## IMPACT OF FLOW OF FLUIDS IN FOOD PROCESSING AND PRESERVATION

## Lecture39

## LECTURE 39 : PRESSURE DROP IN PIPES FOR COMPRESSIBLE LIQUIDS

Good afternoon, my dear boys and girls, my students, and friends. In the packed bed or flow through porous media, we have already covered the basic equations on which packed bed or porous media calculations are made. So, we have done both Reynolds number less than 10, Reynolds number more than 10, and between 10 and 1000, right. Reynolds number less than 10, Reynolds number greater than 1000, and Reynolds number between 10 and 1000, all these we have already done and



we arrived at the Ergun's equation, okay. Now, if we go to that Ergun's equation application. So, here it is 1. Air at 303 Kelvin is flowing through a packed bed of spheres having a diameter of 12 millimeters. The void fraction of the bed is 0.4, and the bed has a diameter of 0.6 meters and a height of 2.5 meters.



The air enters the bed at 1.2 atmospheres at the rate of 0.4 kilograms per second. Calculate the pressure drop of air in the packed bed. Given, Mw, molecular weight of the gas is 28.97 kilograms per kilogram mole, and viscosity 2 times 10 to the power of minus 5 Pascal seconds, right. So, what have we been asked to do? We have been asked to calculate the pressure drop of air in the packed bed.

What is the pressure drop, right? If you remember, This was our Ergun's equation, and we said that if we take, we said if we take, sorry, if we take leaving this side, this part aside. Then the rest of the thing that is taken care of for energy less than 10, right. And if we take the other one, if we take the other one, that is,



This part, if we delete and take this, then it is valid for  $N_{Re}$  greater than 1000, right. So, in between, that is between 10 and 1000, The entire relation is valid, right? The entire relation is valid. So, that is what we will do here, okay. In this problem, we are asked to find out what is the pressure drop if air at 303 Kelvin



Is flowing through a packed bed of spheres having a diameter of 12 millimeters, The void fraction of the bed is 0.4, and the bed has a diameter of 0.6 meters and a height of 2.5 meters. Air enters the bed at 1.2 atmospheres at the rate of 0.4 kg per second. Calculate the pressure drop of air in the packed bed, given the molecular weight of air is 28.97 kg per kg mole. And viscosity mu is 2 times 10 to the power of minus 5 Pascal seconds, right. So, the first thing, what do we do?



Let us find out the cross-sectional area of the bed. So, to do that, what do we do? From our given data, the diameter is 12 millimeters. So, we can say pi by 4 R square, right? That is the area. So, pi by 4 into 0.6 whole square is 0.238 meters square.

So, we find out the cross-sectional area of the bed. Now, we find out G prime (G'), which is the mass velocity. Based on the empty cross-section, it was given that the void fraction of the bed is 0.4, that is, epsilon ( $\epsilon$ ) is 0.4. So, it is epsilon by your area, that is, 0.4 by no, The rate at which the air enters the bed is 1.2 atmospheres at the rate of 0.4 kg per second, yes.

So, G prime is 0.4. This is not the epsilon; G prime is 0.4, which is the rate in kg per second. Divided by the area in meters square. So, kg per meter square per second is 0.4 by 0.283. So, that is 1.413 kg per meter square second, based on the empty cross-sectional area of the bed. So, now, let us assume a delta P. So, let us assume a delta P of 0.051 times 10 to the power of 5 Pascal.



So, if you assume delta P of 0.05 times 10 to the power 5 Pascal, we have already been given  $P_1$ . So,  $P_1$  is 1.2 times 1.01325 times 10 to the power 5. So, that is 1.21590 times 10 to the power 5 Pascal. Therefore, P2 can be found out. That is 1.21590 times 10 to the power 5 minus 0.05 times 10 to the power 5 because  $P_1$  is higher than  $P_2$ ; otherwise, there will not be any flow from  $P_1$  to  $P_2$ .

And this is equal to 1.16590 times 10 to the power 5 Pascal. Obviously, you will try with your calculator and see whether the numbers are matching or not. Hence, Paverage can be written as (P<sub>1</sub> plus P<sub>2</sub>) divided by 2, which is 1.19090 times 10 to the power 5 Pascal. So, if the average pressure is found out, then the average density, rhoaverage, can also be found out as Paverage times M divided by RT, that is, 1.19090 times 10 to the power 5 times the molecular weight, which is given as 28.97, divided by 8314, which is R. And T is 305 Kelvin.

So, it is 1.3605 kg per meter cube, which is the average density, right? Therefore, energy can be written as  $D_P$  times G prime divided by mu times (1 minus epsilon). So,  $D_P$  we have seen is 0.012, G prime we have found out is 1.413, mu is 2 times 10 to the power minus 5, and (1 minus epsilon) is 1 minus 0.4, which comes to 1413. So, it is more than 1000 N<sub>Re</sub>, right.





