

IMPACT OF FLOW OF FLUIDS IN FOOD PROCESSING AND PRESERVATION

Lecture38

LECTURE 38 : PROBLEMS AND SOLUTIONS OF PACKED BED

Good afternoon my dear boys and girls and students and friends right. We have done based on the laminar and turbulent flow in pipes, the very famous equation that equation through equation for fluid flow in pipe right and the unique famous equation that is called your what that is for the pipe flow Hagen Poiseuille's equation right. Hagen Poiseuille's equation and subsequently the equation for flow subsequently for equation



turbulent liquid fluid and with this many people have worked like Blake and Plummer or Blake and Cozzini and and and Berger and Plummer. They have made the use of both Hagen-Poiseuille equation and the equation which is for the turbulent flow right. And with these two equations another scientist called Ergun who rearranged these two equations And then found out a new equation that is called Ergun's equation, very famous in pipe flow or sorry in packed bed or packed bed or your flow through porous media.

Adding eq. (1) and (2), we get,

$$\Delta p = \frac{150(G')^2 \frac{\Delta L}{\phi_s D_p} \frac{(1-\varepsilon)}{\varepsilon^3} \frac{1}{\rho}}{(1-\varepsilon)\mu} + 1.75(G')^2 \frac{\Delta L}{\phi_s D_p} \frac{(1-\varepsilon)}{\varepsilon^3} \frac{1}{\rho}$$

$$\text{or, } \frac{\Delta p \rho}{(G')^2} \frac{\phi_s D_p}{\Delta L} \frac{\varepsilon^3}{(1-\varepsilon)} = \frac{150}{N_{Re}} + 1.75$$

This eq. is known as Ergun's equation for flow through packed bed.

Burke-Plummer eq. Can be written as

$$\Delta p = 1.75 \rho \frac{\Delta L}{\phi_s D_p} \frac{v'^2 (1-\varepsilon)}{\varepsilon^3}$$

$$= 1.75 \rho \frac{\Delta L}{\phi_s D_p} \frac{v'^2 (1-\varepsilon)}{\varepsilon^3} \frac{\rho}{\rho}$$

$$= 1.75 (v' \rho)^2 \frac{\Delta L}{\phi_s D_p} \frac{(1-\varepsilon)}{\varepsilon^3} \frac{1}{\rho}$$

$$= 1.75 (G')^2 \frac{\Delta L}{\phi_s D_p} \frac{(1-\varepsilon)}{\varepsilon^3} \frac{1}{\rho} \dots (2)$$

And the equation they predicted is like this. that delta P rho by G prime square into phi_s D_p by delta L into phi into epsilon by 1 minus epsilon that is equal to 150 by N_{Re} plus 1.75. Now, the limitation of Blake-Kozeny and Barker Plummer equation was that, none of them were able to tell in all types of Reynolds number. They were that is Blake and Kozeny equation was valid for

N_{Re} less than 10 and Barker Plummer equation was valid for N_{Re} greater than 1000. So, to fill up this void of N_{Re} application this Ergun's equation came forward and really it can look into both less than 10 above 1000 and in between right. So, if it is in between then the whole equation as we are able to see here that delta P by G (G') prime square delta P rho by G prime into phi_s D_p by delta L into epsilon (ε) cube by 1 minus epsilon is equal to 150 by N_{Re} plus 1.75.

This whole equation is very good to predict at any N_{Re} between 10 or even 1000. make it valid for less than 10 or greater than 1000 that is Blake and Kozeny and Barker Plummer equation. What if proposed that you see if on the left side yes it is a number because all are

known Δp ρ G prime $\phi_s D_p$ ΔL everything is known. ϵ unknown. So, if N_{Re} is known and if it is less than 10, then it becomes 150 by 10 that is 15.

Now, 15 is much more than 1.75, at least more than 10 times. So, he said that the prediction of ΔP or any other like velocity etc. can be made easily between 10 to 1000 with the whole equation and if N_{Re} is 10 or less than 10 then 150 by N_{Re} is much more than 1.75. So, neglect 1.75, then whatever is getting predicted is good for that and it has been found to be so.

