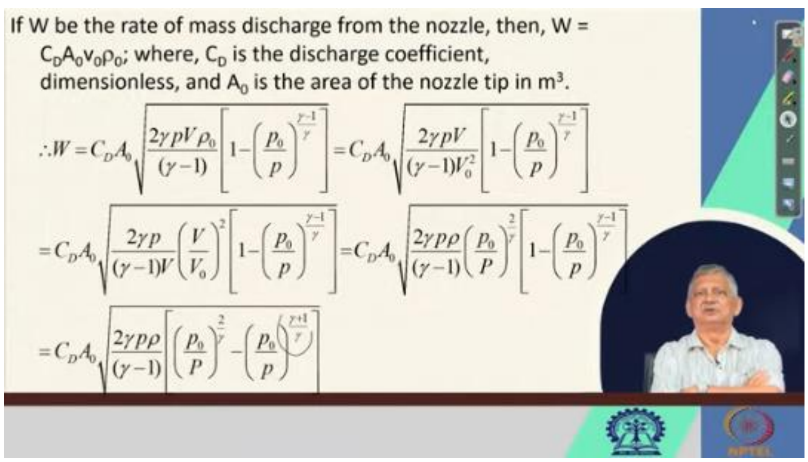




Right, and also that rho as rather 1 by V as rho, right, and the rest remains the same as 1 minus P_o by P to the power gamma minus 1 by gamma, right. Now, we enter or we take this P_o by P to the power 2 by gamma inside, then, we get 2 gamma P rho by gamma minus 1 as it is P_o by P to the power 2 by gamma into 1 is that and P_o by P to the power gamma minus 1 by gamma plus 2 by gamma that means, we have gamma minus 1, sorry, we have that means, gamma minus 1 by gamma plus 2 by gamma, that means, gamma is common, that means, gamma minus 1 plus 2 that means gamma plus 1 by gamma. So, it has become gamma plus 1 by gamma, right.



So, we could have made it and that is the W, discharge rate, right, that is nothing but the discharge. This class time is over. So, we thank you for listening to this. We will come out from discharge rate to some other things in the subsequent classes. Thank you.



Thank you.