So, if you use Ergun's equation, then we can write, delta P rho by G prime square into  $D_P$  by delta L is equal to epsilon cube by 1 minus epsilon. That is equal to 150 by  $N_{Re}$  plus 1.75, right. So, delta P into 1.3605 divided by G prime we have found out to be 1413 whole square delta P into rho by G prime square and  $D_P$  is 0.012 by delta L is 2.5 into epsilon cube is 0.4 cube divided by 1 minus epsilon is 1 minus 0.4.

So, this is equal to 150 by 1413 that is the  $N_{Re}$  plus 1.75. This tells that delta P is equal to 5320.23 Pascal, that is 0.0532023 10 to the power 5 Pascal. That means, by chance our assumption of delta P was very close to the real, right, but you try, I tell you also try that this value has come up right or rather this value ok, where we have taken both 150 by  $N_{Re}$  plus 1.75.

If you remember we said earlier that if  $N_{Re}$  is less than 10, then this dominates and if  $N_{Re}$  is more than 1000, then this dominates this we told. Now, we have taken for this solution both of them, but what I tell you that you please try and see taking out this what is the value of delta P coming right that is one. You also try taking out this, taking out this what is the value of delta P coming right.

P is less than or equal to delta P actual or calculated rather is less than or equal to delta P. In one case, say this, or in the other case, say this if delta P calculated is much greater than delta P assumed. In either of the cases, if you see this is happening, that means, under this situation, you have to take the whole thing. Otherwise, if that is 1000, more than 1000, that means, this one is invalid, only this one should give us some positive result, right. So, that you try and see, then you can differentiate.

 $N_{Re} = D_p G' / \mu (1 - \epsilon)$ = 0.012 X 1. 413 / (1 - 0.4) X 2 X 10-5 = 1413 **Using Ergun's Equation**  $\frac{\Delta p \rho}{\left(G'\right)^2} \frac{D_p}{\Delta L} \frac{\varepsilon^3}{\left(1 - \varepsilon\right)} = \frac{150}{N_{\text{Re}}}$ Alar  $\frac{\Delta p X 1.3605}{\left(1.413\right)^2} X \frac{0.012}{2.5} X \frac{\left(0.4\right)^3}{\left(1-0.4\right)} = \frac{150}{1413}$ +1.75or,  $\Delta p = 5320.23 Pa = 0.0532023X10^5 Pa$ This is close to the assumed value

How good the Egun's equation, this one is applicable, right. Our  $N_{Re}$  is 1413, much greater than 1000, right. So, typically it should be that this one should not have much effect compared to this one. So, that is my prediction, you just check whether that is true or false, right, ok. Then this is one problem which we have dealt with a single assumption, right.



Then let us look into the other, maybe. That will not be so easy. That air at 390 Kelvin flows through a packed bed of cylinders having a diameter of 0.0127 meter and length, the same as diameter. This means the cylinder can be treated as a sphere.



The bed void fraction is 0.45, and the length of the packed bed is 4 meters. The air enters the bed at 2.5 atmospheres. Absolute, at the rate of 3 kg per meter square second, based on the empty cross-section of the bed. Now, calculate the pressure drop of air in the bed, given the molecular weight equals 29 and viscosity, mu, is equal to 1.5 into 10 to the power minus 5 Pascal second.

So, again here, we have to find out the pressure drop, right? So, for understanding the problem, the problem says air at 390 Kelvin flows through a packed bed of cylinders having a diameter of 0.0127 meters and length the same as the diameter. This means the cylinder can be treated as a sphere. Okay, in this case, let me also tell you what it is saying.

Having a diameter of 0.0127 meters, right? So, this is the diameter, 0.0127 meters, and the length is the same as the diameter, which means if this is 0.0127, then its length is also the same as the diameter. So, it appears to be theoretically, if length and diameter are both the same, that means it is a sphere. A sphere also has a diameter D, and the other side is also D. So, two sides having d, and the third side is also D, which means it is a sphere.

So, similarly here also since the length is the same as that of the diameter, we can say that the cylinder appears to be the same as the sphere. Then let us continue. It says that air at 390 Kelvin flows through a packed bed of cylinders having a diameter of 0.0127 meter and length the same as the diameter. That means the cylinder can be treated as a sphere. The bed void fraction is 0.45 and the length of the packed bed is 4 meters.



Air enters the bed at 2.5 atmospheres absolute at the rate of 3 kg per meter square second based on the empty cross section. of the bed, calculate the empty cross section of the bed, calculate the pressure drop of air in the bed given molecular weight is equal to 29 and viscosity is 1.5 into 10 to the power minus 5 Pascal right. Now, here we start with that sphere or cylinder with this having the same diameter and height. So, since the length of the cylinder L is equal to the diameter of the cylinder, that is  $D_c$ , the volume of the cylinder,  $V_c$  can be written as pi by 4  $D_c$  square



So, that must be the same as the volume of the particle, which is pi by 6  $D_p$  cube, right? Here, what did you do? We have taken the area of the cylinder multiplied by the diameter of the cylinder, or length, whatever we call it. So, pi by 4  $D_c$  square multiplied by  $D_c$ . of the cylinder, it is pi by 4  $D_c$  cube, whereas, if it would have been  $D_p$ , then it would have been pi by 6  $D_p$  cube. So, the relation between  $D_c$  and  $D_p$  is like this: the relation between  $D_c$  and  $D_p$  is like this,  $D_p$  is under the cube root of 3 by 2  $D_c$ , right? Therefore, we can say phis is 6  $V_p$  by  $D_p$  S<sub>p</sub>, right? So, the relation between  $D_p$  and  $D_c$  we have found out to be