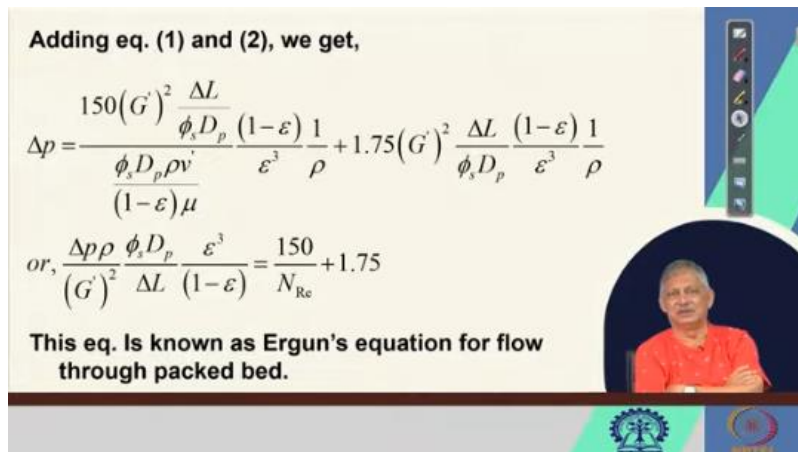
That is why this Ergun's equation is so popular in packed bed and a packed bed and porous media. Whereas, if N_{Re} is greater than 1000, then it is 150 by 1000 right side, 150 by 1000. So, 150 by 1000 means 0.15. So, 0.15 is much less than 1.75. So, predicted that you neglect this 150 by N_{Re} when N_{Re} is greater than 1000, take 1.75 and find out ΔP , or any other parameter, like velocity etc., right.

Adding eq. (1) and (2), we get,

$$\Delta p = \frac{150(G')^2 \frac{\Delta L}{\phi_s D_p} \frac{(1-\epsilon)}{\epsilon^3} \frac{1}{\rho}}{\frac{\phi_s D_p \rho v}{(1-\epsilon)\mu}} + 1.75(G')^2 \frac{\Delta L}{\phi_s D_p} \frac{(1-\epsilon)}{\epsilon^3} \frac{1}{\rho}$$

$$\text{or, } \frac{\Delta p \rho}{(G')^2} \frac{\phi_s D_p}{\Delta L} \frac{\epsilon^3}{(1-\epsilon)} = \frac{150}{N_{Re}} + 1.75$$

This eq. is known as Ergun's equation for flow through packed bed.



So, this is how the goodness of the Ergun's equation is that. it encompasses N_{Re} , both less than 10 and greater than 1000 as well as in between. So, it takes care of both Black Kozeny, Barker Plummer and normal their gap all are filled up by this Ergun's equation. For example, if we do a problem. That, air at 303 Kelvin is flowing through a packed bed of spheres having diameter of 12 millimeter.

Air at 303 K is flowing through a packed bed of spheres having a Diameter of 12 mm. The void fraction of the bed is 0.4 and the bed has a diameter of 0.6 m and a height of 2.5 m. The air enters the bed at 1.2 atm at the rate of 0.4 Kg/s. Calculate the pressure Drop of air in the packed bed. Given, $M_w = 28.97 \text{ kg / kg Mole}$. $\mu = 2 \times 10^{-5} \text{ Pa.s}$.

Solution:

$$\text{Cross Sectional Area of Bed} = (\pi/4)(0.6)^2 = 0.283 \text{ m}^2$$

$$G' = 0.4 / 0.283 = 1.413 \text{ Kg/m}^2\text{-s (Based on empty cross section of bed)}$$



The void fraction of the bed is 0.2 and the bed has a diameter of 0.6 meter and a height of 2.5 meter. The air enters the bed at 1.2 atmosphere at the rate of 0.2 kg per second, calculate the pressure drop of air in the packed bed given molecular weight of air as 28.97 kg per kg mole. and μ as 2×10^{-5} Pascal second right. So, what we have to find out? Calculate the pressure drop ΔP of air in the packed bed right. So, as usual we repeat the problem so that understanding is better.