Dp is equal to under the cube root of 3 by 2 D<sub>c</sub>. So, phi<sub>s</sub> is 6 V<sub>p</sub> by D<sub>p</sub> S<sub>p</sub>. So, that is equal to 6 multiplied by pi by 4 D<sub>c</sub> cube over D<sub>p</sub>, which is under the cube root of 3 by 2 D<sub>c</sub>. multiplied by 3 by 2 into pi D<sub>c</sub> square, which is 0.874, right? It is 0.874. So, that means, the N<sub>Re</sub>, that is

The Reynolds number can be said to be  $phi_s D_p G$  prime by mu into 1 minus epsilon is equal to 0.874 into 0.0127 into 3 over 1.5 into 10 to the power minus 5 into 1 minus 0.45. It is equal to 4036, which is a very high Reynolds number, right. So, to use Ergun's equation, we need to calculate the average value of density between the inlet and outlet. The outlet pressure also depends on delta P. Then, if we assume The first approximation, the density is equal to that at the inlet and outlet, right.



So, if the density is equal and equal to the inlet pressure, then we can write rho is equal to P M by RT because P is given as 0.5. into 101325, M is 29 divided by 8314 into 390. So, this is equal to 2.266 kg per meter cube, right. So, from Ergun's equation, we can write that delta P rho by Gg prime square into  $phi_s D_p$  by delta L into epsilon cube by 1 minus epsilon is equal to 150 by  $N_{Re}$  plus 1.75.

Assuming, as a first approximation, the density is equal to that at the inlet pressure, i.e., .....  $\rho = \frac{pM}{RT} = \frac{2.5X101325X29}{8314X390} = 2.266 \text{ kg} / \text{m}^3$ :. from Ergun's eq. we can write,  $\frac{\Delta p \rho}{\left(G'\right)^2} \frac{\phi_s D_p}{\Delta L} \frac{\varepsilon^3}{\left(1 - \varepsilon\right)} = \frac{150}{N_{\text{Re}}} + 1.75$ or,  $\Delta p = \frac{(G')^2}{\rho} \frac{\Delta L}{\phi_e D_p} \frac{(1-\varepsilon)}{\varepsilon^3} \left[ \frac{150}{N_{\text{Re}}} + 1.75 \right]$  $\frac{3^2}{2.266} X \frac{4}{0.874 X 0.0127} X \frac{(1-0.45)}{0.45^3} \bigg[ \frac{150}{4036} + 1.75 \bigg]$ =15.439 kPa

Therefore, we can write delta P is equal to G prime whole square by rho into delta L by  $phi_s D_p$  into 1 minus epsilon by epsilon cube into 150 by  $N_{Re}$  plus 1.75. So, this means this is equal to 3 square by 2.266 into 4 by 0.874 into 0.0127 into 1 minus 0.05, 0.045 by 0.045 cube. by 4036 plus 1.75, that is, 15.439 kilo Pascal, right. So, that is the delta P we got.

Assuming, as a first approximation, the density is equal to that at the inlet pressure, i.e., .....  $\rho = \frac{pM}{p} = \frac{2.5X101325X29}{2.266 \text{ kg}/m^3} = 2.266 \text{ kg}/m^3$ RT 8314X390 :. from Ergun's eq. we can write,  $\frac{\Delta p\rho}{\left(G'\right)^2}\frac{\phi_s D_p}{\Delta L}\frac{\varepsilon^3}{\left(1-\varepsilon\right)} = \frac{150}{N_{\rm Re}} + 1.75$ or,  $\Delta p = \frac{(G')^2}{\rho} \frac{\Delta L}{\phi_s D_\rho} \frac{(1-\varepsilon)}{\varepsilon^3} \left[ \frac{150}{N_{\text{Re}}} + 1.75 \right]$  $\frac{3^2}{2.266} X \frac{4}{0.874 X 0.0127} X \frac{(1\!-\!0.45)}{0.45^3} \bigg[ \frac{150}{4036} \!+\! 1.75 \bigg]$ =15.439 kPa

Next, we have to check whether it is true or false. Since P inlet was 2.5 into 1000, I mean 101325, is 253.3125 kilo Pascal. P outlet is 253.3125 plus this delta P is 0.15439, that is equal to 268.7515 kilo Pascal, and Paverage is 261.032 kilo Pascal. So, rhoaverage we can write Paverage m by RT is 261032 into 29 by 8314 into 390, is equal to 2.334 kg per meter cube. And delta P is in the same process 14.589 kPa.

That means, this value of density is within 5 percent less than the previous one. After repeating the process, we get delta P is 15.069 kilo Pascal, right. So, this is the way how we can find out the pressure drop and how we can use the Ergun's equation, ok. So, thank you very much.



Thank you.