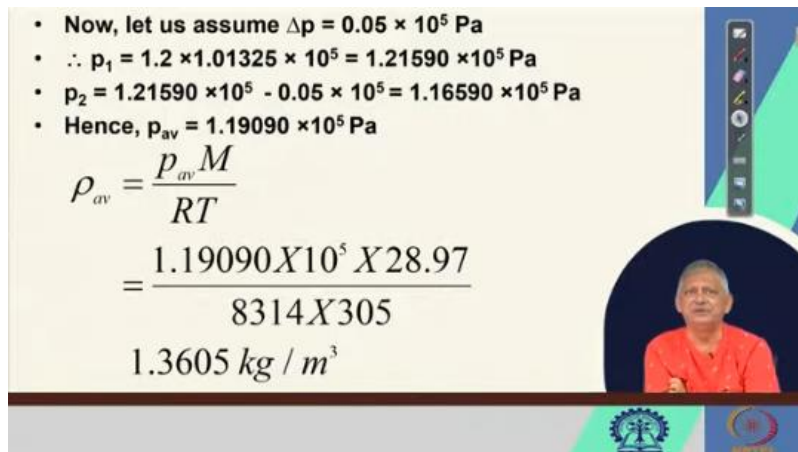
The problem says that air at 303 Kelvin is flowing through a packed bed of spheres having a diameter of 12 millimeter. The void fraction of the bed, I don't know how many of you have played with during your childhood with marble. And I hope you have seen that those marbles, if you put in a glass, that all the marbles are not getting filled up within the glass. There are certain spaces, because of the spherical nature of the marble, it is making some void, right?

So, here also the same thing. So, your the packed bed is full of spheres having diameter of 12 millimeter. Void fraction of the bed is 0.4 and the bed has a diameter of 0.6 meter and the height of 2.5 meter. The air enters the bed at 1.2 atmosphere at the rate of 0.4 kg per second. Calculate the pressure drop of air in the packed bed given

M_w is equal to 28.97 kg per kg mole and viscosity is μ , equals to 2×10^{-5} Pascal second right. So, if you solve it, first what do we do? sectional area of the bed, let us find out. So, cross sectional area of the bed is π by 4 into D square that is 0.5 by 4 into 0.6 whole square. You see, it was said 0.12 millimeter diameter right. So, we have taken π by 4 R square that means, 0.6 meter square into π by 4 is 0.283 meter square. G' prime that is the mass velocity, or fraction or the mass flux, right. G' prime is 0.4 over this sectional area that is 0.283. So, it is 1.413 kg per meter square per second, again based

on empty cross section of bed. Yes, if it is not based on empty cross section of the bed, then you would not be able to take that ok.

The sectional area and 0.4 void fraction, right. So, because of that you have taken that based on the empty cross section is 1.413 kg per meter square second. Now, let us assume delta P is equal to 0.05 into 10 to the power 5 Pascal. It is the first assumption.



- Now, let us assume $\Delta p = 0.05 \times 10^5 \text{ Pa}$
- $\therefore p_1 = 1.2 \times 1.01325 \times 10^5 = 1.21590 \times 10^5 \text{ Pa}$
- $p_2 = 1.21590 \times 10^5 - 0.05 \times 10^5 = 1.16590 \times 10^5 \text{ Pa}$
- Hence, $p_{av} = 1.19090 \times 10^5 \text{ Pa}$

$$\rho_{av} = \frac{p_{av} M}{RT}$$

$$= \frac{1.19090 \times 10^5 \times 28.97}{8314 \times 305}$$

$$1.3605 \text{ kg / m}^3$$


Therefore, P_1 is equal to 1.2 into 1.01325 into 10 to the power 5 is equal to 1.21590 into 10 to the power 5 Pascal. Similarly, P_2 , we have taken delta P as 0.05 into 10 to the power 5. Similarly, P_2 can be written as 1.21590 into 10 to the power 5 which is already given minus 0.05 into 10 to the power 5 that is delta P. So, this is equal to 1.6 into 10 to the power 5 Pascal. Therefore, ρ_{average} can be said to be equal to 1.19090 into 10 to the power 5, sorry p and not rho p_{average} of this and that is 1.19090 10 to the power 5 Pascal.

So, ρ_{average} can be written as P_{average} into M by RT that is equal to 1.19090 into 10 to the power 3 into 28.97 divided by 8314 into 305. This is equal to 1.3605 kg per meter cube, what we have found out only ρ_{average} right. How did we find out? ρ_{average} we have said to be equals to P_{average} into M by RT, P_{average} already we have done earlier. and we have put that p_{average} and molecular weight we also do not know.

- Now, let us assume $\Delta p = 0.05 \times 10^5 \text{ Pa}$
- $\therefore p_1 = 1.2 \times 1.01325 \times 10^5 = 1.21590 \times 10^5 \text{ Pa}$
- $p_2 = 1.21590 \times 10^5 - 0.05 \times 10^5 = 1.16590 \times 10^5 \text{ Pa}$
- Hence, $p_{av} = 1.19090 \times 10^5 \text{ Pa}$

$$\rho_{av} = \frac{p_{av} M}{RT}$$

$$= \frac{1.19090 \times 10^5 \times 28.97}{8314 \times 305}$$

$$1.3605 \text{ kg / m}^3$$


So, take 28.975 over 8314 into 305, 8314 into 305 that means, it is 1.3605 kg per meter cube. If it is so, then we find out N_{Re} . So, N_{Re} , from the definition of N_{Re} , that is $D G$ into rather D_p into G prime over μ into 1 minus ϵ is the N_{Re} that is 0.012 into 1.413 divided by 1 minus 0.4 into 2 into 10 to the power 5 . So, that is equal to 1413 right. So, it is much more than the Barker Plummer equation which is 1000 , 1413 .

$$N_{Re} = D_p G' / \mu(1 - \epsilon)$$

$$= 0.012 \times 1.413 / (1 - 0.4) \times 2 \times 10^{-5} = 1413$$


Using Ergun's Equation

$$\frac{\Delta p \rho D_p \epsilon^3}{(G')^2 \Delta L (1 - \epsilon)} = \frac{150}{N_{Re}} + 1.75$$

$$\frac{\Delta p \times 1.3605}{(1.413)^2} \times \frac{0.012}{2.5} \times \frac{(0.4)^3}{(1 - 0.4)} = \frac{150}{1413} + 1.75$$

or, $\Delta p = 5320.23 \text{ Pa} = 0.0532023 \times 10^5 \text{ Pa}$

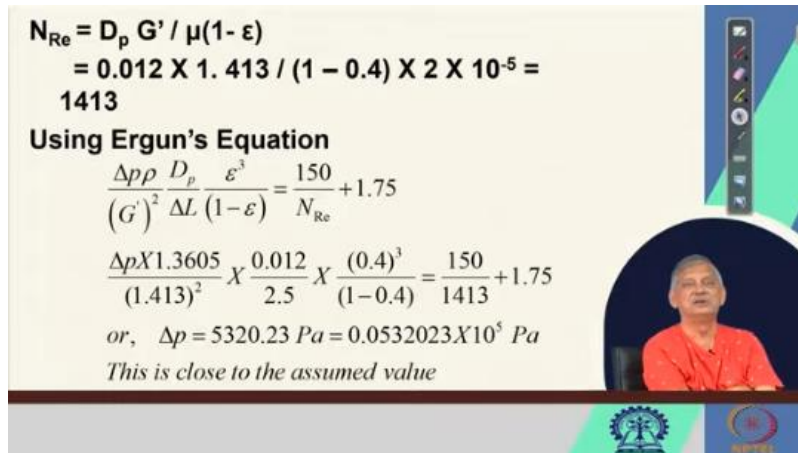
This is close to the assumed value



So, therefore, Ergun's equation can be applied. What is that? It is saying that ΔP into ρ by G prime square into D_p by ΔL into ϵ cube divided by 1 minus ϵ is equal to 150 by N_{Re} plus 1.75 .

I repeat using Ergun's equation from the given N_{Re} , we find out ΔP over G prime square into D_p over dL into ϵ cube by 1 minus ϵ is equal to 150 by N_{Re} plus 1.75 . So, we found out that ΔP into 1.305 into by 1.413 or 413 . divided by 1.0504 into 2 into 10 that is equal to 1413 . Now, using Ergun's equation we can write ΔP ρ by g prime square.

is D_p over ΔL times ϵ^3 by $1 - \epsilon$ is equal to 150 by N_{Re} plus 1.75 . Now, ΔP into 1.3605 divided by 1413 whole square. plus 0.012 divided by 2.5 into 0.4 cube divided by 0.4 that is equal to 150 by 1413 plus 1.73 right. So, 150 by 1413 plus 1.75 is ok. Then we calculate this and find out what is the ΔP and that comes out to be 5320.23 Pascal.



$$N_{Re} = D_p G' / \mu(1 - \epsilon)$$

$$= 0.012 \times 1.413 / (1 - 0.4) \times 2 \times 10^{-5} = 1413$$

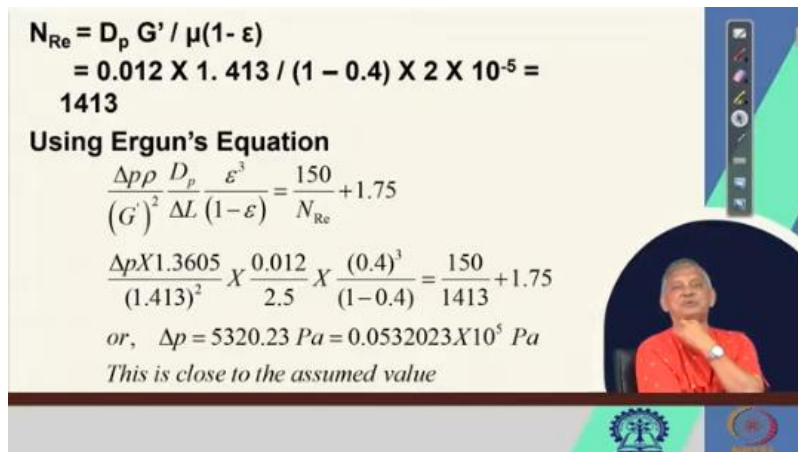
Using Ergun's Equation

$$\frac{\Delta p \rho D_p}{(G')^2 \Delta L (1 - \epsilon)} = \frac{150}{N_{Re}} + 1.75$$

$$\frac{\Delta p \times 1.3605}{(1.413)^2} \times \frac{0.012}{2.5} \times \frac{(0.4)^3}{(1 - 0.4)} = \frac{150}{1413} + 1.75$$

or, $\Delta p = 5320.23 \text{ Pa} = 0.0532023 \times 10^5 \text{ Pa}$

This is close to the assumed value



$$N_{Re} = D_p G' / \mu(1 - \epsilon)$$

$$= 0.012 \times 1.413 / (1 - 0.4) \times 2 \times 10^{-5} = 1413$$

Using Ergun's Equation

$$\frac{\Delta p \rho D_p}{(G')^2 \Delta L (1 - \epsilon)} = \frac{150}{N_{Re}} + 1.75$$

$$\frac{\Delta p \times 1.3605}{(1.413)^2} \times \frac{0.012}{2.5} \times \frac{(0.4)^3}{(1 - 0.4)} = \frac{150}{1413} + 1.75$$

or, $\Delta p = 5320.23 \text{ Pa} = 0.0532023 \times 10^5 \text{ Pa}$

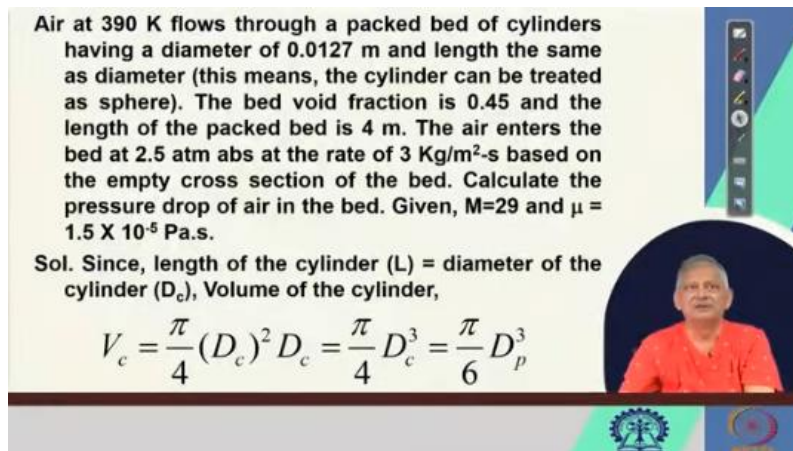
This is close to the assumed value

is ΔP . Therefore, P_A is 0.0532 into 10 to the power 5 Pascal. This value of ΔP that is 0.0532023 into 10 to the power 5 is very very very closer to the assumed value of 0.05 . You see ΔP you have assumed 0.05 into 10 to the power 5 . 0.05 into 10 to the power 5 Pascal.

Now, we are saying that it has also come to 0.0532023 into 10 to the power 5 Pascal and this is very close to the assumed value right. Now, this ΔP is 5320.23 Pascal. So, we have seen that the this calculated value of ΔP and the assumed value which was also 0.05 is very close to each other. That the Ergun's equation is leading to whatever be the N_{Re} is very close to the real value that should be said very close to the real value.

So, it gets established that ΔP predicted and ΔP actual they are very very close to each other. That means, whatever assumptions they had taken are all definitely correct because otherwise predicting ΔP and in reality ΔP are not the same. ok.

So, what should we say now? We should say that yeah for packed bed Ergun's equation can be very well utilized and prediction of ΔP pressure drop. It is not only pressure drop subsequently we will also see that this can be used for finding out the velocity, not only ΔP . If you know ΔP , then you can find out velocity. Like the behaviour of the flow of the fluid for which we are maintaining, rather for which we are very very helpful for the prediction to be very close to the actual. And Ergun's equation gets established in the scientific community so that in future this equation application is not in any problem ok. Then we will go to because this problem we cannot solve it now because our time is not there. So, one problem we have solved.



Air at 390 K flows through a packed bed of cylinders having a diameter of 0.0127 m and length the same as diameter (this means, the cylinder can be treated as sphere). The bed void fraction is 0.45 and the length of the packed bed is 4 m. The air enters the bed at 2.5 atm abs at the rate of 3 Kg/m²-s based on the empty cross section of the bed. Calculate the pressure drop of air in the bed. Given, $M=29$ and $\mu = 1.5 \times 10^{-5}$ Pa.s.

Sol. Since, length of the cylinder (L) = diameter of the cylinder (D_c), Volume of the cylinder,

$$V_c = \frac{\pi}{4} (D_c)^2 D_c = \frac{\pi}{4} D_c^3 = \frac{\pi}{6} D_p^3$$

Another problem and subsequent things we will be observing in the coming class and I wish you that you please derive from your side from the right from the this equation that is our Hagen-Poiseuille's equation from there and using the fanning friction factor pressure drop equation, these two separately for the two cases. Let us see what happens. So, ΔP predicted, ΔP actual they are absolutely enclosed. So, I wish I can I can complete it here because the time is also not much available.

So, if you take any new problem may not be able to complete it. So, may not be able to end it. So, let us say thank you and let us say goodbye.

Thank you.